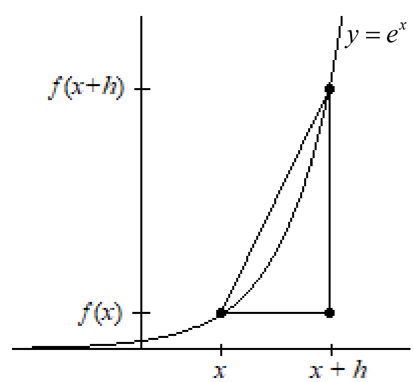
# Derivatives of Exponential Functions

Each of these limits is $f'$ for some function $f$ of $h$ and some value $a$.
Report $f$. Report $a$. Report the value of $f'(a)$.
Use the TANGENT feature of a grapher. If $f'(a)$ is undefined, write DNE.

$$\lim_{h\to 0} \frac{e^h - 1}{h} = \_\_\_\_ . \quad f(h) = \_\_\_\_\_\_; \; a = \_\_\_$$

$$\lim_{h\to 0} \frac{e^{kh} - 1}{h} = \_\_\_\_ . \quad f(h) = \_\_\_\_\_\_; \; a = \_\_\_$$

Use the limit definition to show the derivative of $y = e^x$ is itself.




$f(x) = \square \qquad f(x+h) = \square$

$$\frac{d}{dx} e^x = \lim_{h\to 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \lim_{h\to 0} \square$$

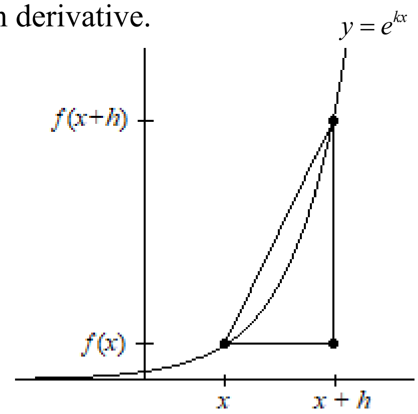
$= \lim_{h\to 0} \frac{1}{h} \cdot \left(e^x \cdot \square - e^x\right)$ by the law of exponents.

$= \lim_{h\to 0} \frac{1}{h} \cdot \left(e^x \left(\square\right)\right)$ by factoring out $e^x$.

$= \lim_{h\to 0} e^x \cdot \lim_{h\to 0} \frac{\square}{h}$ by writing as a product of limits.

$= \square \bullet \square$ Why? ____________________________________

1. Use the limit definition to show the derivative of $y = e^{kx}$ is proportional to its own derivative.
What is the constant of proportionality?




$f(x) = \square$ $\quad f(x+h) = \square$

$$\frac{d}{dx}e^{kx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{(x+h)-x} = \lim_{h\to 0}\square$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\left(e^{kx+\square} - e^{kx}\right)$$ Distribute the $k(x+h)$.

$$= \lim_{h\to 0}\frac{1}{h}\cdot\left(e^{kx}\cdot\square - e^{kx}\right)$$ by the law of exponents.

$$= \lim_{h\to 0}\frac{1}{h}\cdot\left(e^{kx}\left(\square\right)\right)$$ by factoring out $e^{kx}$.

$$= \lim_{h\to 0}e^{kx}\cdot\lim_{h\to 0}\frac{\square}{h}$$ by writing as a product of limits.

$$= \square \cdot \square$$ Why? ____________________________________________

The derivative of $y = e^{kx}$ is proportional to its own derivative with constant of proportionality equal to ______, i.e.

if $y = e^{kx}$, $\quad y' = \square$

2. Use the previous result to show the derivative of $y = b^x$ is proportional to its own derivative. Hint: Set $b = e^k$.
What is the constant of proportionality?

The derivative of $y = b^x$ is proportional to its own derivative with a constant of proportionality of ________, i.e.

if $y = b^x$, $\quad y' = \square$