MA 15400 Student Learning Outcomes

The learning outcomes listed in the Purdue online catalog for MA 15400 are as follows:

1. understand and be able to use trigonometric functions, inverse trigonometric functions, analytic geometry, laws of sines and cosines, vectors and the dot product to solve application problems.
2. learn about conic sections (ellipses, hyperbolas and parabolas) and rational functions.

Additional learning outcomes are listed below.

The Graph of the Height of a Ferris Wheel Car with Time

1. Identify if a graph represents a periodic function.
2. Determine period, amplitude and midline.
3. Use a graph to find and interpret y if given t or vice versa

The Sine Function (and its Sidekick, Cosine)

1. Find the (x, y) coordinates of a point on the circle if given its angle.
2. Find angles between 0° and 360° which have the same sine or cosine of a given angle.
3. Determine in which quadrant an angle lies if given certain conditions.

Radians

1. Interpret the radian measure of the central angle of a circle of radius r as the number of radius lengths, r, that you need to wrap around the rim of the circle on the arc spanned by the angle.
2. Find the location of an angle θ that is a fraction of π, such as $\pm\pi/6$, $\pm\pi/4$, $\pm\pi/3$, $\pm\pi/2$, or multiples of these.
3. Convert an angle from degrees to radians and vice versa.
4. If given two of the arc length s, radius r, or an angle θ, find the third by using the relationship between them.

Special Right Triangles

1. Use proportional reasoning to find the exact values of the 3 sides of an isosceles right 45°- 45°-90° triangle if given one of the sides.
2. Use proportional reasoning to find the exact values of the 3 sides of a 30°- 60°-90° right triangle if given one of the sides.

Trigonometric Values of Special Angles

1. Find exact values of sine and cosine for multiples of 30°, 45°, and 60° without a calculator.
2. Find exact values of sine and cosine for multiples of $\pi/6$, $\pi/4$, or $\pi/3$ without a calculator
3. Given a sketch of an angle θ that is a multiple of 30°, 45°, and 60° or $\pi/6$, $\pi/4$, or $\pi/3$, report the value of θ as well as its sine and cosine.

Graphs of Sine and Cosine and Outside Changes to the Function

1. Report the main characteristics (period, amplitude, midline, domain, range, odd/even symmetry, when it is positive, negative, increasing, decreasing, if it starts at or above the midline) of the graph of $y = \sin\theta$, and $y = \cos\theta$.
2. Relate the graph of the sine/cosine function to the unit circle as the x-coordinate (cosine) or the y-coordinate (sine) of the point.
3. For $y = A\sin(x) + k$ or $y = A\cos(x) + k$, identify the period, amplitude, and midline.

Inside multiplicative change

1. Report the period of the graph of $y = A\sin Bx + k$.
2. If you have found the period, p, of $y = \sin Bx$, check your value is correct by verifying that $B\cdot\pi$.= 2π.
3. Given a graph, report the midline, amplitude and period and use them to find the formula $y = A\sin Bx + k$ or $y = A\cos Bx + k$.

Inside Additive Change - Phase Shift

1. Write the formula of function $y = f(x)$ which has been shifted h units to the right as $y = f(x - h)$.
2. Write the formula of function $y = f(x)$ which has been shifted h units to the left as $y = f(x + h)$.
3. Given a graph and a model choice (regular or upside down sine or cosine) report the phase shift φ and the horizontal shift h.
4. Given a graph with a nonzero phase shift φ, find a formula $y = A\sin(Bx-\varphi)$ or $y = A\cos(Bx-\varphi)$.
5. Sketch the graph of a function if given its formula $y = A\sin(B(x-h)) + k$ or $y = A\sin(Bx-\varphi) + k$.
6. Given a graph with a nonzero phase shift φ or horizontal shift h, find a formula $y = A\sin(B(x-h)) + k$ or $y = A\sin(Bx-\varphi) + k$.
7. If you have found the phase shift, φ, of the graph of $y = \sin(Bx-\varphi)$, check that your value is correct by multiplying out $y = \sin(B(x-h))$ and verifying that $B\cdot h = \varphi$, where h is the horizontal shift.

The Tangent Function

1. Report the main characteristics (period, domain, range, symmetry, value at $\pi/4$, when it is positive, negative, zero, undefined. increasing) of the graph of $y = \tan\theta$.
2. Relate the graph of $y = \tan\theta$ as the slope y/x of the terminal side of θ that corresponds to the point (x, y) on the unit circle.
3. Given the graph of $y = A\tan Bx$ and its intercepts and vertical asymptotes, find A and B.
4. Given the formula of $y = A\tan Bx$, report its intercepts and vertical asymptotes (exact). Solve $A\tan Bx = A$.

The Reciprocal Functions

1. Find exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$ if given the angle θ as a multiple of 30°, 45°, and 60° (or $\pi/6$, $\pi/4$, or $\pi/3$).
2. Find exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$ if given the value of one of these trig functions using the Pythagorean Theorem.
3. If given the location of θ use the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$ to find the value of $\sin\theta$ (and its sign) if given $\cos\theta$ or vice versa.

Inverse Trig Functions

1. Solve simple trig equations over a requested interval; for example $[0, 2\pi)$ or [0,360°) or other intervals, providing **a.** exact values of angles measured in radians (when given special angles which are multiples of 30°, 45°, 60° or their radian equivalents.) **b.** decimal approximations using the inverse trig functions.
2. Distinguish the meaning of the notation $\sin^{-1}x$, $\arcsin x$, $\sin^2 x$, $\sin x^2$, $\csc x$, etc.

Right Triangle Trigonometry

1. If given any two of the side lengths of a right triangle, find the remaining parts.
2. If given a side length and an angle of a right triangle, find the remaining parts.
3. Solve application problems which involve right triangles.
4. Interpret $\tan\theta$ as the slope of the angle of inclination.

The Law of Sines

1. Solve for sides and angles of a triangle using the Law of Sines.
2. Determine when you can use right triangle trigonometry (SOHCAHTOA) paired with Pythagoras and when you can use Law of Sines.
3. Solve application problems which involve the Law of Sines.

The Law of Cosines.

1. Solve for sides and angles of a triangle using the Law of Cosines.
2. Determine when you can use the Law of Cosines and when you can use the Law of Sines.
3. Solve application problems which involve the Law of Cosines

The Ambiguous Case of the Law of Sines

1. Solve problem situations involving the ambiguous case of the Law of Sines.
2. If given a situation which requires finding an angle with the Law of Sines, determine if one, two, or no triangles exist.

Solving Trig Equations Graphically

1. Solve a trig equation which involves sine or cosine graphically.
2. Use symmetry and the period of the function to find all solutions to a trigonometric equation.

Trig Identities

1. Rewrite trigonometric expressions.
2. Build fractional fluency.

The Pythagorean Identities and the Double Angle Identities

1. Use the double angle trig identities for $\sin 2\theta$ and $\cos 2\theta$ to rewrite trig expressions.
2. Use the Pythagorean Identities $\cos^2\theta + \sin^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $\cot^2\theta + 1 = \csc^2\theta$.
3. Write trigonometric expressions into algebraic expressions.
4. Continue to build fractional fluency.

Solving Trig Equations
1. Use the trig identities to solve trig equations algebraically using the unit circle.
2. Use a grapher to determine the solutions to a trig equation.

Using Sum and Difference Identities
1. Recognize the sum and difference identity for sine and use it to simplify expressions.
2. Recognize the sum and difference identity for cosine and use it to simplify expressions.
3. Recognize the sum and difference identity for tangent and use it to simplify expressions.

Polar Coordinates
1. Convert coordinates from polar to rectangular.
2. Convert coordinates from rectangular to polar.
3. Sketch graphs of polar equations.

Function Composition and Decomposition
1. If given the formulas for $f(x)$ and $g(x)$, find the formula for $f(g(x))$, i.e., f composed with g.
2. If given the formula for $h(x) = f(g(x))$, find possible formulas for $f(x)$ and $g(x)$, i.e., decompose h.
3. If given the formula for $h(x) = f(g(x))$ and $f(x)$, find the formula for $g(x)$.
4. If given the formula for $h(x) = f(g(x))$ and $g(x)$, find the formula for $f(x)$.

Inverse Functions
1. Determine if a function is invertible.
2. Find the formula for the inverse function.

Graphical Representation of Vectors
1. Given the sketch of a vector, use a grid to sketch the scalar multiplication or the opposite of the vector.
2. Given the sketch of vectors, use a grid and the "head to tail" method to sketch the result of vector addition or subtraction.

Component Form of Vectors
1. Resolve a vector into horizontal and vertical components given its magnitude and direction.
2. Given the horizontal and vertical components of a vector, report its magnitude and direction.
3. Perform vector arithmetic.

Applications of Vectors
1. Perform vector arithmetic for a vector of n-dimensions, $n > 2$.
2. Use vectors to combine forces.

Introduction to Sequences
1. If given some terms of a sequence, find the next term.
2. Classify a sequence as arithmetic, geometric, or neither.

Finding the Formula for the nth Term of a Sequence
1. If given some terms of a sequence, find the next term.
2. Classify a sequence as arithmetic, geometric, or neither.
3. If given a plot of the terms (n, a_n) of an arithmetic sequence, find the equation of the linear function $y = mx + b$ which passes through the points.
4. Relate the slope m to the common difference $d = a_2 - a_1$ of the sequence. Relate b to what would be* the term a_0 prior to the first term, i.e., $a_0 + d = a_1$.
5. If given a plot of the terms (n, a_n) of a geometric sequence, find the equation of the exponential function $y = ab^x$ which passes through the points. Relate the growth factor b to the common ratio $r = \frac{a_2}{a_1}$ of the sequence. Relate a to what would be the term a_0 prior to the first term, i.e., $a_0 \cdot r = a_1$.

Introduction to Series and Sigma Notation
1. Find the sum of an arithmetic series using an efficient strategy (like what Gauss did).
2. If given a series in expanded notation, write it in sigma notation or vice versa.

The Sum of a Finite Geometric Series
1. Determine the sum of a finite geometric series using the **formula**.
2. Solve application problems involving finite geometric series.

The Sum of an Infinite Geometric Series (for 0 < r < 1)
1. Determine the sum of a infinite geometric series using the **formula**.
2. Solve application problems involving infinite geometric series.

Parametric Equations
1. Given graphs of two parametric equations $x = f(t)$ and $y = g(t)$, identify the path of the object (y vs. x).
2. Given equations of two parametric equations $x = f(t)$ and $y = g(t)$, eliminate the parameter to write y in terms of x.

How to Use a Grapher to Sketch Parametric Equations
1. Change the mode on a grapher from function mode to parametric mode.
2. Use the window settings on a grapher to produce a sketch of a set of parametric equations.

Circles
1. Given the implicitly defined formula of a circle, report its center and radius
2. Given the graph of a circle, report its center, radius, and formula in implicit and parametric form.

Parabolas, Ellipses and Hyperbolas (neither rotated)
*RUN = horizontal distance from the center to the farthest point on the graph.
*RISE = Vertical distance from the center to the farthest point on the graph.
1. Given the implicitly defined formula of an ellipse, report its center, vertices, domain, range, RUN*, RISE*, and length of the major and minor axes. Sketch its graph.
2. Given the graph of an ellipse, report its center, vertices, RUN, RISE, and length of the major and minor axes.
3. Given the graph of an ellipse, write a formula in implicit and parametric form.
4. Given the implicitly defined formula of a hyperbola, report its center, vertices, RUN, and RISE.Sketch its graph.
5. Given the implicitly defined formula of a hyperbola, report its formula in parametric form.
6. Given the graph of a hyperbola, report its center, vertices, RUN, RISE, and slopes and equations of the asymptotes.
7. Given the graph of a hyperbola, write a formula in implicit and parametric form.
8. Report the foci of a parabola, ellipse, and hyperbola if given its formula in implicit form.

Complex Numbers
1. Plot a complex number on the complex plane if given the number in rectangular $x + yi$ or polar form $r(\cos\theta + i\sin\theta) = re^{i\theta}$.
2. Convert a complex number from rectangular form $x + yi$ to polar form $r(\cos\theta + i\sin\theta) = re^{i\theta}$ or vice versa.
3. Perform arithmetic with complex numbers and express the answer in rectangular form $x + yi$.
4. Use Euler's Formula $r(\cos\theta + i\sin\theta) = re^{i\theta}$ to deduce De Moivre's Theorem $(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$.