Derivatives of Trig Functions

Important Ideas:

The derivative of the sine is $\cos\theta$ and the derivative of the cosine is $-\sin\theta$

The derivative of the tangent is $\sec^2\theta$ and the derivative of the cotangent is $-\csc^2\theta$

The derivative of the secant is $\sec\theta\tan\theta$ and the derivative of the cosecant is $-\csc\theta\cot\theta$

cofunctions of

$\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Tip: Simplify First!

Memorize These

$\frac{1}{\sec\theta} = \cos\theta$

$\frac{1}{\csc\theta} = \sin\theta$

$\frac{1}{\cot\theta} = \tan\theta$

$\frac{1}{\sec t} = \frac{1}{\left(\frac{1}{\cos t}\right)} = \cos t$

Check Your Understanding!

In 1 – 5: find the derivative of each function.

1. $f(\theta) = \csc\theta + \sec\theta$

$= \boxed{-\csc\theta\cot\theta + \sec\theta\tan\theta}$

2. $W(t) = \frac{1}{\sec t} - \frac{2}{3\csc t}$

$= \cos t - \frac{2}{3}\cdot\frac{1}{\csc t}$

$= \cos t - \frac{2}{3}\cdot(\sin t)$

$W'(t) = -\sin t - \frac{2}{3}\cdot(\cos t) = \boxed{-\sin t - \frac{2}{3}\cos t}$

3. $F(y) = \frac{\sin y}{\tan y \cdot \csc y} = \frac{\sin y}{\tan y \cdot \frac{1}{\sin y}}$

$= \frac{\sin y}{\frac{\sin y}{\cos y}\cdot\frac{1}{\sin y}} = \frac{\sin y}{\frac{1}{\cos y}}$

$= \sin y \cos y$

Use Product Rule: $F'(y) = \sin y \frac{d}{dy}\cos y + \cos y \frac{d}{dy}\sin y$

$= \sin y\cdot(-\sin y) + \cos y\cdot(\cos y)$

$= \boxed{-\sin^2 y + \cos^2 y}$ or $= \boxed{\cos^2 y - \sin^2 y}$

4. $P(x) = \cos x \cdot \tan x = \cos x \cdot \frac{\sin x}{\cos x} = \sin x$

$P'(x) = \frac{d}{dx}\sin x = \boxed{\cos x}$

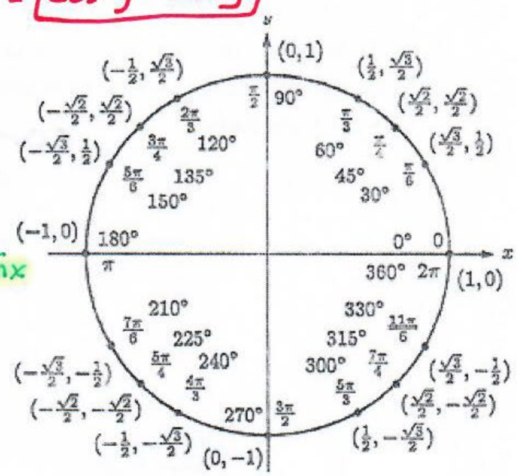
$=$

5. $g(\alpha) = \cot\alpha \cdot \cot\alpha$

Use Product Rule

$g'(\alpha) = \cot\alpha \frac{d}{d\alpha}\cot\alpha + \cot\alpha \frac{d}{d\alpha}\cot\alpha$

$= 2\cdot\cot\alpha\cdot(-\csc^2\alpha) = \boxed{-2\cot\alpha\csc^2\alpha}$



6. Find $H'\left(\frac{\pi}{3}\right)$ when $H(x) = \cos x \cdot \tan x + \frac{\sin x + \tan x}{\sin x}$

Simplify first: $H(x) = \underbrace{\cos x \cdot \frac{\sin x}{\cos x}} + \frac{\sin x + \frac{\sin x}{\cos x}}{\sin x}$

$= \sin x + \frac{\sin x}{\sin x} + \frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}$

"1 out"

$= \sin x + 1 + \frac{1}{\cos x}$

$= \sin x + 1 + \sec x$

$H'(x) = \cos x + 0 + \sec x \tan x$

$H'\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{1}{\cos \pi/3}\frac{\sin \pi/3}{\cos \pi/3}$

$H'(\pi/3) = \cos \pi/3 + \sec \pi/3 \tan \pi/3$

$= \cos \pi/3 + \frac{1}{\cos \pi/3}\frac{\sin \pi/3}{\cos \pi/3} = \frac{1}{2} + \frac{1}{\frac{1}{2}}\cdot\frac{\frac{\sqrt{3}}{2}}{1/2}$

$= \frac{1}{2} + 4\cdot\frac{\sqrt{3}}{2}$

$= \boxed{1/2 + 2\sqrt{3}}$