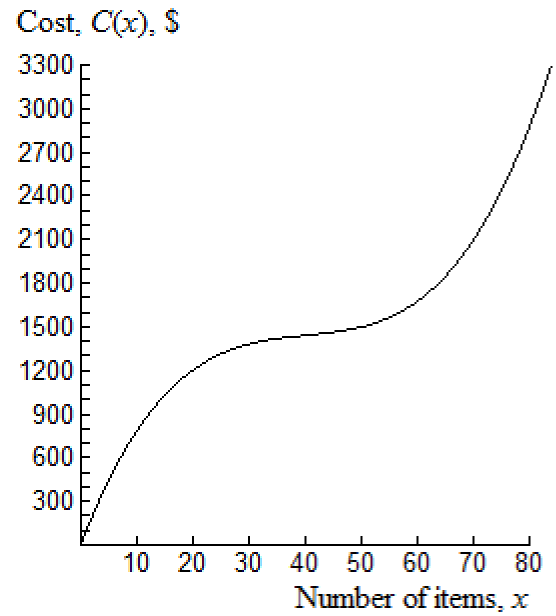
4.6 *Marginal Analysis, the MVT, and Linear Approximation*



Some cost functions increase quickly at first and then more slowly because producing larger quantities usually is more efficient (due to economy of scale).

At still higher production levels, the cost function increases faster again as resources become scarce (or new factories must be built).

Such a cost function may look like the one to the right.

1. The cost to produce an additional item is the *marginal cost.* For simplicity, economists define the marginal cost as the derivative $C'(x)$.

Consider the production levels at $x = 10$ and $x = 60$.
For which production level, $x = 10$ and $x = 60$,
is the marginal cost greater? The marginal cost is greater at $x =$ ______ items.

2. Interpret your answer to the previous question in practical terms. Select one.

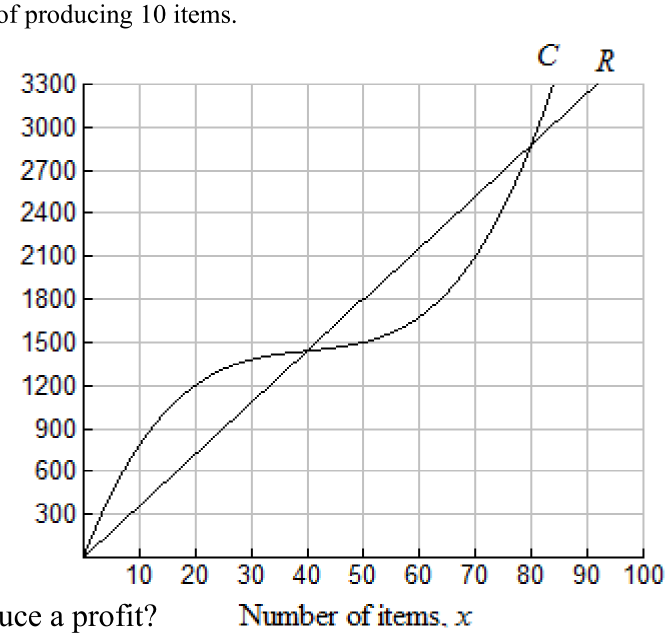
A. The cost to go from producing 10 items to 11 items is greater than the cost to go from producing 60 to 61 items.
B. The cost to go from producing 60 items to 61 items is greater than the cost to go from producing 10 to 11 items.
C. The cost of producing 60 items is greater than the cost of producing 10 items.

3. The graph of R(x) and C(x) is shown.
The cost function is

$C(x) = 0.02x^3 - 2.4x^2 + 100x$

If items sell for \$36, write the revenue function

$R(x) =$ [ ]



4. Use the graph to complete the blanks.
Once production begins, the company will first break even when they make _____ items and then again when they make ______ items.

5. For what production values will the company produce a profit?

6. The company will choose to produce more items when marginal revenue exceeds marginal costs. Why?

Report: $R'(x) =$ [ ]

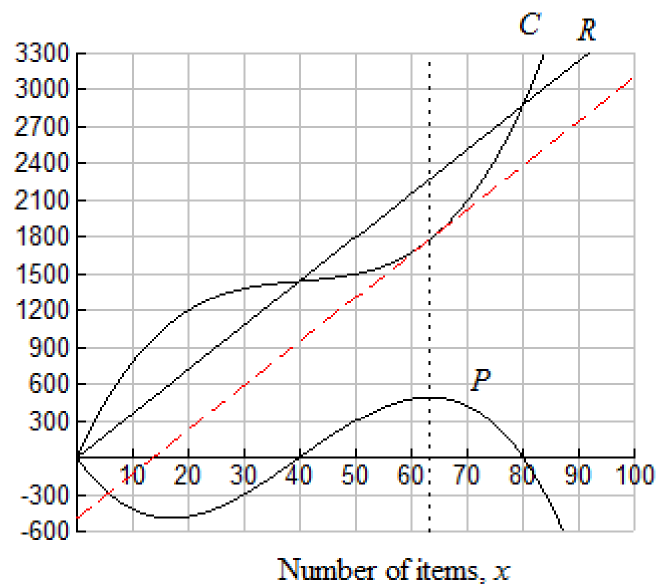
7. Consider production levels at $x = 50$ and $x = 70$. Use the graph to answer the following:

a. Does the marginal revenue exceed marginal cost if they produce x = 50 items? A. Yes B. No
b. Does the marginal revenue exceed marginal cost if they produce x = 70 items? A. Yes B. No

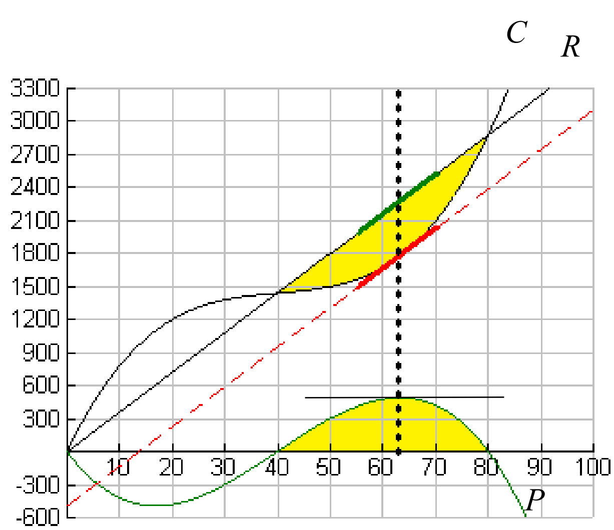
8. The curve $C(x) = 0.02x^3 - 2.4x^2 + 100x$, the line $R(x) = 36x$, the profit function $P(x)$, and the tangent line to $C(x)$ at the value of $x \approx 63$ are sketched below.

a. No calculations are necessary to answer these. At a production level of $x \approx 63$ where P is maximum,
what is the exact slope of the tangent line to $P(x)$? $P'(63) =$ _______
what is the exact slope of the tangent line to $R(x)$? $R'(63) =$ _______
what is the exact slope of the tangent line to $C(x)$? $C'(63) =$ _______

b. Explain the relationship between the marginal profit $P'(63)$, the marginal revenue $R'(63)$, and the marginal cost $C'(63)$ at the production level $x \approx 63$ when the profit is maximum. Sketch tangent lines to each at $x \approx 63$.



A man named Archimedes looked at shaded areas like these and wondered, "Are these shaded areas the same?"
What do you think?



Although it is not necessary to answer the questions in this activity, we have by subtraction:

$$P(x) = R(x) - C(x) = 36x - (0.02x^3 - 2.4x^2 + 100x)$$
$$= -0.02x^3 + 2.4x^2 - 64x$$
$$= -0.02x(x-40)(x-80)$$