Rhino Bonus: Derivatives of Exponential Functions Using the Limit Definition

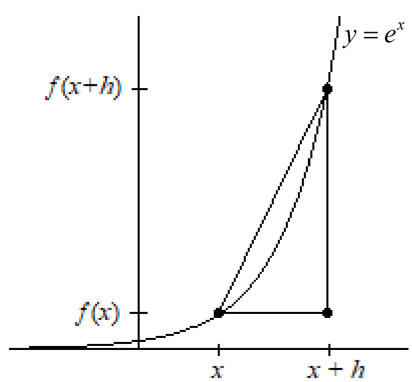
Each of these limits is $f\,'$ for some function f of h and some value a.
Report f. Report a. Report the value of $f\,'(a)$.
Use the activity at https://www.geogebra.org/calculator/vwb5rmqd

$$\lim_{h\to 0}\frac{e^h-1}{h} = \_\_\_\_ . \quad f(h) = \_\_\_\_\_\_;\ a = \_\_\_$$

$$\lim_{h\to 0}\frac{e^{kh}-1}{h} = \_\_\_\_ . \quad f(h) = \_\_\_\_\_\_;\ a = \_\_\_$$

Complete the remaining boxes and blanks below and on the reverse for +1 Rhino Participation Bonus.

1. Use the limit definition to show the derivative of $y = e^x$ is itself.



$f(x) = \square \qquad f(x+h) = \square$

$$\frac{d}{dx}e^x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{(x+h)-x} = \lim_{h\to 0}\square$$

$= \lim_{h\to 0}\frac{1}{h}\cdot\left(e^x\cdot\square - e^x\right)$ by the law of exponents.

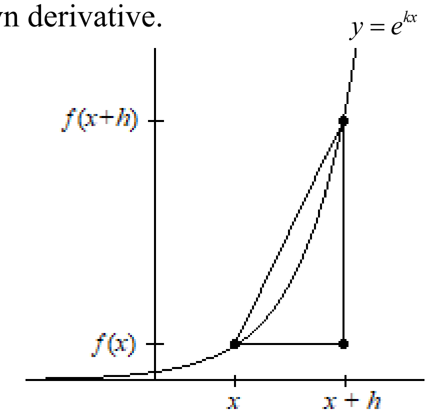
$= \lim_{h\to 0}\frac{1}{h}\cdot\left(e^x\left(\square\right)\right)$ by factoring out e^x.

$= \lim_{h\to 0} e^x \cdot \lim_{h\to 0}\frac{\square}{h}$ by writing as a product of limits.

$= \square \cdot \square$ Why? ____

(OVER PLEASE)

2. Use the limit definition to show the derivative of $y = e^{kx}$ is proportional to its own derivative. Then report the constant of proportionality below.



$f(x) = \boxed{\qquad\qquad}$ $\quad f(x+h) = \boxed{\qquad\qquad\qquad\qquad}$

$$\frac{d}{dx} e^{kx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \lim_{h \to 0} \boxed{\qquad\qquad\qquad\qquad\qquad}$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \left(e^{kx + \boxed{\quad}} - e^{kx} \right)$$ Distribute the $k(x+h)$.

$$= \lim_{h \to 0} \frac{1}{h} \cdot \left(e^{kx} \cdot \boxed{\qquad} - e^{kx} \right)$$ by the law of exponents.

$$= \lim_{h \to 0} \frac{1}{h} \cdot \left(e^{kx} \left(\boxed{\qquad\qquad} \right) \right)$$ by factoring out e^{kx}.

$$= \lim_{h \to 0} e^{kx} \cdot \lim_{h \to 0} \frac{\boxed{\qquad\qquad}}{h}$$ by writing as a product of limits.

$$= \boxed{\qquad} \cdot \boxed{\qquad\qquad}$$ Why? ____

The derivative of $y = e^{kx}$ is proportional to its own derivative with constant of proportionality equal to _______, i.e.

if $y = e^{kx}$, $\qquad y' = \boxed{\qquad\qquad\qquad}$