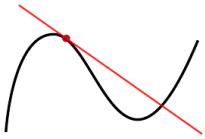
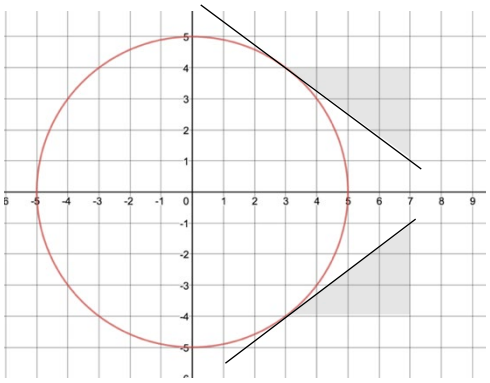
3.8 Implicit Differentiation (as opposed to Explicit Differentiation)



So far, we have found tangent lines for all kinds of functions: polynomials, exponentials, logarithmic, and trig functions. What other kinds of graphs might we be able to find tangent lines for?

1. Write the equation of the circle given.



2. What is the slope of the tangent line to the circle at $x = 0$? How do you know?

3. What is the slope of the tangent lines at $x = 3$? Use the grid to help.

4. When we do the same thing to both sides of an equation, the two sides remain equivalent. Let $y = 3x^2$.
   a. The left hand side, y , represents a function.
      Take the derivative of both sides of the equation with respect to x.

   b. $\frac{dy}{dx}$ or y'represents the derivative of the function y with respect to x.

5. We can apply this thinking to our tangent line problem. Let's take the derivative of both sides of the circle equation, $x^2 + y^2 = 25$.
   a. We have $\frac{d}{dx}(x^2)$ = ____.
      We have $\frac{d}{dx}(25)$ = ____.

   b. Even though *y* is not by itself, it still represents a function of *x*.
      Why is y^2 a composite function?

   c. Use the chain rule to find $\frac{d}{dx}(y^2)$. $\frac{d}{dx}(y^2)$ = _____

6. Give an equation for the slope of the tangent line at *any* point on this circle. Does this agree with your answer for questions 2 and 3? What is the slope when y = 0?

Section 3.8—Implicit Differentiation

Important Ideas:

Check Your Understanding!

1. Find y' for $\frac{x}{y^3} = 1$ by first solving for *y*. Then find y' using implicit differentiation. Are the answers the same?

2. Find y' for $2y^3 + 4x^2 - y = x^6$

3. Write the equation of the tangent line to $x^4 + y^2 = 3$ at $(1, -\sqrt{2})$

4. (Multiple Choice) If $y = \ln(x^2 + y^2)$, then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is
   A. 0 B. -1 C. 1 D. 2 E. Undefined

5. Calculate y'' for $x^2 + 4y^2 = 1$ using implicit differentiation. Simplify.