KEY

1. $$\lim_{x\to 0} \frac{k\sin ax}{bx} \underset{\text{L'H}}{=} \lim_{x\to 0} \frac{k(\cos ax)\cdot a}{b} = \frac{k(\cos 0)\cdot a}{b} = \boxed{\frac{ka}{b}}$$

chain rule

L'H since of form $\frac{0}{0}$

evaluate by direct substitution

2. $$\lim_{x\to 0} \frac{e^{x}-1}{\sin x} \underset{\text{L'H}}{=} \lim_{x\to 0} \frac{e^x}{\cos x} = \frac{e^0}{\cos 0} = \frac{1}{1} = \boxed{1}$$

L'H since of form

$$\frac{e^0-1}{\sin 0} = \frac{1-1}{0} = \frac{0}{0}$$

evaluate by direct substitution

3. $$\lim_{x\to -7} \frac{x^2+10x+21}{x^2-49} \underset{\text{L'H}}{=} \lim_{x\to -7} \frac{2x+10}{2x} = \boxed{\frac{2}{7}}$$

L'H since of form $\frac{0}{0}$

evaluate by substitution

4. $$\lim_{x\to 0} \frac{4x^3-6x^2}{-5x^2} \underset{\text{L'H}}{=} \lim_{x\to 0} \frac{12x^2-12x}{-10x} \underset{\text{L'H}}{=} \lim_{x\to 0} \frac{24x-12}{-10} = \boxed{1.2}$$

L'H since of form $0/0$

L'H since still in form $\frac{0}{0}$

direct substitution

Alternatively, without L'Hôpital:

$$\lim_{x\to 0} \frac{4x^3-6x^2}{-5x^2} = \lim_{x\to 0} \frac{x^2(4x-6)}{x^2\,(-5)} = \lim_{x\to 0} \frac{4x-6}{-5} = \frac{-6}{-5} = \boxed{1.2}$$

5. $$\lim_{x\to \infty} \frac{x^3\ln x}{x^5} = \lim_{x\to \infty} \frac{\ln x}{x^2} \underset{\text{L'H}}{=} \lim_{x\to \infty} \frac{1/x}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = \boxed{0}$$

L'H since of form $\frac{\infty}{\infty}$

check with the graph $y = \frac{x^3\ln x}{x^5}$

6. $$\lim_{x\to\infty} \frac{x^{999}}{x^{1000}} = \boxed{0}$$ since $y = x^{1000}$ outpaces $y = x^{999}$ as $x \to \infty$.

You could also apply L'Hôpital's Rule 999 times.

7. $\lim_{x\to 0} x^3 \cot x$ Since $\cot x = \frac{\cos x}{\sin x}$ has a vertical asymptote at $x=0$, this is of the form $0 \cdot \infty$. Rewrite this to make it in the form $\frac{0}{0}$ $\Rightarrow \lim_{x\to 0} \frac{x^3}{\tan x} \underset{\text{L'H}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{3x^2}{\sec^2 x} = \frac{0}{1} = \boxed{0}$

(use $\cot x = \frac{1}{\tan x}$)

Since $\sec x = \frac{1}{\cos x}$, then at $x=0$ since $\cos x = 0$, $\sec 0 = 1$

8. $\lim_{x\to\infty} \frac{e^{10/\sqrt{x}} - 1}{\frac{10}{\sqrt{x}}} \underset{\text{L'H}}{\overset{\uparrow}{=}} \lim_{x\to\infty} \frac{e^{10/\sqrt{x}} \cdot \frac{d}{dx}\frac{10}{\sqrt{x}}}{\frac{d}{dx}\frac{10}{\sqrt{x}}} \underset{1 \text{ out}}{\overset{\uparrow}{=}} \lim_{x\to\infty} e^{10/\sqrt{x}} = \boxed{1}$

since of form $\frac{0}{0}$

Since $e^0 = 1$

This is like #2 but uglier. $\lim_{x\to\infty} \frac{10}{\sqrt{x}} = 0$

9. $\lim_{x\to\infty} \left(1 + \frac{r}{x}\right)^x$ Use your notes from April 8, 2025 class.

You can also see below

① Let $y = \left(1+\frac{r}{x}\right)^x$ and find $\lim_{x\to\infty} \ln y$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \ln\left(1+\frac{r}{x}\right)^x = \lim_{x\to\infty} x \ln\left(1+\frac{r}{x}\right)$

which is of the form $\infty \cdot 0$ since $r/x \to 0$ and $\ln 1 = 0$

Rewrite this to make it in the form $\frac{0}{0}$:

$\lim_{x\to\infty} x\ln\left(1+\frac{r}{x}\right) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{r}{x}\right)}{\frac{1}{x}} \underset{\text{L'H}}{\overset{\uparrow}{=}} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{r}{x}} \cdot r \cdot \frac{d}{dx}\frac{1}{x}}{\frac{d}{dx}\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{1}{1+\frac{r}{x}} \cdot r = \frac{1}{1+0} \cdot r = r$. This is $\lim_{x\to\infty} \ln y$

② $\lim_{x\to\infty} \left(1+\frac{r}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{r}{x}\right)^x} = \lim_{x\to\infty} e^{\ln y} =$

$e^{\lim_{x\to\infty} \ln y} = e^r$ So the answer is $\boxed{e^r}$

10. $\lim_{x\to 0} (1+2x)^{5/x}$

Because the power contains x, let's first set $y=(1+2x)^{5/x}$ then find $\lim_{x\to 0} \ln y$

① $\lim_{x\to 0} \ln y = \lim_{x\to 0} \ln (1+2x)^{5/x}$

$= \lim_{x\to 0} \frac{5}{x} \cdot \ln(1+2x)$ using Bob Barker Property

this is in the form $\infty \cdot 0$ since $\frac{5}{x} \to \infty$ and $\ln 1 = 0$

$= \lim_{x\to 0} \frac{5\ln(1+2x)}{x}$ ← this is in the form $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{5 \cdot \frac{1}{1+2x} \cdot 2}{1}$

$= \lim_{x\to 0} \frac{10}{1+2x} = \frac{10}{1+0} = 10$ but we are not done yet.

② Now to the original problem:

$\lim_{x\to 0} (1+2x)^{5/x} = \lim_{x\to 0} e^{\ln (1+2x)^{5/x}}$

$= e^{\lim_{x\to 0} \ln (1+2x)^{5/x}} = \boxed{e^{10}}$.