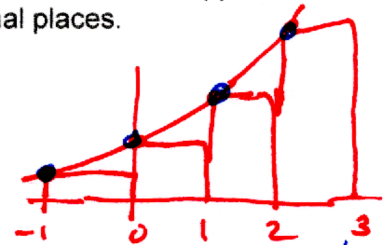
2. Consider the region enclosed between the x-axis and the curve $y = e^x$.

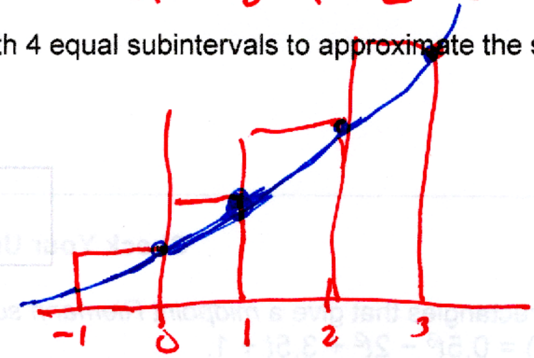
   a. Use a *left Riemann sum* approximation with 4 equal subintervals to approximate the area of the region between $x = -1$ and $x = 3$. Report to 3 decimal places.

   $$\sum_{k=-1}^{2} e^k \cdot 1 \approx 11.475$$



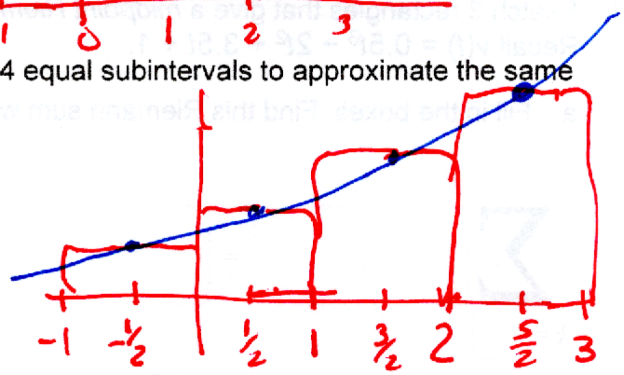
   b. Use a *right Riemann sum* approximation with 4 equal subintervals to approximate the same region. Report to 3 decimal places.

   $$\sum_{k=0}^{3} e^k \cdot 1 \approx 31.193$$



   c. Use a *midpoint Riemann sum* approximation with 4 equal subintervals to approximate the same region. Report to 3 decimal places.

   $$\sum_{k=-1/2}^{5/2} e^k \cdot 1 \approx 18.919$$

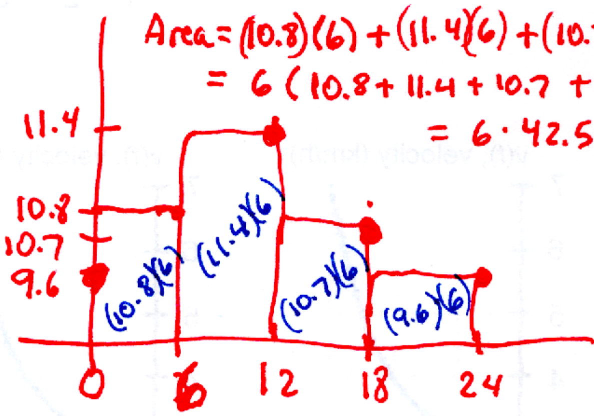


3. The rate at which water flows out of a pipe in gallons per hour is given by $R(t)$. Selected values of $R(t)$ are shown in the table below.

   a. Use a *right Riemann sum* approximation with 4 equal subintervals to approximate the area underneath $R(t)$ from $t = 0$ to $t = 24$. Show your calculations.

   Each rectangle has a width $\Delta t = 6$

   Area $= (10.8)(6) + (11.4)(6) + (10.7)(6) + (9.6)6$
   $= 6(10.8 + 11.4 + 10.7 + 9.6)$
   $= 6 \cdot 42.5 = \boxed{255}$



t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

   b. What does this area represent?

   Since we have $R(t)$ in $\frac{\text{gallons}}{\text{hr}}$ and Δt in hours

   the area is in $\frac{\text{gallons}}{\cancel{\text{hr}}} \cdot \cancel{\text{hr}} =$ gallons,

   which gives the volume of water, 255 gallons in 1 day.

Quick Notes

To approximate the area on $[a, b]$ of f with n equal subdivisions we have the options of

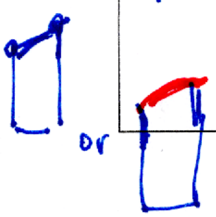
Riemann rectangles {
① left approximation – use left endpts of the interval
② right " – " right endpts " " "
③ midpt " – use middle of the interval
} to find the height of the rectangle computed by the output of f

or trapezoid

$$S_n = \sum_{i=1}^{n} f(x_i)(\Delta x)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{\infty} f(x_i)\Delta x$$

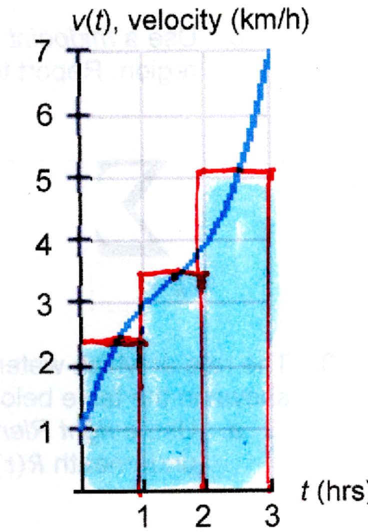
$= \text{total area } \int_a^b f(x)\,dx$



or parabola Simpson's Rule

Check Your Understanding

1. Sketch 3 rectangles that give a *midpoint Riemann sum* approximation for Jayda's distance for $0 \le t \le 3$. Recall $v(t) = 0.5t^3 - 2t^2 + 3.5t + 1$.



a. Fill in the boxes. Find this Riemann sum with your grapher. Do not round off.

$$\sum_{k=1/2}^{5/2} v(k)\cdot 1 = 10.8125$$

Y₁■.5X³−2X²+3.5X+1

$\sum_{K=.5}^{2.5}(Y_1(K))$

10.8125

b. Use 6 rectangles of equal width to approximate Jayda's distance.

i. The width of each rectangle is $\Delta t = \underline{0.5}$.

ii. Use a *left Riemann sum* approximation. Sketch the 6 rectangles below.

$\sum_{K=0}^{5}(Y_1(\frac{K}{2})*\frac{1}{2})$

9.40625

$$\sum_{k=0}^{5} v\left(\frac{k}{2}\right)\cdot\frac{1}{2} = 9.40625$$

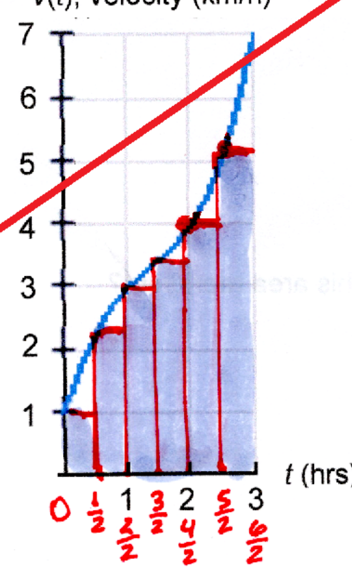
iii. Use a *right Riemann sum* approximation. Sketch the 6 rectangles to the right.

$\sum_{K=1}^{6}(Y_1(\frac{K}{2})*\frac{1}{2})$

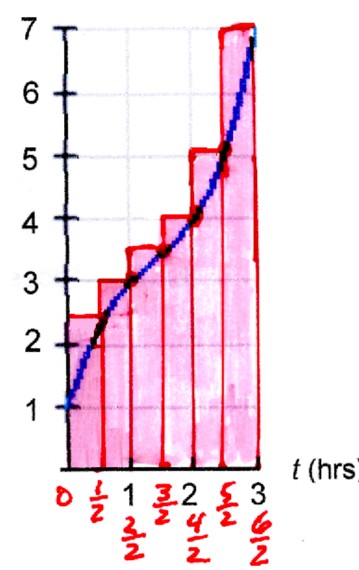
12.40625

$$\sum_{k=1}^{6} v\left(\frac{k}{2}\right)\cdot\frac{1}{2} = 12.40625$$

left Riemann sum



right Riemann sum



We will be able to find the *exact* area soon.
The exact value is 87/8 km or 10.875 km.