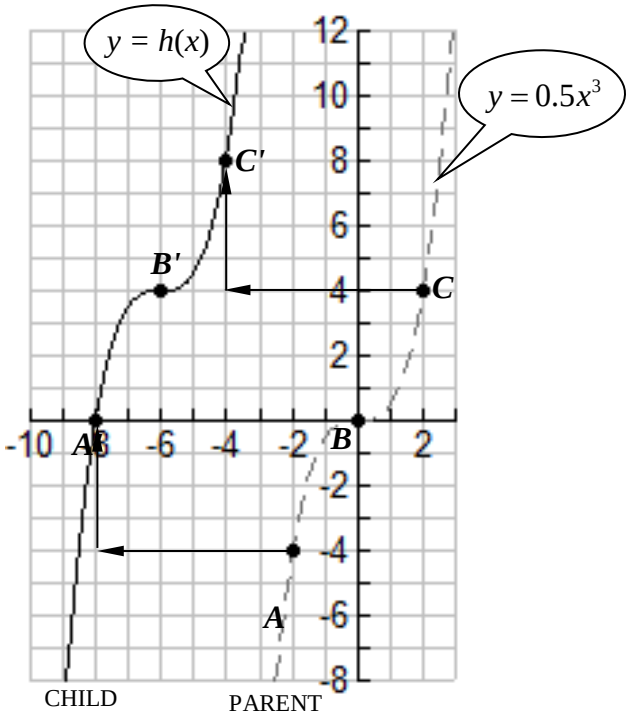
Practice Questions to Check Prerequisite Skills

1. The graph of $y = 0.5x^3$ is shown (dashed), along with the graph of $h(x)$ on the set of axes below. The graph of $h(x)$ is a translation of $y = 0.5x^3$, which has been shifted both horizontally and vertically. Points ***A***, ***B***, and ***C*** on $y = 0.5x^3$ correspond to ***A'***, ***B'***, and ***C'*** on $h(x)$, respectively.



a. Describe in words the translation of $y = 0.5x^3$ to $h(x)$.
Example: a shift left or right <*some specified number of* > units and a shift up or down <*some specified number of* > units.

b. Write the equation of $h(x)$ as a function of x.

c. At what value does the graph of $h(x)$ cross the x-axis? (This should be consistent with your formula in part **b**.)

d. At what value does the graph of $h(x)$ cross the y-axis? (You can use your formula or a grapher. No work need be shown.)

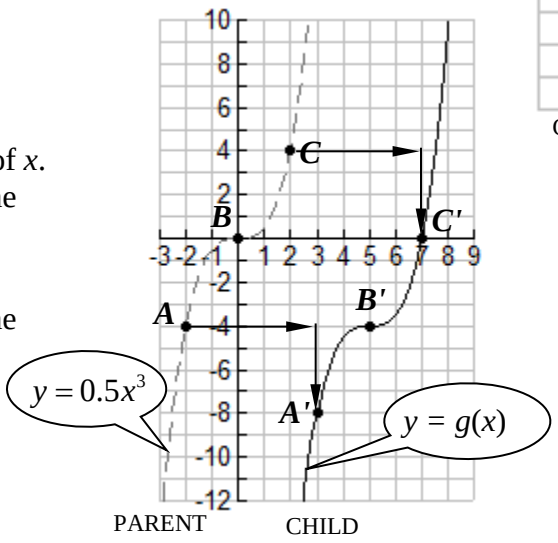
2. The graph of $y = 0.5x^3$ is shown (dashed), along with the graph of $g(x)$ on the set of axes below.

a. Describe in words the translation of $y = 0.5x^3$ to $g(x)$.

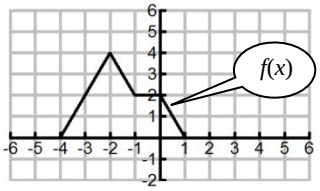
b. Write the equation of $g(x)$ as a function of x.

c. At what value does the graph of $g(x)$ cross the x-axis?

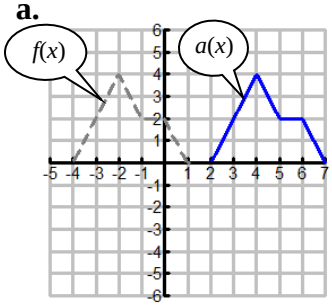
d. At what value does the graph of $g(x)$ cross the y-axis?

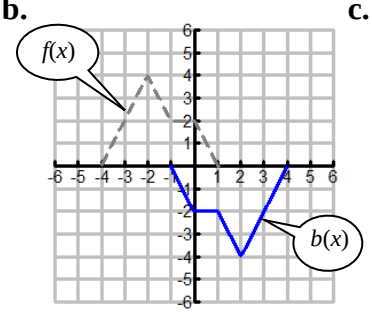


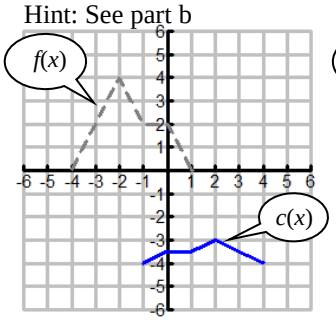
3. The graph of $y = f(x)$ is shown. The functions shown below are transformations of $f(x)$.

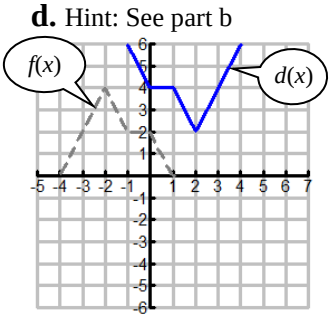


Describe each transformation and write a formula for each function in terms of $f(x)$.

a.


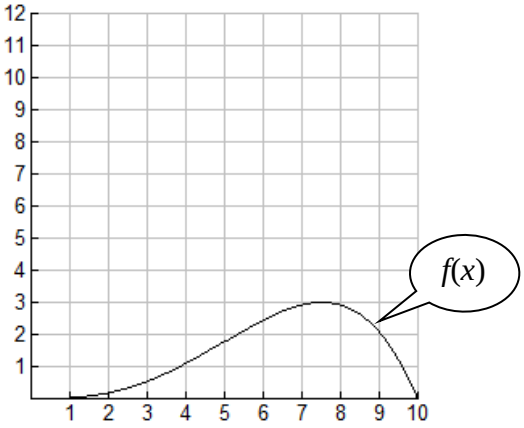
b.


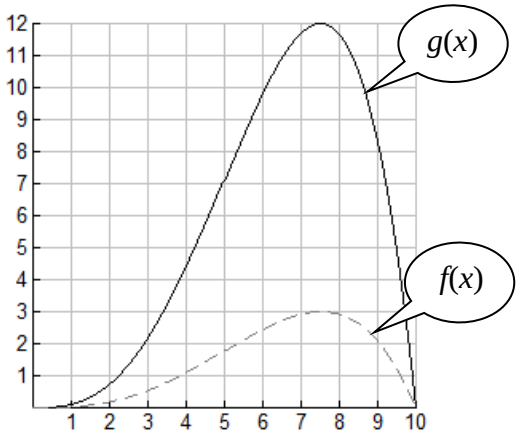
c. Hint: See part b


d. Hint: See part b


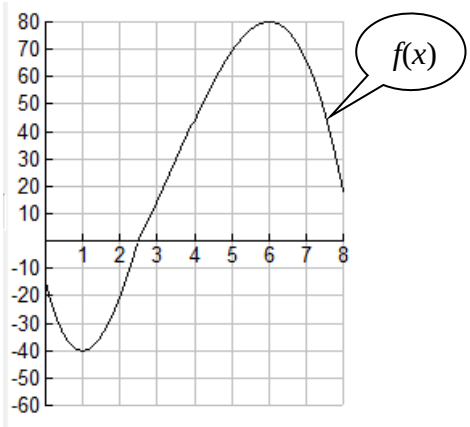
4. The graph of $y = f(x)$.is shown. Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$.
Find a formula for $g(x)$ in **terms of $f(x)$.**

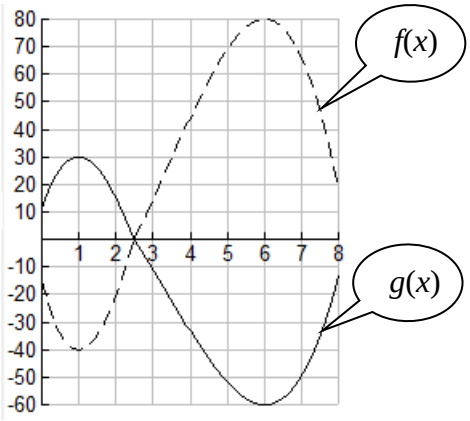
a.





b.





5. Which of these is $\ln\sqrt[3]{x^2}$? Circle one.

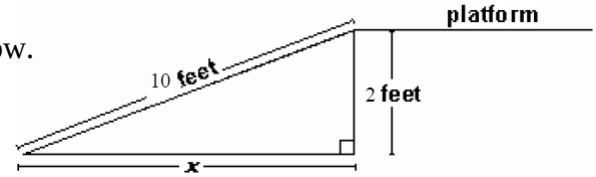
A. $3\ln\sqrt{x}$ B. $3\ln x$ C. $x\ln 3$ D. $\frac{2}{3}\ln x$ E. $\frac{3}{2}\ln x$ F. $3\ln x^2$ G. $2\ln x^3$ H. None of these.

6. Solve the equation. $e^x = 17.3$
Report both an exact solution (involving a logarithm) and an approximate solution to 2 decimal places.

7. Solve the equations. Report both an exact solution and an approximate solution to 3 decimal places.
a. $5\ln(3x) = 20$ **b**. $5\log x + 7 = 10$

8. Solve the equations.
a. $4u(u-2)=0$ **b**. $25u^2 = 4$ **c**. $25u^2 = 4u$ **d**. $13x - 4x^2 = 0$ **e**. $13x - 4x^2 = 3$
f. $2u^2 = u + 1$

9. Use the diagram to the right to answer the question below.
The figure is not drawn to scale.



A ramp feet 10 long is leaning against a raised platform which is 2 feet above the ground.
What is the distance from the ramp's contact point with the ground and the base of the platform?

A. 8 feet B. $\sqrt{104}$ feet C. $\sqrt{96}$ feet D. 6 feet E. None of these.

10. Jonesville and Smithville each have a population of 5000 size at year $t = 0$.
Suppose Jonesville's population grows by 200 people per year.
Suppose the population of Smithville grows by 2% per year.

a. Which is true? (Circle one)

A. Both towns are growing exponentially.
B. Jonesville is growing linearly and Smithville is growing exponentially.
C. Jonesville is growing exponentially and Smithville is growing linearly.
D. Both towns are growing linearly.

b. Find a formula for the population of the town of Jonesville at year t. $P =$ _____

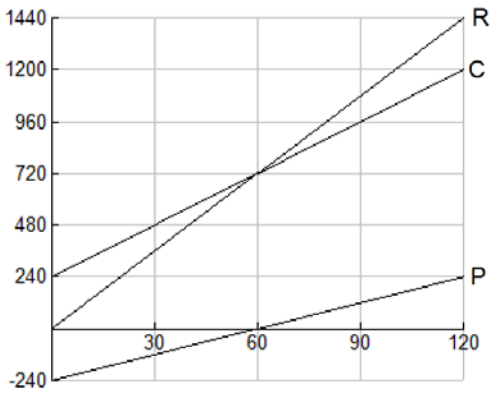
c. Find a formula for the population of the town of Smithville at year t. $P =$ _____

11. In the year 1900 the population P of a town was 200. The town grew by 23% every year.
In the year 1900 the population Q of a town was 400 people but it grew by 200 people every year.

a. Write formulas for P and Q.

b. Find how many years it will take after 1900 for the population of Q to overtake the population of P. Report your solution to 2 decimal places.

12. The revenue $R(x)$, cost $C(x)$, and profit $P(x)$ for a product are graphed in the figure to the right, where x is the quantity produced and sold. **Note:** $P(x) = R(x) - C(x)$



a. Determine the number of items that must be sold to break even, i.e., revenue is equal to costs.
The break-even quantity is ☐ units sold.

b. Find the formulas of the three functions.

$P(x) =$ _____

$R(x) =$ _____

$C(x) =$ _____

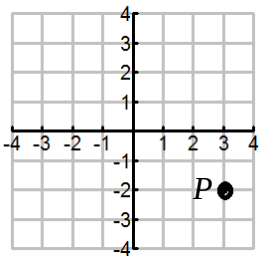
13. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.
Suppose that you have \$6000 to invest. Which investment yields the greater return over 13 years: 8.07% compounded **continuously** or 8.1% compounded **monthly**? (Select one)

A. Investing \$6000 at 8.07% compounded **continuously** over 13 years yields the greater return.
B. Investing \$6000 at 8.1% compounded **monthly** over 13 years yields the greater return.
C. Both investments yield the same return.

14. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.
Suppose that you have \$9000 to invest. Which investment yields the greater return over 19 years: 7.88% compounded **continuously** or 7.9% compounded **monthly**? (Select one)

A. Investing \$9000 at 7.88% compounded **continuously** over 19 years yields the greater return.
B. Investing \$9000 at 7.9% compounded **monthly** over 19 years yields the greater return.
C. Both investments yield the same return.

15. Suppose the point $P(3,-2)$ is a point on the graph of $y = f(x)$



a. Suppose $f(x)$ is **even:**

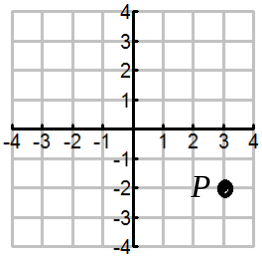
i. Report the coordinates of another point Q, which corresponds to P. (____ , ____)

ii. Plot the point Q on the grid provided.

b. Suppose $f(x)$ is **odd**:

i. Report the coordinates of another point Q, which corresponds to P. (____ , ____)

ii. Plot the point Q on the grid provided.



16. **a**. Assume the table represents a **linear** function.

i. Complete the box in the first row and the last row.

x	y
0	
1	125
2	85
3	45
4	

ii. Report the **equation** of the line in slope-intercept form:

$y =$ ☐ $x +$ ☐

b. Assume the table represents an **exponential** function.

i. Complete the box in the first row and the last row.

x	y
0	
1	128
2	32
3	8
4	

ii. If we report the equation of the exponential function in the form $y = ab^x$, then we have

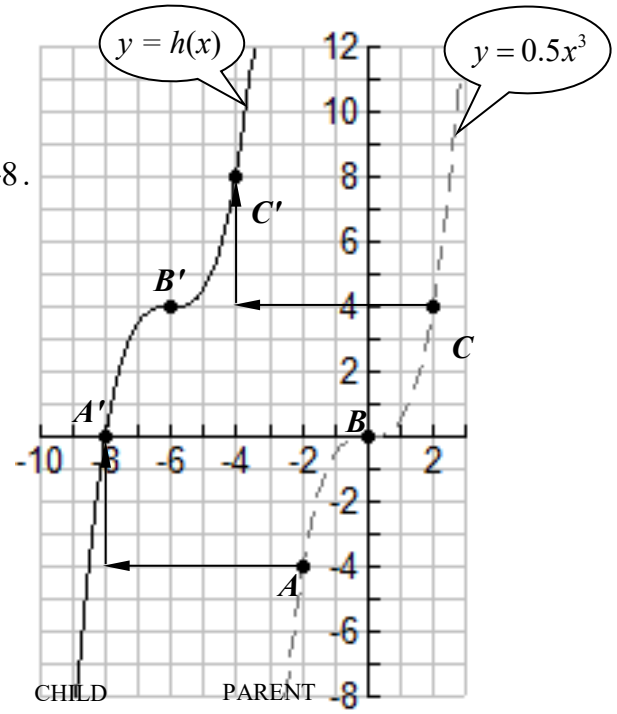
$a =$ ☐ and $b =$ ☐

1. a. Horizontal shift 6 left and vertical shift 4 up.
Notice B' is $(-6, 4)$ and B is $(0, 0)$.

b. $h(x) = 0.5(x + 6)^3 + 4$ (Enter in a grapher to check.)

c. Use the graph. Notice A' to see $h(x)$ crosses the x-axis at -8.
Check with the formula.
If $x = -8$, $h(x) = 0.5(x + 6)^3 + 4$
$= 0.5(-8 + 6)^3 + 4$
$= 0.5(-2)^3 + 4$
$= 0.5(-8) + 4 = 0$.

d. Use the formula. It crosses the y-axis when $x = 0$.
$h(0) = 0.5(0 + 6)^3 + 4 = 112$. You can also use the table.

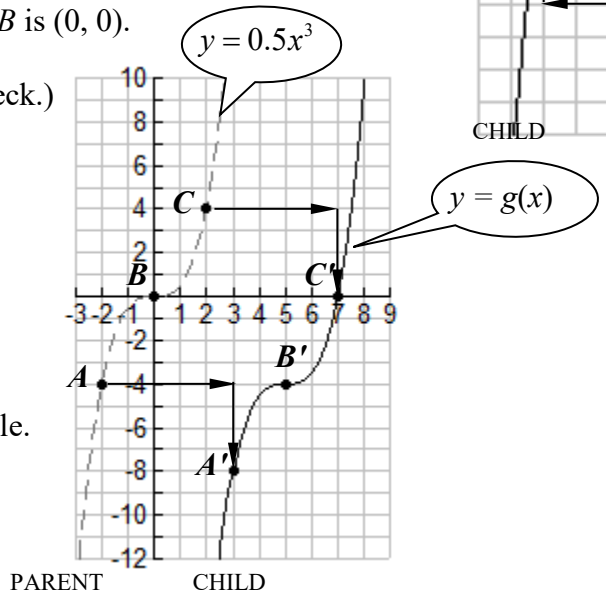


2. a. Horizontal shift 5 right and vertical shift 4 down.
Notice B' is $(5, -4)$ and B is $(0, 0)$.

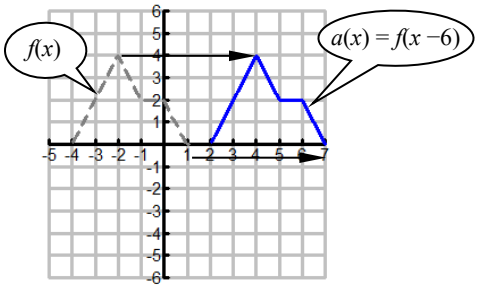
b. $g(x) = 0.5(x - 5)^3 - 4$
(Enter in a grapher to check.)

c. Notice C' to see $g(x)$ crosses the x-axis at 7.

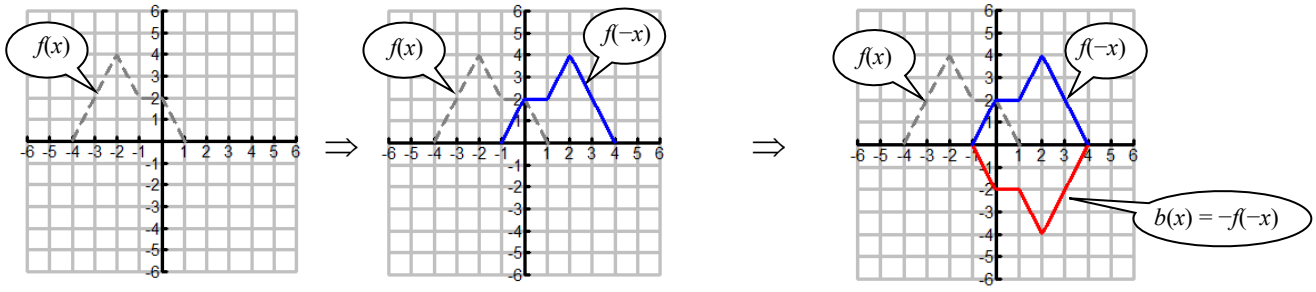
d. Use the formula.
It crosses the y-axis when $x = 0$.
$x = 0$.
$g(0) = 0.5(0 - 5)^3 - 4$
$= -66.5$
You can also use the table.



3. a. The graph of $y = a(x)$ is a horizontal shift of the graph of $y = f(x)$ to the right 6 so $a(x) = f(x - 6)$.



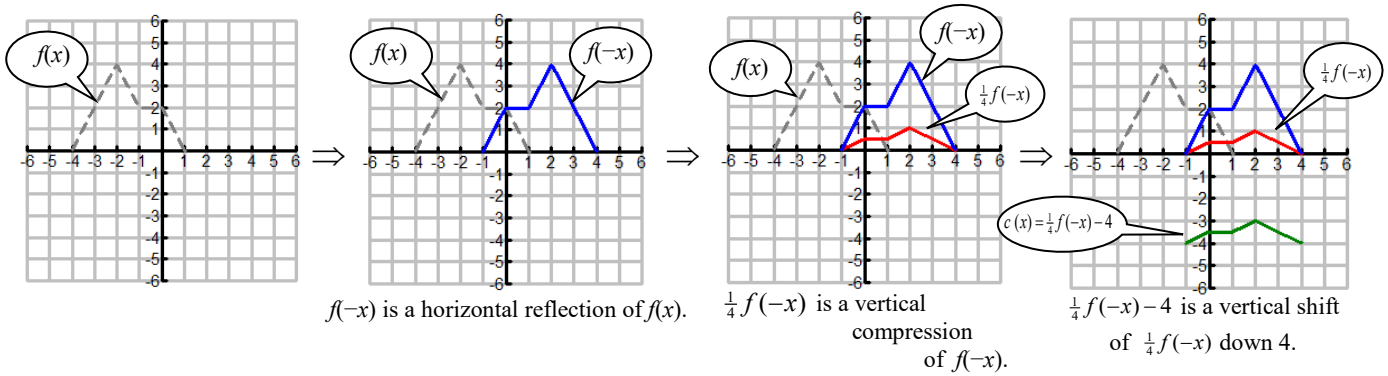
b. The graph of $y = b(x)$ is a horizontal and vertical reflection of the graph of $y = f(x)$ so $b(x) = -f(-x)$.



$f(-x)$ is a horizontal reflection of $f(x)$.

$-f(-x)$ is a vertical reflection of $f(-x)$.

c. The graph of $y = c(x)$ is a horizontal reflection, followed by a vertical compression by a factor of $\frac{1}{4}$, followed by a vertical shift down 4 units, so $c(x) = \frac{1}{4}f(-x) - 4$.

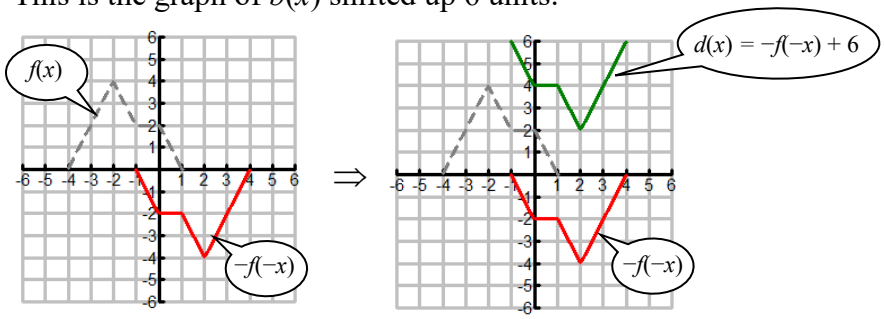


$f(-x)$ is a horizontal reflection of $f(x)$.

$\frac{1}{4}f(-x)$ is a vertical compression of $f(-x)$.

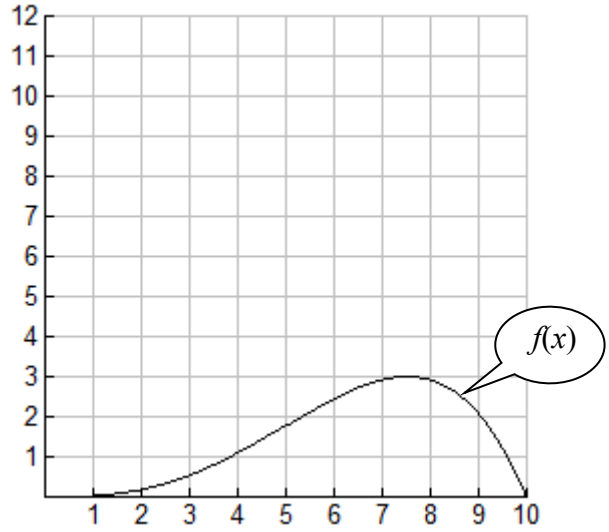
$\frac{1}{4}f(-x) - 4$ is a vertical shift of $\frac{1}{4}f(-x)$ down 4.

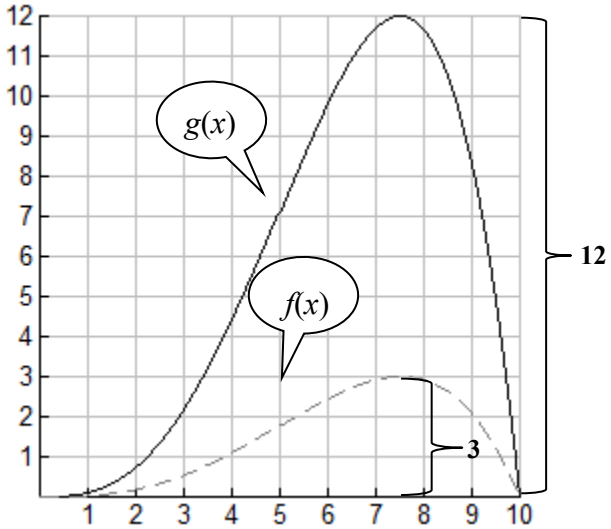
d. The graph of $y = d(x)$ is a horizontal and vertical reflection, followed by a vertical shift up 6. This is the graph of $b(x)$ shifted up 6 units.



4. The graph of $y = f(x)$.is shown. Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$. Find a formula for $g(x)$ in **terms of $f(x)$.**

a.



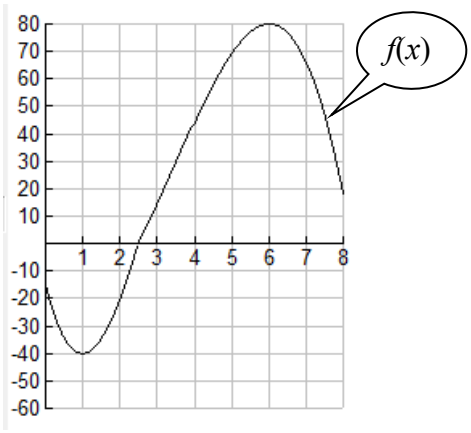


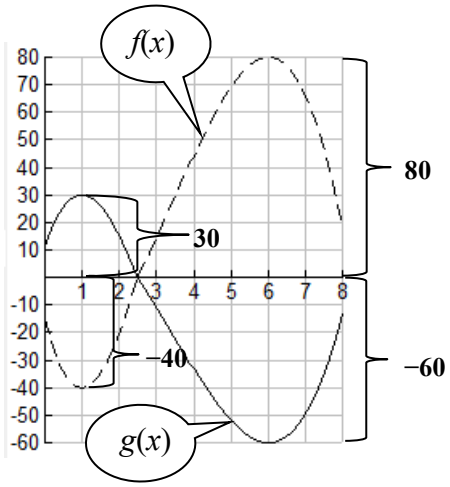
The outputs of $g(x)$ are *larger* than those for $f(x)$ so it is a *vertical stretch*. Compare maximum points.

The graph of $g(x)$ is a vertical stretch of the graph of $f(x)$ by a factor of k, where $3k = 12$.

Thus $k = 4$ and $g(x) = 4f(x)$.

b.





The outputs of $g(x)$ are *smaller* than those for $f(x)$ so it is a *vertical shrink*. Compare maximum points. The graph of $g(x)$ is a vertical compression of the graph of $f(x)$ by a factor of k, where $80k = -60$. You could also compare minimum points: $-40k = 30$. In either case, $k = -0.75$ and $g(x) = -0.75\,f(x)$.

5. Which of these is $\ln\sqrt[3]{x^2}$? Circle one.

A. $3\ln\sqrt{x}$ B. $3\ln x$ C. $x\ln 3$ D. $\frac{2}{3}\ln x$ E. $\frac{3}{2}\ln x$ F. $3\ln x^2$ G. $2\ln x^3$ H. None of these.

$\ln\sqrt[3]{x^2} = \ln x^{2/3} = \frac{2}{3}\ln x$ so Choice D.

6. Solve $e^x = 17.3$

EXACT: $x = \ln 17.3$

APPROXIMATE: $x \approx 2.85$

We have $e^x = 17.3$ Since the base is e, take natural logarithms of both sides.

$\ln e^x = \ln 17.3$ Use the inverse property $\ln e^Q = Q$

$x = \ln 17.3$

Check: If $x \approx 2.85$ and $e^x = 17.3$, then $e^{2.85} \approx 17.3$

7. **a**. $5\ln(3x) = 20$

EXACT: $x = \frac{1}{3}e^4$ or $\dfrac{e^4}{3}$

APPROXIMATE: $x \approx 18.199$

We have $5\ln(3x) = 20$ Divide both sides by 5.

$\ln(3x) = 4$ Make both sides a power of e.

$e^{\ln(3x)} = e^4$ Use inverse property

$3x = e^4$ Divide both sides by 3

$x = \frac{1}{3}e^4$ or $\dfrac{e^4}{3}$

Check: If $x \approx 18.199$ and $5\ln(3x) = 20$, then $5\ln(3\cdot 18.199) \approx 20$

b. $5\log x + 7 = 10$

EXACT: $x = 10^{3/5}$

APPROXIMATE: $x \approx 3.981$

We have $5\log x + 7 = 10$ Subtract 7 from both sides.

$5\log x = 3$ Divide both sides by 5.

$\log x = \frac{3}{5}$ Make both sides a power of 10.

$10^{\log x} = 10^{3/5}$ Use inverse property.

$x = 10^{3/5} \approx 3.981$

Check: If $x \approx 3.9815$ and $5\log x + 7 = 10$, then $\log(3.981) + 7 \approx 10$

8 a. $4u(u-2) = 0$

This is a quadratic equation, but how nice! It is already in factored form.
This can be solved by setting each factor equal to 0 and
applying the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$.

We have $4u = 0$ and $u - 2 = 0$.
To solve $4u = 0$ we just divide both sides by 4 to get $u = 0$. To solve $u - 2 = 0$, add 2 to both sides.
The two solutions are $u = 0, 2$. (This can also be done just by inspection.)

A common error is to multiply these out to get $4u^2 - 8u = 0$.
This is similar to part d. But unfortunately that takes you into the wrong direction.
(Multiplying out is like returning clean clothes out of the dryer and putting them back in the washer. Oops!☺)
Since it is already factored, you do not want to reverse the factorization by distributing.

b. $25u^2 = 4$

This is a quadratic equation, but since it contains only u^2 we can divide by 25 and take square roots.

We have $25u^2 = 4$ Divide both sides by 25.

$u^2 = \frac{4}{25}$ Take square roots of both sides. Remember there are two square roots.

$u = \pm\sqrt{\frac{4}{25}} = \pm\frac{2}{5}$

Alternatively, you can get 0 on one side and factor, and
use the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$

$25u^2 = 4$

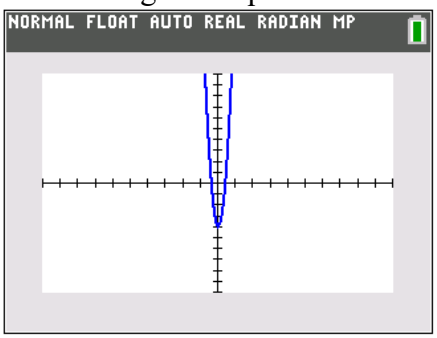
$25u^2 - 4 = 0$ Then set each factor equal to 0.

$(5u+2)(5u-2) = 0$

$5u + 2 = 0$ || $5u - 2 = 0$

$u = -\frac{2}{5}$ || $u = \frac{2}{5}$

A common error is to only report the positive solution and forget there is a negative square root.
A quick sketch of $y = 25x^2 - 4$ confirms there are two zeros.
A sketch may just be done with pencil and paper
using knowledge of transformations.
The graph of $y = 25x^2 - 4$ is a vertical shift
of the graph of $y = 25x^2$ down 4 units.
The graph of $y = 25x^2$ is a vertical stretch
of the graph of $y = x^2$ by a factor of 25.



c. $25u^2 = 4u$
This is a quadratic equation.
Get 0 on one side of the equation so you can apply the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$.
Factor.

We have $25u^2 - 4u = 0$ Factor out the greatest common factor, which here is u.
$u(25u - 4) = 0$ Set each factor equal to 0 using the zero product property.
$u = 0 \;\|\; 25u - 4 = 0$
$u = \frac{4}{25}$
There are two solutions, namely $u = 0, \frac{4}{25}$.

A common error is to divide both sides of $25u^2 = 4u$ by u.
Don't do this. You will lose a solution.

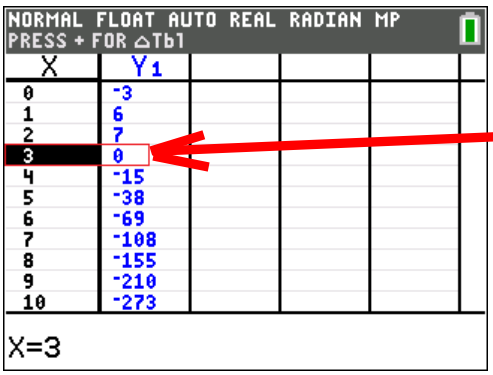
d. $13x - 4x^2 = 0$
This is a quadratic equation similar to part c. Happily 0 is already on one side of the equation. Factor.

We have $13x - 4x^2 = 0$ Factor out the greatest common factor, which here is x.
$x(13 - 4x) = 0$ Set each factor equal to 0 using the zero product property.
$x = 0 \;\|\; 13 - 4x = 0$
$x = \frac{13}{4}$ or 3.25
There are two solutions, namely $u = 0, \frac{13}{4}$.

e. $13x - 4x^2 = 3$
This is a quadratic equation. Get 0 on one side of the equation.

$13x - 4x^2 - 3 = 0$

Enter $Y_1 = 13X - 4X^2 - 3$ into a grapher. A table can quickly unveil if there are any integer zeros.



The table shows that
one zero of $y = 13x - 4x^2 - 3$ is **3**.

Since 3 is a zero, then $(x-3)$ is a factor of the equation.
Let's arrange terms in descending powers of x: $-4x^2+13x-3=0$

Multiply both sides by -1 so that the quadratic term is positive. $4x^2-13x+3=0$
This will make it easier to work with.

$(\_\_x\ -\ \_\_\_)(x-3)=4x^2-13x\ +\ 3\ =0$

LAST TERMS

Ask $(\boxed{?})\cdot(-3)=3$. The product of both of these is **3** so we have –1 and –3.

$(\_\_x\ -\ \_1\_)(x-3)=4x^2-13x\ +\ 3\ =0$

FIRST TERMS

Ask $(\boxed{?})\cdot(x)=4x^2$. The product of both of these is $4x^2$ so we have $4x$ and x.

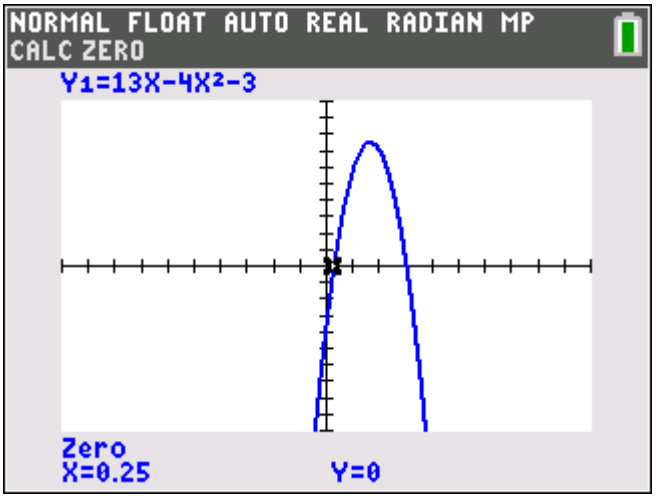
Thus we have $(4x-1)(x-3)=0$. Use the zero product property $A\cdot B=0 \Leftrightarrow A=0$ or $B=0$
Set each factor equal to 0 and solve.

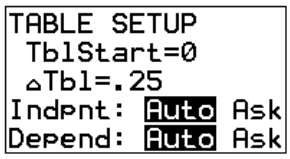
$$(4x-1)\times(x-3)=0$$

$4x-1=0$	$x-3=0$
$x=\frac{1}{4}$	$x=3$

The solutions are $x=\frac{1}{4}, 3$.

You can also find the zeros using the graph of $Y_1=13X-4X^2-3$ or
This can also be confirmed with a table if the increment is set to 0.25.





X	Y1
0	-3
0.25	0
0.5	2.5
0.75	4.5
1	6
1.25	7
1.5	7.5
1.75	7.5
2	7
2.25	6
2.5	4.5
2.75	2.5
3	0

8. f. $2u^2 = u + 1$

Since this is a quadratic equation with three terms, get 0 on one side of the equation.

$2u^2 - u - 1 = 0$

We have a trinomial. One way to solve this equation is to try to factor.

$(\_\_\_ + \_\_\_)(\_\_\_ + \_\_\_\_) = \mathbf{2u^2} - u - 1 = 0$

FIRST TERMS — The product of both of these is $2u^2$ so we have $2u$ and u in the blanks.

$(2u + \_\_\_)(u + \_\_\_\_) = 2u^2 - u - 1 = 0$

LAST TERMS — The product of both of these is -1 so we have 1 and -1 in the blanks.
But which goes where?

The sum of the product of the inner and the product of the outer is the middle term $-u$.
One of u and $2u$ must be negative, and the sum must be $-u$.
This is only possible if $2u$ is negative and u is positive. This means $2u$ is multiplied by -1.

$(2u \quad 1)(u \quad 1) = 2u^2 - u - 1 = 0$

u
$+ -2u$
$-u$

Thus we have $(2u + 1)(u - 1) = 0$. Use the zero product property $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$

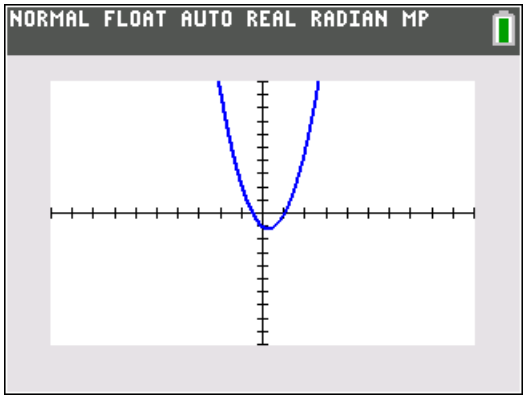
Now set each factor equal to 0 and solve.

$(2u + 1) \times (u - 1) = 0$

$2u + 1 = 0$	$u - 1 = 0$
$u = -\frac{1}{2}$	$u = 1$

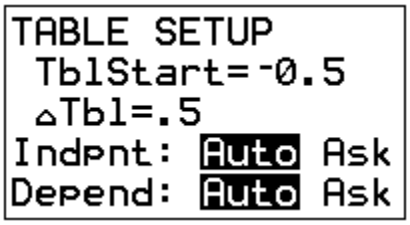
The solutions are $u = -\frac{1}{2}, 1$.

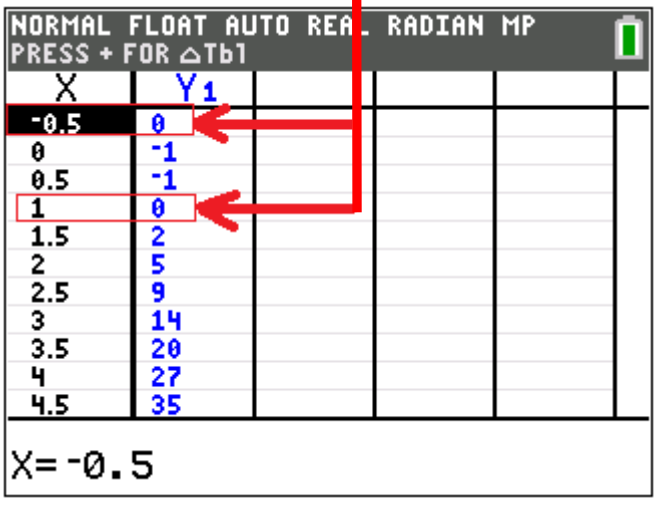
The solution process can be enhanced by using a grapher.
In the Y= Editor, enter $Y_1 = 2X^2 - X - 1$.
The zeros of the graph appear to be 1 and $-\frac{1}{2}$.



The zeros of $y = 2x^2 - x - 1$ are the solutions to $2x^2 - x - 1 = 0$

This can be confirmed with a table:





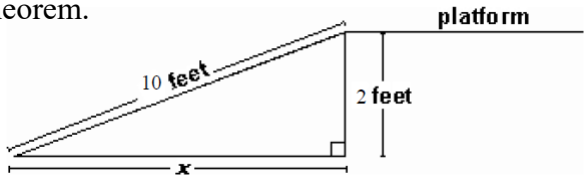
9. Use the Pythagorean Theorem.

$x^2 + 2^2 = 10^2$

$x^2 + 4 = 100$

$x^2 = 96$

$x = \pm\sqrt{96}$



Since x is the length of a side, it is the positive square root $\sqrt{96}$ and Choice C is correct.

10. **a**. Choice B is true.
Linear functions grow by a constant rate, and
exponential functions grow by a constant percent rate.

b. Jonesville: $P = 5000 + 200t$

c. Smithville: $P = 5000(1.02)^t$

11. **a.** The function P is exponential. $P = 200(1.23)^x$.
The function Q is linear. $Q = 400 + 200x$

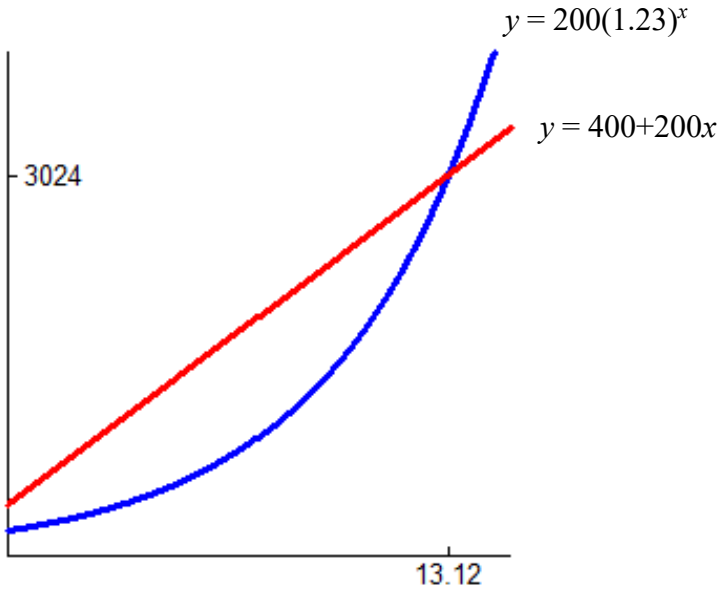
b. $x \approx 13.12$ years

The equation $200(1.23)^x = 400 + 200x$ is not possible to solve algebraically.

Method 1: Using a table, enter the formulas Y1= **200(1.23)^x and** Y2 = **400 + 2x** in Y= and scroll. Eventually set your step size to 0.01

X	Y1	Y2
13.1	3011.5	3020
13.11	3017.8	3022
13.12	3024	3024
13.13	3030.3	3026
13.14	3036.6	3028

Method 2: Using a graph, set a viewing window (aided by the table) and then find the intersection point.

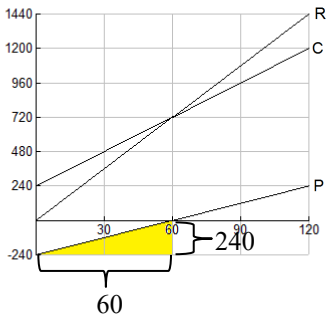


12. a. The x-coordinate of the intersection point of R and C is $x = 60$.

b. The slope of $P(x)$ is $\frac{\Delta y}{\Delta x} = \frac{240}{60} = 4$.

The vertical intercept is $(0, -240)$.

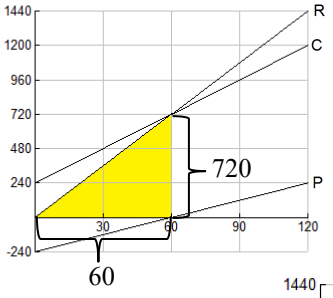
So $P(x) = 4x - 240$.



The slope of $R(x)$ is $\frac{\Delta y}{\Delta x} = \frac{720}{60} = 12$.

The vertical intercept is $(0, 0)$.

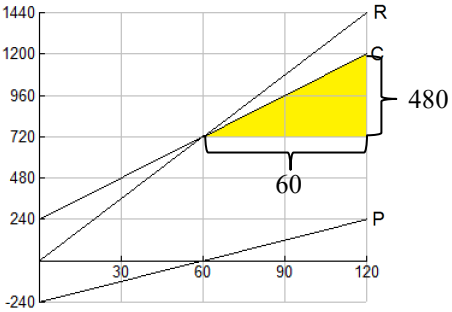
So $R(x) = 12x$.



The slope of $C(x)$ is $\frac{\Delta y}{\Delta x} = \frac{480}{60} = 8$.

The vertical intercept is $(0, 240)$.

So $C(x) = 8x + 240$.



Another method: Since $P = R - C$,
you can also find C once you know P and R.
$P + C = R$ so $4x - 240 + C = 12x$
Add 240 to both sides and subtract $4x$ from both sides.
Thus $C = 8x + 240$.

13. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.

Suppose that you have \$6000 to invest. Which investment yields the greater return over 13 years: 8.07% compounded **continuously** or 8.1% compounded **monthly**?

\$6000 at 8.07% compounded **continuously** for 13 years returns $A = 6000e^{0.0807\times13} \approx \$17{,}130.48$

\$6000 at 8.1% compounded **monthly** for 13 years returns $A = 6000(1+\frac{.081}{12})^{12\times13} \approx \$17{,}136.69$

Choice B. Investing \$6000 at 8.1% compounded **monthly** over 13 years yields the greater return.

14. Use the compound interest formulas $A = P(1+\frac{r}{n})^{nt}$ and $A = Pe^{rt}$ as appropriate.

Suppose that you have \$9000 to invest. Which investment yields the greater return over 19 years: 7.88% compounded **continuously** or 7.9% compounded **monthly**? (Select one)

\$9000 at 7.88% compounded **continuously** for 19 years returns $A = 9000e^{0.0788\times19} \approx \$40,222.42$

\$9000 at 7.9% compounded **monthly** for 19 years returns $A = 9000(1+\frac{.079}{12})^{12\times19} \approx \$40{,}177.43$

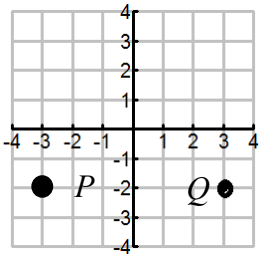
Choice A. Investing \$9000 at 7.88% compounded **continuously** over 19 years yields the greater return.

15. Suppose the point $P(3,-2)$ is a point on the graph of $y = f(x)$

a. Suppose $f(x)$ is **even:**

i. Report the coordinates of another point Q, which corresponds to P. (-3 , -2)

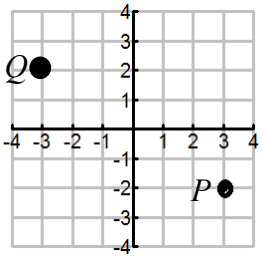
ii. Plot the point Q on the grid provided.



b. Suppose $f(x)$ is **odd**:

i. Report the coordinates of another point Q, which corresponds to P. (-3 , 2)

ii. Plot the point Q on the grid provided.



16. a. Assume the table represents a **linear** function.

i. Complete the box in the first row and the last row.

For a linear function, the difference of consecutive outputs is constant. We see from the table that difference is **−40**.
Therefore the output when $x = 4$ is $45 - 40 = \mathbf{5}$.
Similarly, the output when $x = 0$ is $125 + 40 = \mathbf{165}$.

x	y	
0	**165**	
1	$125 = (\boxed{?}) - 40$	
2	$85 = 125 - 40$	
3	$45 = 85 - 40$	
4	**5**	$= 45 - 40$

ii. Report the **equation** of the line in slope-intercept form: $y =$ **−40** $x +$ **165**

b. Assume the table represents an **exponential** function.

i. Complete the box in the first row and the last row.

For an exponential function, the ratio of consecutive outputs is constant. We see from the table that ratio is **1/4**.
Therefore the output when $x = 4$ is $8 \times \frac{1}{4} = \mathbf{2}$.
Similarly, the output when $x = 0$ is $128 \times 4 = \mathbf{512}$.

x	y	
0	**512**	
1	$128 = (\boxed{?}) \times \frac{1}{4}$	
2	$32 = 128 \times \frac{1}{4}$	
3	$8 = 32 \times \frac{1}{4}$	
4	**2**	$= 8 \times \frac{1}{4}$

ii. If we report the equation of the exponential function in the form $y = ab^x$, then we have

$a =$ **512** and $b =$ $\frac{\mathbf{1}}{\mathbf{4}}$