Section 5.5 —Integrating using Substitution

= Chain Rule For Derivatives in Reverse

Important Ideas:

$$\frac{d}{dx} f(u(x)) = \underbrace{f'(u(x))}_{\text{derivative of the outside}} \cdot \underbrace{u'(x)}_{\text{derivative of the inside}}$$

$$\int \underbrace{f'(u(x))}_{\text{deriv of outside}} \cdot \underbrace{\boxed{u'(x)dx}}_{\text{deriv of inside}} = f(u(x)) + C$$

$$\int f'(u)\,du$$

1. Set $u=$ inner function
2. Find $\frac{du}{dx}$ to get $du = \frac{du}{dx}\cdot dx$
3. "correct" the integrand so it matches du exactly. Be Legal. We can only multiply by a form of 1 OR solve for dx in terms of du
4. Rewrite the integral in terms of u.
5. Antidifferentiate using basic formulas
6. For indefinite integrals substitute the expression for u to get it back into x
7. If desired, check mentally by differentiation

Check Your Understanding!

For questions 1-6, integrate.

1. $\int (3x-4)^5 dx$

$= \frac{1}{3}\int (3x-4)^5\, 3dx$ method 1 $\frac{1}{3}\int u^5 du$

$u = 3x-4$

$\frac{du}{dx} = 3$ $du = 3dx$

$\frac{du}{3} = dx$

$\int (3x-4)^5 dx = \int u^5 \cdot \frac{du}{3} = \frac{1}{3}\frac{u^6}{6} + C$ method 2

$= \frac{u^6}{18} + C$

$\frac{u^6}{18} + C = \frac{(3x-4)^6}{18} + C$

$= \boxed{\frac{(3x-4)^6}{18} + C}$

2. $\int \sec(7x)\tan(7x)\, dx$

$\int \sec u \tan u\, du = \sec u + C$

$u = 7x$

$du = 7dx$

$dx = \frac{du}{7}$

$\int \sec 7x \tan 7x\, dx = \int \sec u \tan u \frac{du}{7}$

$= \frac{1}{7}\sec u + C = \boxed{\frac{1}{7}\sec 7x + C}$

If you were to drop the dx on the du it leads to the flow of red ink.

3. $\int (4x^3-2)e^{x^4-2x} dx$

~~$u = 4x^3-2$~~

$u = x^4-2x$

$du = (4x^3-2)dx$

$\int e^u du = e^u + C$

$= \boxed{e^{x^4-2x} + C}$

4. $\int x^2\sqrt{x^3+1}\, dx$

$u = x^3+1$

$du = 3x^2 dx$

$= \frac{1}{3}\int \sqrt{x^3+1}\, 3x^2 dx = \frac{1}{3}\int u^{1/2} du = \frac{1}{3}\frac{u^{3/2}}{(3/2)} + C$

$= \frac{2}{9}u^{3/2} + C$

$= \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$

Check by differentiation using the chain rule.

5. $\int 3x\sin(3x^2)\, dx$ $= \frac{1}{2}\int \sin(3x^2)\cdot 2\cdot 3x dx$

$u = 3x^2$

$du = 6x dx$

$= \frac{1}{2}\int \sin u\, du$

$= \frac{-1}{2}\cos u + C$

$= \boxed{\frac{-1}{2}\cos(3x^2) + C}$

6. $\int \tan x\, dx$ (Hint: re-write the integrand in terms of sine and cosine).

$\int \tan x\, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{1}{\cos x}\cdot \sin x\, dx$

$u = \cos x$

$du = -\sin x\, dx$

$dx = \frac{du}{-\sin x}$

$= -1\int \frac{1}{u}\cdot \underbrace{(-\sin x dx)}$

$= -\int \frac{1}{u} du = -\ln|u| + C = \boxed{-\ln|\cos x| + C}$

① Multiply by $(-1)(-1)$

②

$\int \frac{1}{\cos x}\cdot \sin x\, dx = \int \frac{1}{u}\cdot \frac{\sin x\cdot du}{-\sin x} = $

$= \boxed{\ln|\sec x| + C}$