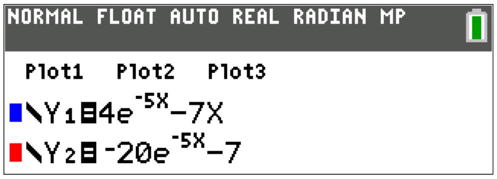
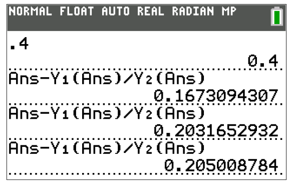
KEY to QUIZ 11 (HW 28-30)

1. **a.** Since $f'(x)=4e^{-5x}-7x$ we have $f'(x)=-20e^{-5x}-7$ so $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}=\boxed{x_n-\dfrac{4e^{-5x_n}-7x_n}{-20e^{-5x_n}-7}}$



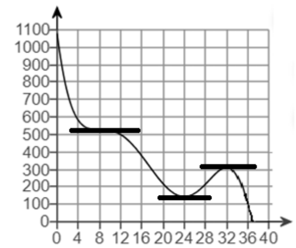


$x_0=0.4$

$x_1\approx 0.1673$

$x_2\approx 0.203$

$x_3\approx 0.205$

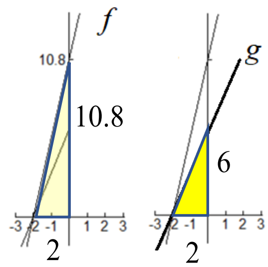


2. **a.** $x = 8, 24, 32$.

b. There are horizontal tangents at $x = 8, 24, 32$ so the tangent line approximating $f(x)$ will never touch the x-axis to produce the value for the next iteration. You could also just point out that at these values $f'(x)=0$.

3. Using slope triangles,

a. $f'(x)=\dfrac{RISE}{RUN}=\dfrac{10.8}{2}=\boxed{5.4}$ **b.** $g'(x)=\dfrac{RISE}{RUN}=\dfrac{6}{2}=\boxed{3}$



c. $\lim\limits_{x\to-2}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}$ so $\lim\limits_{x\to-2}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to-2}\dfrac{f'(x)}{g'(x)}=\dfrac{5.4}{3}=\boxed{1.8}$

d. $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}=\dfrac{\infty}{\infty}$ so $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to\infty}\dfrac{f'(x)}{g'(x)}=\dfrac{5.4}{3}=\boxed{1.8}$

You could verify this using formulas since $f(x)=5.4(x+2)$ and $g(x)=3(x+2)$ so $\dfrac{f(x)}{g(x)}=\dfrac{5.4(x+2)}{3(x+2)}$.

4. **a.** The exponential function $f(x)=e^{0.0001x}$ will eventually outpace the power function $g(x)=x^{10000}$.

We can apply L'Hôpital's Rule multiple times to show $\lim\limits_{x\to\infty}\dfrac{e^{0.0001x}}{x^{10000}}=\infty$

b. The function $f(x)=e^{0.0001x}$ will be eventually be outpaced by $h(x)=x^{0.0001x}$ for $x > e$ since powers are the same.

5. **a.** We have the form 0/0 so we can apply L'Hôpital's Rule:

$$\lim_{x\to0}\frac{\sin 4x}{5x}\overset{LH}{=}\lim_{x\to0}\frac{4\cos 4x}{5}=\frac{4\cos 0}{5}=\frac{4\cdot 1}{5}=\boxed{\frac{4}{5}}\text{ or }\boxed{0.8}.$$

$$\textbf{OR }\lim_{\theta\to0}\frac{\sin\theta}{\theta}=1\text{ so }\lim_{x\to0}\frac{\sin 4x}{5x}=\lim_{x\to0}\frac{\sin 4x}{5x}\cdot\frac{\frac{4}{5}}{\frac{4}{5}}=\lim_{x\to0}\frac{\sin 4x}{4x}\cdot\frac{\frac{4}{5}}{1}=\boxed{\frac{4}{5}}\text{ or }\boxed{0.8}.$$

b. We have the form ∞/∞ so we can apply L'Hôpital's Rule:

$$\lim_{x\to\infty}\frac{e^{19/x}-1}{\frac{19}{x}}\overset{LH}{=}\lim_{x\to\infty}\frac{e^{19/x}\cdot\frac{d}{dx}\frac{19}{x}}{\frac{d}{dx}\frac{19}{x}}=\lim_{x\to\infty}e^{19/x}=e^{0}=\boxed{1}$$

c. We have the form 0/0 (since $e^0 - 0 - 1 = 1 - 1 = 0$) so we can apply L'Hôpital's Rule.

$$\lim_{x\to0}\frac{e^x-\sin x-1}{7x^2+x^3}\underset{\uparrow\text{LH}}{=}\lim_{x\to0}\frac{e^x-\cos x}{14x+3x^2}$$

$\lim\limits_{x\to0}\dfrac{e^x-\cos x}{14x+3x^2}$ is of the form 0/0 (since $e^0 - \cos 0 = 1 - 1 = 0$) so we can apply L'Hôpital's Rule again.

$\lim\limits_{x\to0}\dfrac{e^x-\cos x}{14x+3x^2}\underset{\uparrow\text{LH}}{=}\lim\limits_{x\to0}\dfrac{e^x+\sin x}{14+6x}$. This is not in indeterminate form so we can't apply L'Hôpital's Rule.

Using direct substitution, $\lim\limits_{x\to0}\dfrac{e^x+\sin x}{14+6x}=\dfrac{e^0+0}{14+0}=\boxed{\dfrac{1}{14}}$

d. We do not have the form 0/0 so we cannot apply L'Hôpital's Rule. But direct substitution works.

$$\lim_{x\to 0} \frac{e^x + 4}{17x + 59} = \frac{e^0 + 4}{0 + 59} = \frac{1+4}{59} = \boxed{\frac{5}{59}}$$

e. Rewrite using the Bob Barker Property since $\ln(1+2x)^{1/x} = \frac{1}{x} \cdot \ln(1+2x) = \frac{\ln(1+2x)}{x}$

$\lim_{x\to 0} \ln(1+2x)^{1/x} = \lim_{x\to 0} \frac{\ln(1+2x)}{x}$ is of the form 0/0 (since ln 1 = 0) so we can apply L'Hôpital's Rule:

$$\lim_{x\to 0} \frac{\ln(1+2x)}{x} \overset{LH}{=} \lim_{x\to 0} \frac{\frac{1}{1+2x} \cdot \frac{d}{dx}(1+2x)}{1} = \lim_{x\to 0} \frac{1}{1+2x} \cdot 2 = \frac{1}{1+0} \cdot 2 = \boxed{2}$$

f. Let $y = (1+2x)^{1/x}$. From part **e**, we have $\lim_{x\to 0} \ln y = 2$ Therefore $\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^{\lim_{x\to 0} \ln y} = \boxed{e^2}$

BONUS

Since $-1 < \sin x < 1$, we have that $\frac{e^x - 1}{7x^2 + x^3} < \frac{e^x - \sin x}{7x^2 + x^3}$.

Since e^x dominates any polynomial, $\lim_{x\to\infty} \frac{e^x - 1}{7x^2 + x^3} = \infty$

Then $\lim_{x\to\infty} \frac{e^x - \sin x}{7x^2 + x^3} = \boxed{\infty}$