Name ______________________________

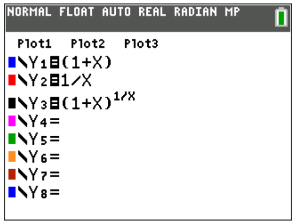
Due Tuesday, February 25

Rhino Bonus: The Derivative of the Natural Logarithm Function $y = \ln(x)$

Complete the steps to show why $\frac{d}{dx}\ln x = \frac{1}{x}$ to earn +1 Rhino Participation Bonus!

1. Fun Fact: The value of $\lim_{Q\to 0}(1+Q)^{\frac{1}{Q}}$ is a famous number. What is the **exact** value of this limit? Fill it in the box below.

   You can explore this limit with a graphing calculator as shown below.



NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1	Y2	Y3	*Exact Value*
0.1	1.1	10	2.5937	$(1.1)^{10}$
0.01	1.01	100	2.7048	$(1.01)^{100}$
0.001	1.001	1000	2.7169	$(1.001)^{1000}$
0.001	1.0001	10000	2.7181	$(1.0001)^{10000}$
0.0001	1.00001	100000	2.7183	$(1.00001)^{100000}$

   Complete: $(1.00000000000000000000000000000000000000000000000000000...0001)^{10000000000000000000000000000000000000000000000000000000000000...000} = \lim_{Q\to 0}(1+Q)^{\frac{1}{Q}} =$ ☐

2. Recall the following properties of logarithms.

   a. Sum Property: $\ln A + \ln B =$ ☐

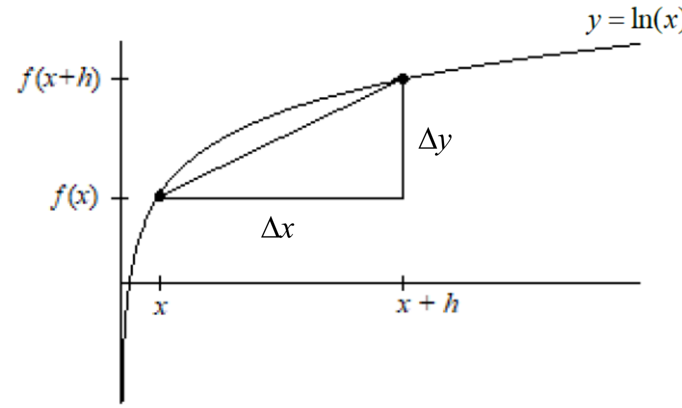
   b. Difference Property: $\ln A - \ln B =$ ☐

   c. Power Property: $k \cdot \ln A =$ ☐ (Bob Barker Property in Reverse)

   d. Can the expression ln (A + B) be simplified? Circle one: YES NO

   If yes, please simplify it below. If not, please leave as is.

(OVER PLEASE)



3. Write in terms of the natural logarithm function and x and h.

$$\frac{d}{dx}\ln x = \lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x} = \lim_{h\to 0}\square$$

4. Rewrite your expression in the box in #3 using the **Difference Property**.

$$= \lim_{h\to 0}\frac{\square}{h}$$

5. Use the property that $\frac{*}{h} = \frac{1}{h}\cdot *$. Follow the remaining steps. Arrows indicate you recopy the previous box.

$$= \lim_{h\to 0}\frac{1}{h}\cdot\ln\left(1+\square\right)$$ Divide your expression in the box in #4 by x.

$$= \lim_{h\to 0}\frac{1}{x}\cdot\square\cdot\ln\left(1+\square\right)$$ Rewrite $\frac{1}{h}$ using "1 in"

$$= \lim_{h\to 0}\frac{1}{x}\cdot\lim_{h\to 0}\square\cdot\ln\left(1+\square\right)$$ Write as a product of limits.

$$= \lim_{h\to 0}\frac{1}{x}\cdot\lim_{h\to 0}\ln\square$$ Use Property in 2c.

6. Let $Q = \frac{h}{x}$. As $h\to 0$, what happens to Q? $Q\to\square$

7. Write the limit in the last line of #5 all in terms of Q. The content of the pink boxes is the same.

$$\frac{d}{dx}\ln x = \lim_{h\to 0}\frac{1}{x}\cdot\lim_{Q\to\square}\ln\left(\square\right)^{\square} = \lim_{h\to 0}\frac{1}{x}\cdot\ln\left(\lim_{Q\to\square}\left(\square\right)^{\square}\right) = \lim_{h\to 0}\frac{1}{x}\cdot\ln\square$$ Use the Fun Fact in #1.

$$= \square$$