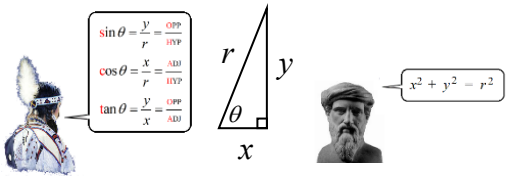
3.10 - I Get By With the Help of My Friends



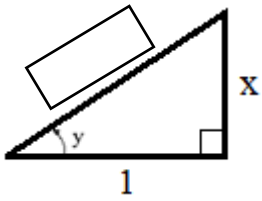
We develop the derivatives of inverses of trigonometric functions with the help of two famous amigos, SOHCAHTOA and Pythagoras. It's a process we will use later in MA 16600. Remember y = $\tan^{-1}$ x means tan y = x and, for **values in their domain and range***, tan ($\tan^{-1}$ x) = x and $\tan^{-1}$ (tan x) = x. Similar relationships hold for the other five trig functions.

*This contingency clause will keep the lawyers at bay.

1. Let $y = \tan x$.
   a. To find the inverse of this function, switch *x* and *y*.

   b. Differentiate both sides of this inverse equation and solve for $\frac{dy}{dx}$.

   c. Recall that the trig functions represent ratios of sides in right triangles. In the triangle $\tan y = x$. What is the third side in this triangle?



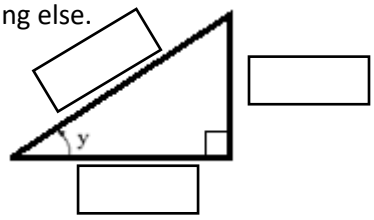
   d. Use the triangle ratios to find $\cos y$ and $\sec y$.

   e. Write $\frac{dy}{dx}$ only in terms of *x* using substitutions from part (d).

2. Let $y = \sin x$.
   a. What's the first step to finding the inverse of this function?

   b. Differentiate both sides of this inverse equation and solve for $\frac{dy}{dx}$.

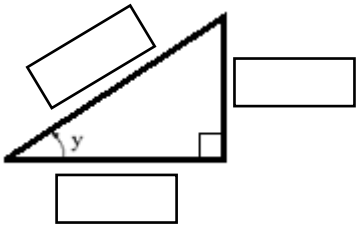
   c. In the triangle below, $\sin y = x$. Fill in the boxes with x, 1, and something else. What is $\cos y$?



   d. Write $\frac{dy}{dx}$ only in terms of *x* using what you learned in part (c).

3. Let $y = \cos x$.
   a. Write the inverse equation, differentiate both sides, and solve for $\frac{dy}{dx}$.

   b. In the triangle below, $\cos y = x$. Label the sides of the triangle. What is $\sin y$?



   c. Write $\frac{dy}{dx}$ only in terms of *x* using what you learned in part (b).

Section 3.10 — Derivatives of Inverse Trigonometric Functions

Important Ideas:

Check Your Understanding!

1. Let $y = \sin^{-1}(3x)$. Find y'.

2. For $y = \tan^{-1}(4x^2)$, find $\frac{dy}{dx}$.

3. If $\arcsin x = \ln y$, find $\frac{dy}{dx}$.

4. For $y = \arccos(e^{7x})$, find y'.

5. Let $g(x) = \operatorname{arccot} x$. Find $g'(2)$.