Check Your Understanding

1. Given that $f(x) = 2x^2(x^3 - 3)$, find $f'(x)$ in two different ways.

① $f'(x) = (x^3 - 3)\cdot \frac{d}{dx}2x^2 + 2x^2 \cdot \frac{d}{dx}(x^3-3)$

$= (x^3-3)\cdot 2\cdot \frac{d}{dx}(x^2) + 2x^2\cdot(3x^2+0)$

$= (x^3-3)2\cdot 2x + 2x^2\cdot(3x^2)$

$= 4x(x^3-3) + 6x^4$

$= 4x^4 - 12x + 6x^4$

$= \boxed{10x^4 - 12x}$

② Simplify first by distributing
"Rewrite"

$f(x) = (2x^2)(x^3-3)$

$= 2x^5 - 6x^2$

$f'(x) = \boxed{10x^4 - 12x}$

2. Find $\frac{dg}{dt}$ using the product rule if $g(t) = \frac{e^t}{t^2}$

$g(t) = \frac{e^t}{t^2} = e^t \cdot t^{-2}$

$\frac{dg}{dt} = (e^t)\frac{d}{dt}(t^{-2}) + (t^{-2})\frac{d}{dt}(e^t)$

$= e^t \cdot -2t^{-3} + t^{-2}\cdot e^t$

$= e^t(-2t^{-3} + t^{-2})$

$= e^t\left(\frac{-2}{t^3} + \frac{1}{t^2}\right)$

$= e^t\left(\frac{-2}{t^3} + \frac{1}{t^2}\cdot\frac{t}{t}\right)$ "1 in"

$= \boxed{e^t\left(\frac{-2+t}{t^3}\right)} = \boxed{\frac{(t-2)e^t}{t^3}}$

3. Find $f'(x)$ if $f(x)=4x^3 \ln x$

$$f'(x) = 4x^3 \cdot \frac{d}{dx}\ln x + \ln x \cdot \frac{d}{dx} 4x^3$$

$$= 4x^3 \cdot \frac{1}{x} + \ln x \cdot 12x^2$$

"1 out" $= 4x^2 \cdot \frac{x}{x} + 12x^2 \ln x$

$= \boxed{4x^2 + 12x^2 \ln x}$ or $\boxed{4x^2(1+3\ln x)}$ both are fine!

4. If $f(x)=xe^x$, report each. Hint: Look back. Look forward.

a. $f'(x) = \frac{d}{dx} xe^x$

$= x \cdot \frac{d}{dx}e^x + e^x \cdot \frac{d}{dx} x$

$= x \cdot e^x + e^x \cdot 1$

$= \boxed{xe^x + e^x}$

b. $f''(x) = \frac{d}{dx} xe^x + \frac{d}{dx} e^x$

$=$ $\boxed{\text{Answer to part a}}$ $+ e^x$

$= xe^x + e^x + e^x = \boxed{xe^x + 2e^x}$

c. $f'''(x) = \frac{d}{dx} xe^x + \frac{d}{dx} 2e^x$

$=$ $\boxed{\text{Answer to part a}}$ $+ 2 \cdot e^x$

$= xe^x + e^x + 2e^x = \boxed{xe^x + 3e^x}$

d. $f^{(100)}(x) = \boxed{xe^x + 100e^x}$ generalizing the above.

e. $f^{(n)}(x) = \boxed{xe^x + ne^x}$