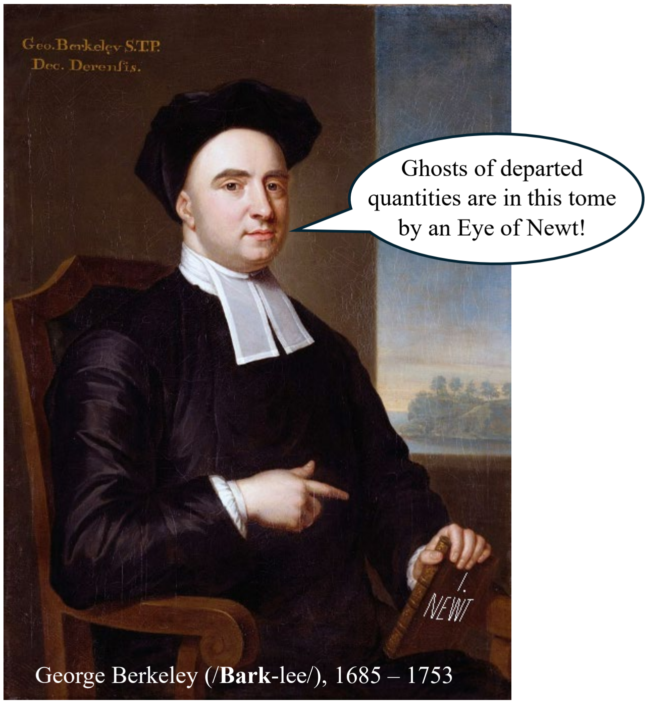
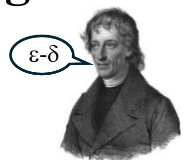
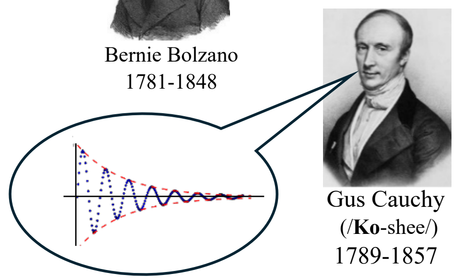
Ghostbusting with Limits!



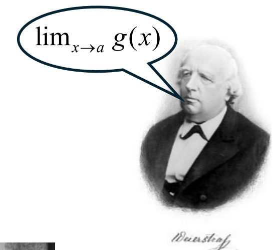
George Berkeley (/**Bark**-lee/), 1685 – 1753



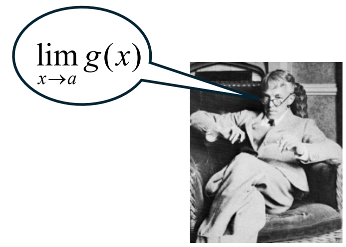
Bernie Bolzano
1781-1848



Gus Cauchy
(/**Ko**-shee/)
1789-1857



Karl Weierstrass
1815-1897



Godfrey H. Hardy
1877 -1947

The following "fluxions" (instantaneous rates of change) shown below can be found using algebraic manipulation. We can thank the above "ghostbusters," fueled over the next 150 years after Berkeley, to give us analytic tools to find these precisely.

1. $\lim_{x\to 5}\frac{4x-20}{x-5} = \lim_{x\to 5}\frac{4(x-5)}{(x-5)} = \lim_{x\to 5} 4 = \boxed{4}$ ("1 out")

2. $\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)} = \lim_{x\to 3}(x+3) = 3+3 = \boxed{6}$ ("1 out")

3. $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ Method 1 (rewrite $x-4$): $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{(\sqrt{x}-2)}{(\sqrt{x}-2)(\sqrt{x}+2)}\ \lim_{x\to 4}\frac{(\sqrt{x}-2)}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4}\frac{1}{(\sqrt{x}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$ ("1 out")

   Method 2 (u-substitution) : Let $u=\sqrt{x}$ and rewrite everything in terms of u: $u=\sqrt{x}\Rightarrow x=u^2$

   As $x\to 4$ we have $u=\sqrt{x}\to\sqrt{4}=2$

   $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{u\to 2}\frac{u-2}{u^2-4} = \lim_{u\to 2}\frac{(u-2)}{(u+2)(u-2)} == \lim_{u\to 2}\frac{1}{(u+2)} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$ ("1 out")

4. $\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2}\frac{(\frac{1}{x}-\frac{1}{2})}{(x-2)}\frac{2x}{2x} = \lim_{x\to 2}\frac{2x(\frac{1}{x})-2x(\frac{1}{2})}{(x-2)2x} = \lim_{x\to 2}\frac{(2-x)}{(x-2)2x} = \lim_{x\to 2}\frac{(2-x)}{(x-2)2x} = \lim_{x\to 2}\frac{-1}{2x} = \boxed{-\frac{1}{4}}$ ("1 in", "–1 out")

5. $\lim_{x\to 0}\frac{\sqrt{5+x}-\sqrt{5}}{x}$ Method 1 (u-substitution): Let $u=\sqrt{5+x}$ and rewrite everything in terms of u.

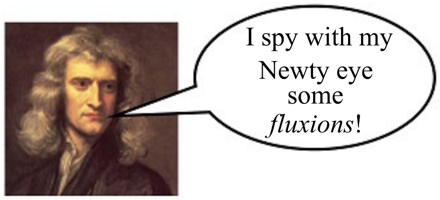
   $u=\sqrt{5+x}$
   $5+x=u^2$
   $x=u^2-5$

   As $x\to 0$ we have $u=\sqrt{5+x}\to\sqrt{5+0}=\sqrt{5}$

   $\lim_{u\to\sqrt{5}}\frac{u-\sqrt{5}}{u^2-5} = \lim_{u\to\sqrt{5}}\frac{(u-\sqrt{5})}{(u+\sqrt{5})(u-\sqrt{5})} = \lim_{u\to\sqrt{5}}\frac{1}{u+\sqrt{5}} = \frac{1}{\sqrt{5}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}.$ ("1 out")

   Method 2 (Multiply by conjugate): Let $\lim_{x\to 0}\frac{\sqrt{5+x}-\sqrt{5}}{x} = \lim_{x\to 0}\frac{(\sqrt{5+x}-\sqrt{5})}{x}\frac{(\sqrt{5+x}+\sqrt{5})}{(\sqrt{5+x}+\sqrt{5})}$ ("1 in")

   $= \lim_{x\to 0}\frac{(5+x-5)}{x(\sqrt{5+x}+\sqrt{5})} = \lim_{x\to 0}\frac{(x)}{(x)(\sqrt{5+x}+\sqrt{5})} = \lim_{x\to 0}\frac{(x)}{(x)(\sqrt{5+x}+\sqrt{5})} = \lim_{x\to 0}\frac{1}{\sqrt{5+x}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$ ("1 out")



Sir Isaac Newton
1642–1726/1727

Through the Eye of Newt

Each of the quantities represents the slope of the tangent line of some function $f(x)$ at some value a (also called the slope of the curve at $x = a$ or the derivative at $x = a$.

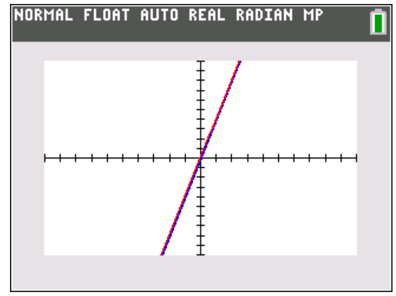
Use the derivative template $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to identify $f(x)$ and a.

Make a sketch to indicate visually what each represents to look at it through the eye of Newt. Interpret $f'(a)$.

1. $\lim_{x \to 5} \frac{4x - 20}{x - 5} = \boxed{4}$

   $f(x) = \boxed{4x}$ The value $a = \boxed{5}$

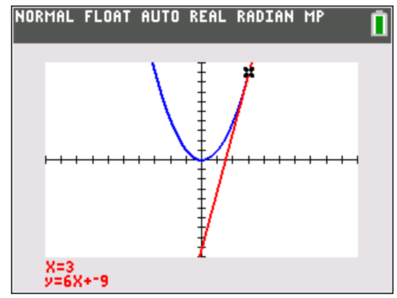
   $f'(\boxed{5}) = \boxed{4}$ says the slope of the curve to $f(x) = 4x$ at $x = 5$ is 4.



2. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \boxed{6}$

   $f(x) = \boxed{x^2}$ The value $a = \boxed{3}$

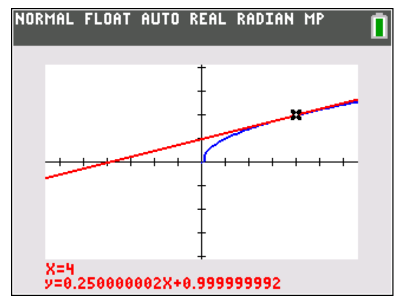
   $f'(\boxed{3}) = \boxed{4}$ says the slope of the curve to $f(x) = x^2$ at $x = 3$ is 6.



3. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \boxed{\frac{1}{4}}$

   $f(x) = \boxed{\sqrt{x}}$ The value $a = \boxed{4}$

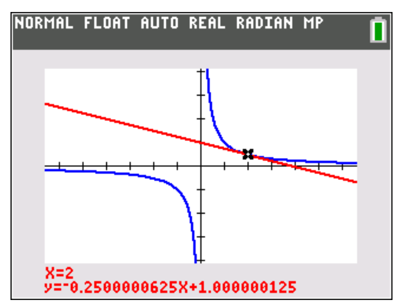
   $f'(\boxed{4}) = \boxed{\frac{1}{4}}$ says the slope of the curve to $f(x) = \sqrt{x}$ at $x = 4$ is $\frac{1}{4}$.



4. $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \boxed{-\frac{1}{4}}$

   $f(x) = \boxed{\frac{1}{x}}$ The value $a = \boxed{2}$

   $f'(\boxed{2}) = \boxed{-\frac{1}{4}}$ says the slope of the curve to $f(x) = \frac{1}{x}$ at $x = 2$ is $-\frac{1}{4}$.



5. $\lim_{x \to 0} \frac{\sqrt{5 + x} - \sqrt{5}}{x} = \boxed{\frac{1}{2\sqrt{5}}}$

   $f(x) = \boxed{\sqrt{5 + x}}$ The value $a = \boxed{0}$

   $f'(\boxed{0}) = \boxed{\frac{1}{2\sqrt{5}}}$ says the slope of the curve to $f(x) = \sqrt{5 + x}$ at $x = 0$ is $\frac{1}{2\sqrt{5}}$.