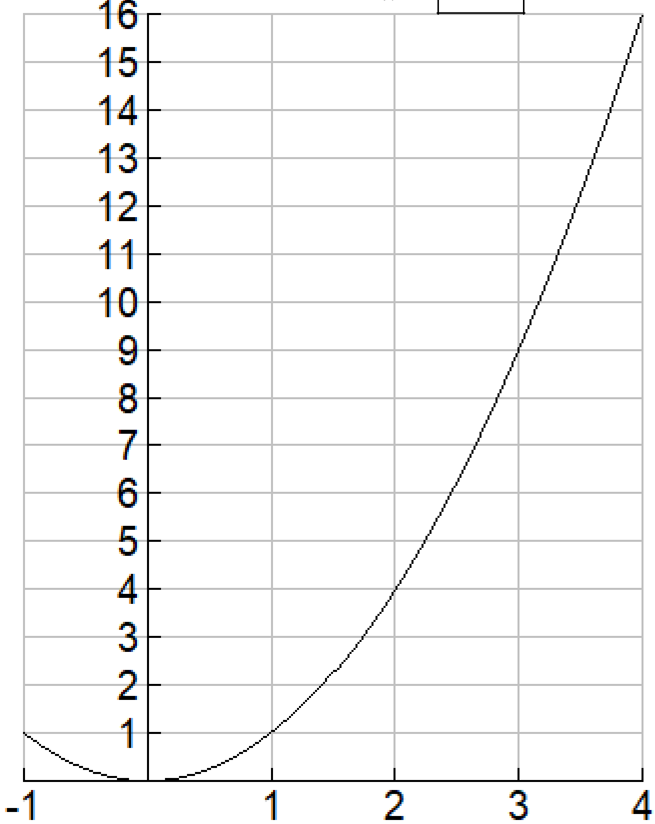
Average Rate of Change vs Instantaneous Rate of Change

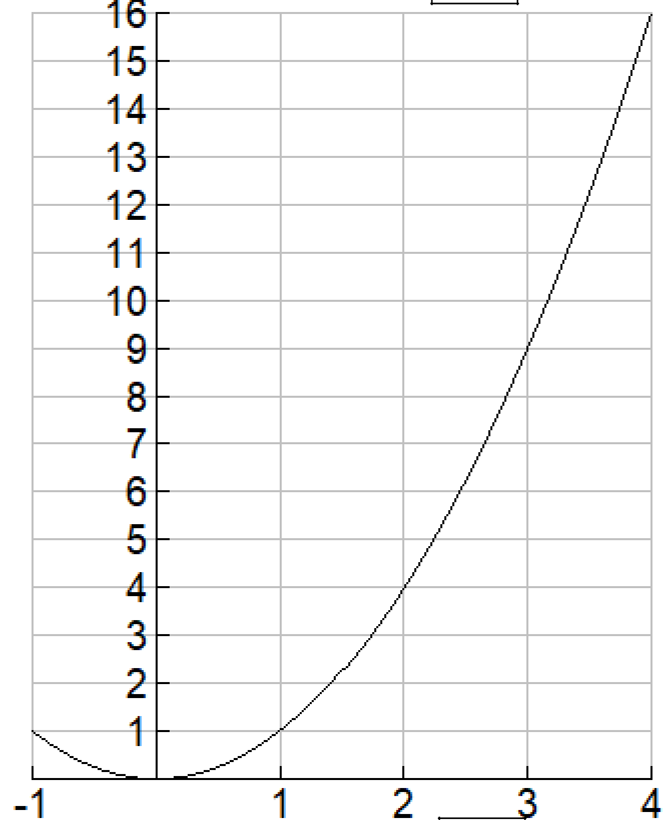
The graphs give $y = x^2$. Sketch line segments to represent and calculate $\frac{\Delta y}{\Delta x}$, the *average rate of change*.

Investigate the *instantaneous rate of change* at (3, 9) by exploring what $\Delta y / \Delta x$ approaches as $\Delta x \to 0$.

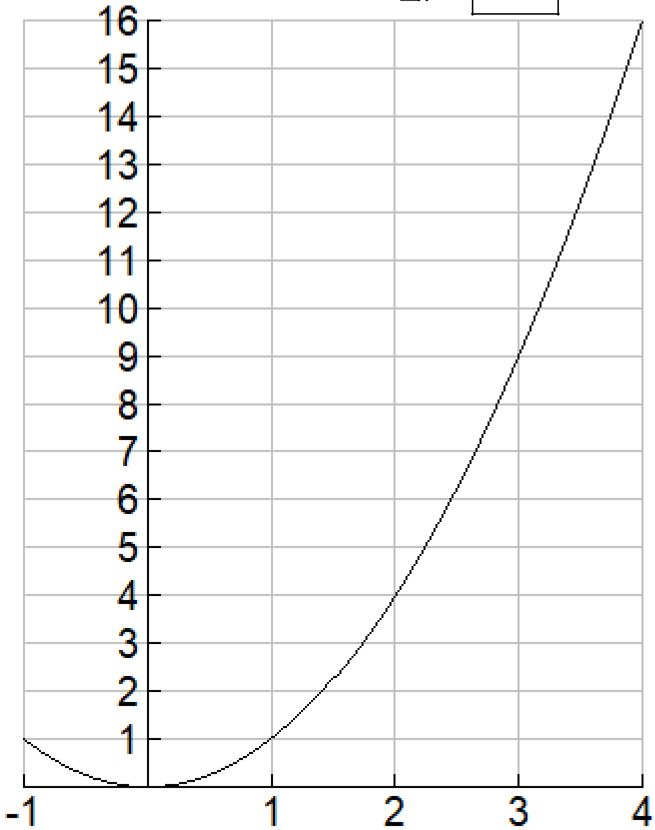
a. From $x = 0$ to $x = 3$, Δx = __, $\frac{\Delta y}{\Delta x}$ =



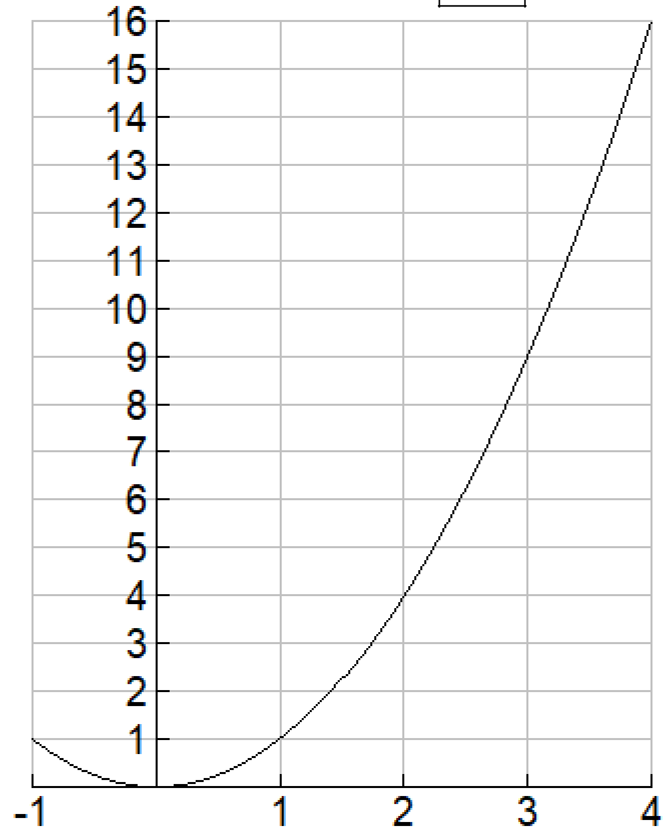
b. From $x = 1$ to $x = 3$, Δx = __, $\frac{\Delta y}{\Delta x}$ =



c. From $x = 2$ to $x = 3$, Δx = __, $\frac{\Delta y}{\Delta x}$ =



d. From $x = 3$ to $x = 4$, Δx = __, $\frac{\Delta y}{\Delta x}$ =



Secant Lines vs the Tangent Line
Average Rate of Change vs *Instantaneous* Rate of Change

A **secant line** to a curve passes through two points on the curve.

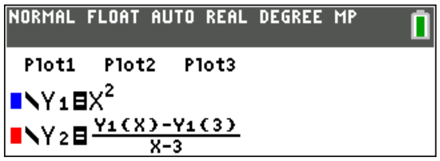
The **tangent line** to a curve at a point *P* is the unique line which just barely touches the curve at *P*.

> *Assume Thomas the Tank Engine travels along the curve at night and we view his headlights from a drone in the sky above. Then the tangent line to a curve at the point P is given by the direction of his headlights (or, more morbidly, the direction he would continue to travel if he flew off the rails.)*

As you look at a smaller and smaller section of the curve, (in fact, *infinitesimally* small)
the curve starts to more and more looks like the tangent line. We say the curve is *locally linear* at this point.

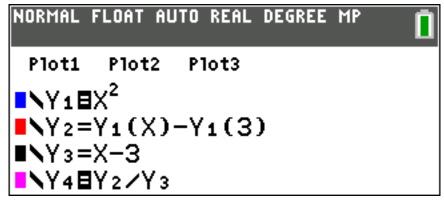
Use ALPHA [f1] for the stacked fraction.

Use ALPHA [f4] to select Y1.



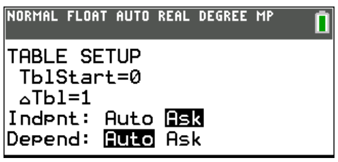
Method 1

or

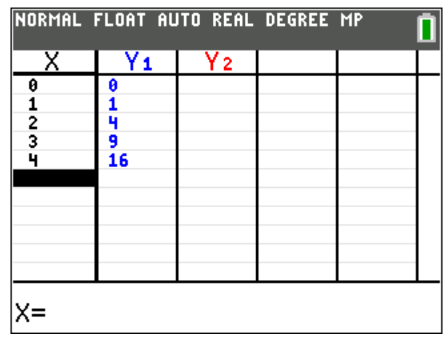


Method 2

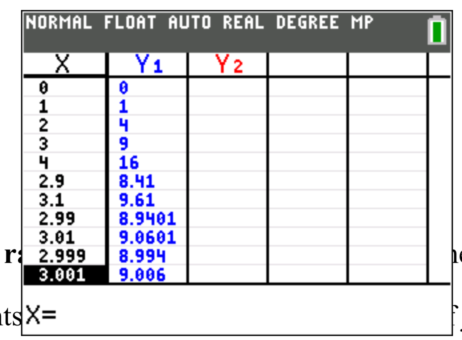
Press 2nd [TBLSET]



Press 2nd [TABLE]. Find the values of Y2. Compare these with the first page.



Find the values of Y2.



If $y = f(x)$, the **average r**… …ned to be $m_{sec} = \frac{\Delta y}{\Delta x}$, the **slope of the secant line** drawn through the points … $y = f(x)$.

The **instantaneous rate of change** at $x = a$ is defined to be m_{tan} , the **slope of the tangent line**.
It is also called: the **slope of the curve** at $x = a$ or the **derivative** of the function evaluated at $x = a$.

This can be written mathematically: As $b \to a$ or as $\Delta x \to 0$ ……………………., then $m_{sec} \to m_{tan}$

Limit Notation: We can also write $\lim_{\Delta x \to 0} m_{sec} = \lim_{b \to a} m_{sec} = m_{tan}$