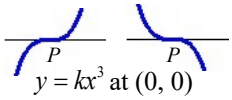
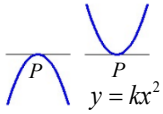
Critical Values and Critical Points of a Function

Examples of **horizontal tangent lines** to a curve at a point P:
We call point P is called a **stationary** point.
Formally: The two sided limit of the derivative at P is zero.



$y = kx^3$ at (0, 0)

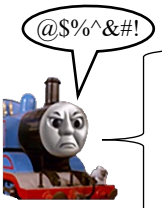


$y = kx^2$ at (0, 0)

Examples of **vertical tangent lines** to a curve at a point P:
Formally: The left sided derivative (LSD) and the right sided derivative (RSD) are infinities of the *same* sign or the one sided limit of the derivative at P is $\pm\infty$.

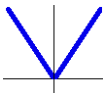
$y = k\sqrt[3]{x}$ at (0, 0)

$y = k\sqrt{x}$ at (0, 0)



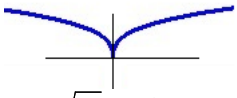
Corners and **cusps** have a pointy needle-like behavior and are not smooth.
(Corners and cusps make Thomas cuss. No train can travel on such tracks.)
Examples: www.geogebra.org/m/yn6xudfs#material/ytuc47zp
Cusps vs Corners: www.wolframalpha.com/examples/mathematics/calculus-and-analysis/applications-of-calculus/cusps-a nd-corners
Formally: The left sided derivative (LSD) and the right sided derivative (RSD) are *not the same*.

corner



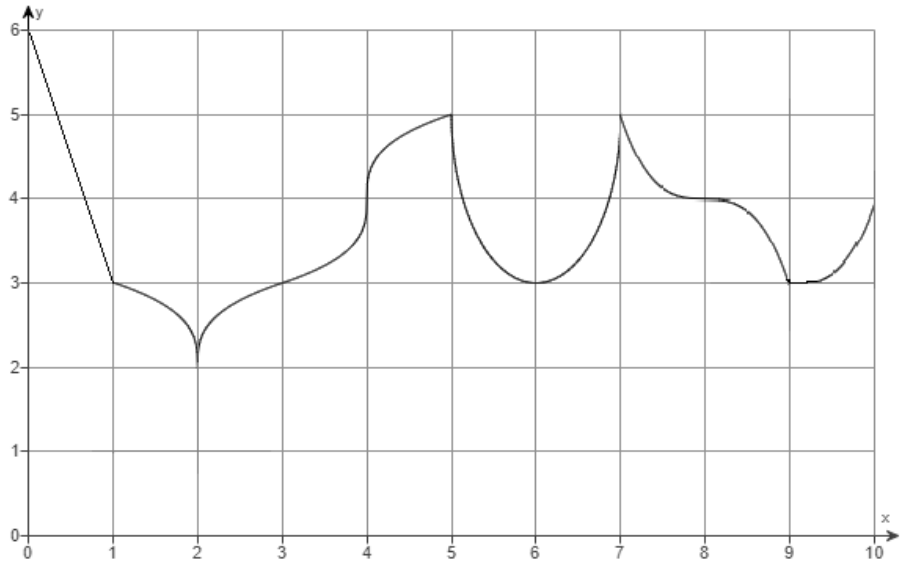
$y = |x|$ at (0, 0)

vertical cusp



$y = \sqrt[3]{x^2} = x^{2/3}$ at (0, 0)

Critical values of a function $y = f(x)$ on an interval $a < x < b$ occur at values of x where its derivative is **zero** or **undefined**.



1. List the **critical values** on the open interval (0, 10) for the graph of $f(x)$ shown above : x = _____

2. At what values of x does the graph of $f(x)$ have any **horizontal tangent lines**: x = _____
   vertical tangent lines at x = _____
   corners and cusps at x = _____

3. We may also be asked to report **critical points** of a function, which are *ordered pairs*. List the **critical points**.
   _____

4. What makes these values (or points) earn the name ***critical***?