

## Does “One Out” Always Occur When Finding the Derivative of a Polynomial or Rational Function?

Due Thursday, February 1 at the start of class for +1 Rhino Bonus Participation Point.

1. When finding the limit  $f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  using algebraic techniques, it would be convenient if

another function,  $g(x)$  existed with the property below:

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{n(x)}{x - a} = \lim_{x \rightarrow a} \frac{g(x)(x - a)}{x - a},$$

where both the numerator  $n(x)$  and the function  $g(x)$  are continuous at  $x = a$ .

- a. Suppose  $f(x)$  is any **polynomial function**.

- (+0.1) i. Explain why  $(x - a)$  is a factor of  $n(x) = f(x) - f(a)$ ,  
i.e., there exists  $g(x)$  such that  $n(x) = f(x) - f(a) = g(x)(x - a)$ .  
Hint: Appeal to the *Factor Theorem* (perhaps worth looking up on Wikipedia).

- (+0.1) ii. Report the value of  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  algebraically using Ghostbuster techniques and part ai.



- (+0.8) b. Suppose  $w(x)$  is any **rational function**  $w(x) = \frac{p(x)}{q(x)}$ , where  $p(x)$  and  $q(x)$  are both polynomials.

Assume  $w(x)$  is continuous at  $x = a$ . It would be convenient if we could use “One Out” to find the following limit algebraically.

$$w'(a) = \lim_{x \rightarrow a} \frac{w(x) - w(a)}{x - a} = \lim_{x \rightarrow a} \frac{\frac{p(x)}{q(x)} - \frac{p(a)}{q(a)}}{x - a}$$

Multiply numerator and denominator  $\frac{\frac{p(x)}{q(x)} - \frac{p(a)}{q(a)}}{x - a}$  by  $\frac{q(x)q(a)}{q(x)q(a)}$  and simplify so that your numerator

is a polynomial function. (We can do this since  $q(x)$  is continuous at  $x = a$ .) Then apply the factor theorem as done in part a to show we can use “One Out”. Then report  $w'(a)$ . Show work below. Use the back if necessary.

