Does “One Out” Always Occur When Finding the Derivative of a Polynomial or Rational Function Using the Limit Definition?

Name ______________________________

+1 Rhino Bonus Participation Point.

1. When finding the limit $f\,'(x) = \lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ using algebraic techniques, it would be convenient if another function, $g(x)$ existed with the property below:

$$f\,'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{x\to a}\frac{n(x)}{(x-a)} = \lim_{x\to a}\frac{g(x)(x-a)}{(x-a)},$$

where both the numerator $n(x)$ and the function $g(x)$ are continuous at $x = a$.

a. Suppose $f(x)$ is any **polynomial function**.

(+0.1) **i.** Explain why $(x-a)$ is a factor of $n(x) = f(x) - f(a)$,
i.e., there exists $g(x)$ such that $n(x) = f(x) - f(a) = g(x)\,(x-a)$.
Hint: Appeal to the *Factor Theorem* (perhaps worth looking up on Wikipedia).

(+0.1) **ii.** Report the value of $f\,'(a) = \lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ algebraically using analytical techniques,.
i..e., Ghostbusting, and part **ai**.

(+0.8) **b.** Suppose $w(x)$ is any **rational function** $w(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are both polynomials.

Assume $w(x)$ is continuous at $x = a$. It would be convenient if we could use “One Out” to find the following limit algebraically.

$$w\,'(a) = \lim_{x\to a}\frac{w(x)-w(a)}{x-a} = \lim_{x\to a}\frac{\frac{p(x)}{q(x)}-\frac{p(a)}{q(a)}}{x-a}$$

Multiply numerator and denominator $\dfrac{\frac{p(x)}{q(x)}-\frac{p(a)}{q(a)}}{x-a}$ by $\dfrac{q(x)q(a)}{q(x)q(a)}$ and simplify so that your numerator is a polynomial function. (We can do this since $q(x)$ is continuous at $x = a$.) Then apply the factor theorem as done in part **a** to show we can use “One Out”. Then report $w\,'(a)$. Show work below. Use the back if necessary.