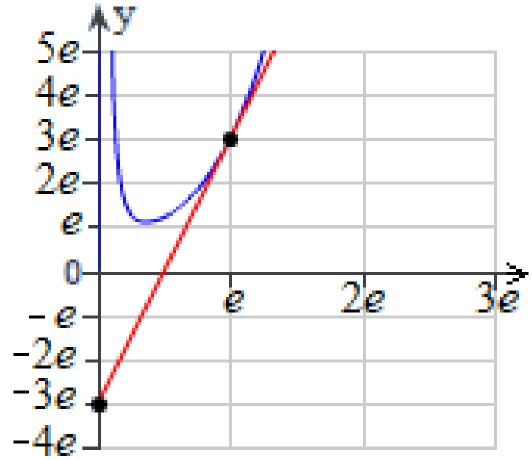
The function $f(x) = 3x^{\ln x}$ and its tangent line at $x = e$ is shown. Each tick mark on the graph is in steps of e. The y-intercept of the tangent line is $(0, -3e)$.



a. Use the formula to find the exact value of $f(e)$.
b. Use the grid to find $f'(e)$.
c. Find the derivative of $y = 3x^{\ln x}$ using logarithmic differentiation.
d. The minimum of the function $f(x)$ occurs when the graph of $f(x)$ has a horizontal tangent line, i.e., when the derivative $\frac{dy}{dx} = 0$.

   Use your formula for $\frac{dy}{dx}$ to find the value of x for which $\frac{dy}{dx} = 0$.
e. Find the equations of any horizontal tangent lines for $y = 3x^{\ln x}$.

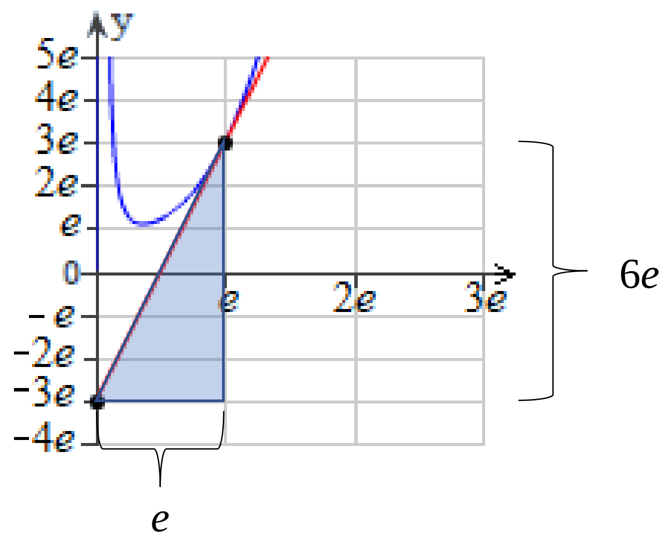
a. Use the formula to find the exact value of $f(e)$.

$f(e) = 3e^{\ln e} = 3e^1 = \boxed{3e}$ This is confirmed by the graph.

b. Use the grid to find $f'(e)$.

Using the grid and the slope triangle, the slope of the tangent line shown is

$\frac{6e}{e} = \boxed{6}$



c. Find the derivative of $y = 3x^{\ln x}$ using logarithmic differentiation.

1. Take natural logarithms of both sides

   $\ln(y) = \ln(3x^{\ln x})$

2. Use properties of logarithms to write as a product

$$\ln(y) = \ln(3x^{\ln x})$$
$$\ln(y) = \ln(3) + \ln x^{\ln x}$$
$$\ln(y) = \ln(3) + (\ln x)(\ln x)$$
$$\ln(y) = \ln(3) + (\ln x)^2$$

3. Differentiate both sides implicitly. Use the power rule and the chain rule.

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln y = \frac{\mathrm{d}}{\mathrm{d}x}\ln(3) + \frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^2$$

$$\frac{1}{y}\frac{dy}{dx} = 0 \;+\; 2\ln(x)\frac{\mathrm{d}}{\mathrm{d}x}\ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}$$

4. Solve for $\frac{dy}{dx}$. Replace y with $3x^{\ln x}$.

$$\frac{dy}{dx} = \frac{2\ln x}{x}\cdot y$$

$$\frac{dy}{dx} = \frac{2\ln x}{x}\cdot 3x^{\ln x}$$

$$\frac{dy}{dx} = \boxed{\frac{6x^{\ln x}\ln x}{x}}$$

Check with part b:

$$f'(e) = \left.\frac{dy}{dx}\right|_{\substack{x=e\\ y=3e}} = \frac{2\ln x}{x}\cdot y = \frac{2\ln e}{e}\cdot 3e = 6$$

d. The minimum of the function *f*(*x*) occurs when the graph of *f*(*x*) has a horizontal tangent line, i.e., when the derivative $\frac{dy}{dx} = 0$.

Use your formula for $\frac{dy}{dx}$ to find the value of *x* for which $\frac{dy}{dx} = 0$.

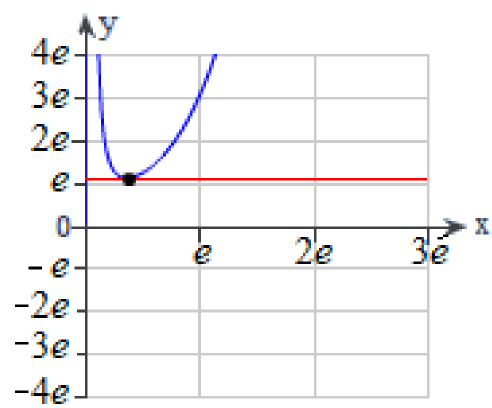
Set $\frac{dy}{dx} = \frac{2\ln x}{x}\cdot 3x^{\ln x} = 0$

$$\frac{2\ln x}{x}\cdot 3x^{\ln x} = 0$$

$$\ln x = 0$$

$$e^{\ln x} = e^0$$

$$x = e^0 = \boxed{1}$$



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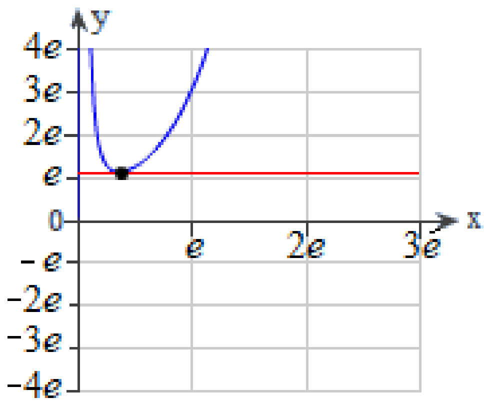
We could also set $y = 3x^{\ln x} = 0$ and solve but since $\ln x$ is defined only for $x > 0$, the function $y = 3x^{\ln x}$ is always positive and undefined at $x = 0$.

As $x \to 0^+$ we have $y = 3x^{\ln x} \to \infty$ as shown on the graph.

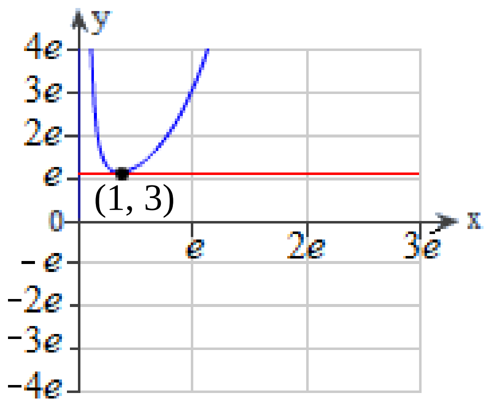
Therefore the minimum of the function $y = 3x^{\ln x}$ occurs at $x = 1$.



e. Find the equation of the horizontal tangent line at the minimum of the function. Substitute $x = 1$ in the formula.

When $x = 1$, we have $y = 3x^{\ln x} = 3 \cdot 1^{\ln 1} = 3 \cdot 1^0 = 3$.

Since 3 is slightly larger than $e \approx 2.718$, this matches the graph



The equation of the horizontal tangent line is $\boxed{y = 3}$.