5.1 Fast and Curious

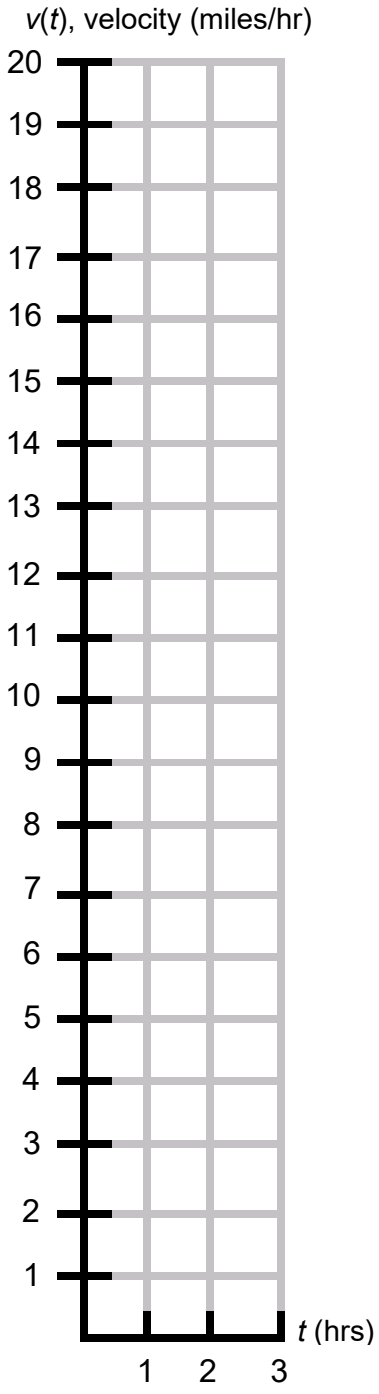
1. On a certain stretch of highway, Bal puts on Cruise Control and drives at 70 miles per hour for 3 hours.
   a. Sketch a graph of Bal's velocity on this stretch of highway.

   b. How much distance did she cover during that time period?

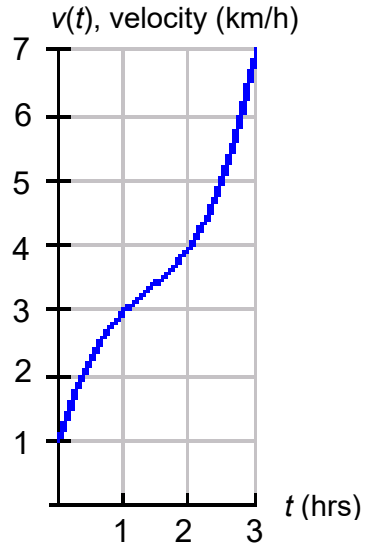
2. Bal's little brother Malik is riding his bicycle with a velocity given by $v(t) = 2 + 6t$ for $0 \le t \le 3$ where $v(t)$ is in miles per hour and t is in hours.

   a. Sketch a graph of Malik s velocity for $0 \le t \le 3$.

   b. How much distance did Malik cover over the three-hour period?



3. Jayda is riding with a velocity given by $v(t) = 0.5t^3 - 2t^2 + 3.5t + 1$ for $0 \le t \le 3$ where $v(t)$ is in km per hour and t is in hours. The graph is shown.



   a. How can we use this graph to determine how far Jayda has traveled?

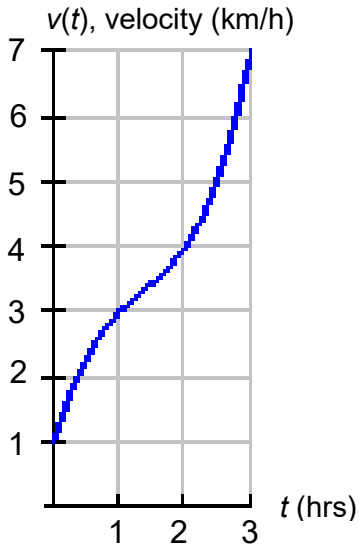
   b. During which 1-hour interval is Jayda accumulating the most kilometers? How do you know?

4. Not all areas can be found using simple geometric formulas, so we'll have to approximate the area. To keep things simple, we'll use 3 rectangles of equal width. How will you determine the height of each rectangle?

5. Record the height, width, and area of each rectangle in the table below. Draw these rectangles on the graph.

   a. Use *left end points* for the height of the rectangles.

Width	Height	Area



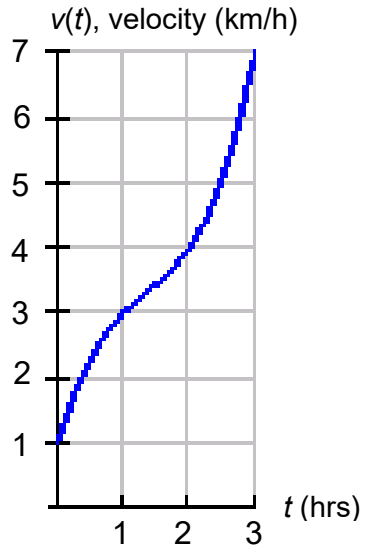
   What is your estimate for the distance Jayda rode during those 3 hours?

$$\sum_{k=\square}^{\square} \square = \square$$

   Is your estimation an overestimate or an underestimate? How do you know?

   b. Use *right end points* for the height of the rectangles.

Width	Height	Area



   What is your estimate for the distance Jayda rode during those 3 hours?

$$\sum_{k=\square}^{\square} \square = \square$$

   Is your estimation an overestimate or an underestimate? How do you know?

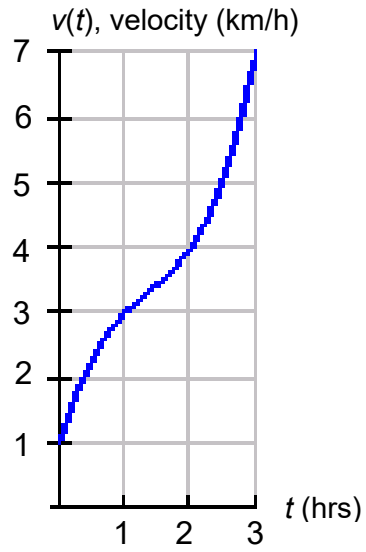
6. How could we get an even better approximation of Jayda's total distance traveled?

5.1 – Approximating Areas with Riemann Sums

Quick Notes

Check Your Understanding

1. Sketch 3 rectangles that give a *midpoint Riemann sum* approximation for Jayda's distance for $0 \le t \le 3$. Recall $v(t) = 0.5t^3 - 2t^2 + 3.5t + 1$.

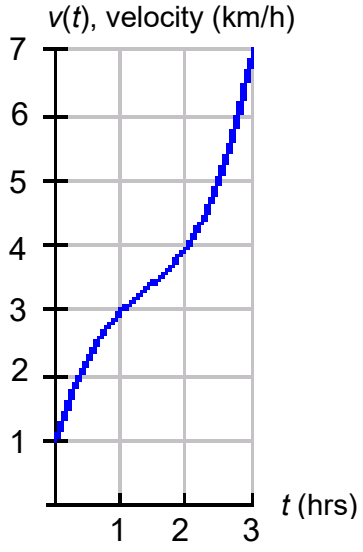


   a. Fill in the boxes. Find this Riemann sum with your grapher. Do not round off.

$$\sum_{k=\square}^{\square} \square = \square$$

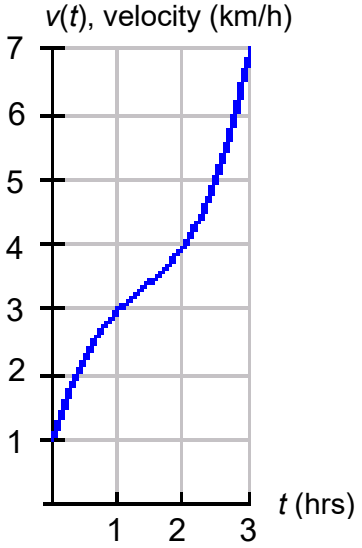
   b. Use 6 rectangles of equal width to approximate Jayda's distance.
      i. The width of each rectangle is $\Delta t =$ _____.
      ii. Use a *left Riemann sum* approximation. Sketch the 6 rectangles below.

$$\sum_{k=\square}^{\square} \square = \square$$



      iii. Use a *right Riemann sum* approximation. Sketch the 6 rectangles to the right.

$$\sum_{k=\square}^{\square} \square = \square$$



We will be able to find the *exact* area soon.
The exact value is 87/8 km or 10.875 km.

2. Consider the region enclosed between the *x*-axis and the curve $y = e^x$.
   a. Use a *left Riemann sum* approximation with 4 equal subintervals to approximate the area of the region between $x = -1$ and $x = 3$. Report to 3 decimal places.

$$\sum_{k=\square}^{\square} \square = \square$$

   b. Use a *right Riemann sum* approximation with 4 equal subintervals to approximate the same region. Report to 3 decimal places.

$$\sum_{k=\square}^{\square} \square = \square$$

   c. Use a *midpoint Riemann sum* approximation with 4 equal subintervals to approximate the same region. Report to 3 decimal places.

$$\sum_{k=\square}^{\square} \square = \square$$

3. The rate at which water flows out of a pipe in gallons per hour is given by $R(t)$. Selected values of $R(t)$ are shown in the table below.
   a. Use a *right Riemann sum* approximation with 4 equal subintervals to approximate the area underneath $R(t)$ from $t = 0$ to $t = 24$. Show your calculations.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

   b. What does this area represent?