KEY to Practice Questions for MA 15300 Test 3 (Sections 2.4, 6.1, 6.2, 3.1, 3.2, and 11.1-11.3)

Open the bookmark panel by selecting the Bookmarks icon along the side margin to easier navigation.

1) The graph of $y = 0.5x^3$ is shown (dashed), along with the graph of $h(x)$ on the set of axes in Figure 1.

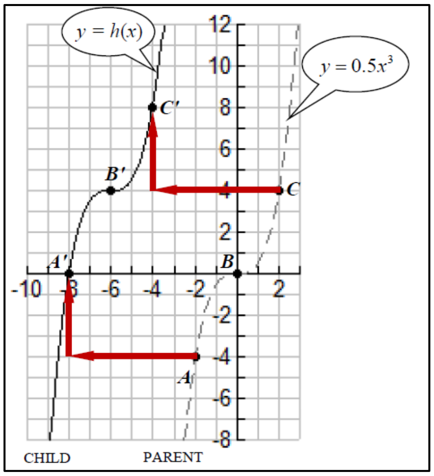


Figure 1: Translation of $y = 0.5x^3$ to $y = h(x)$

a) **Horizontal shift 6 left** and **vertical shift 4 up**.
Notice B' is $(-6, 4)$ and B is $(0, 0)$.

b) $h(x) = \mathbf{0.5(x + 6)^3 + 4}$ (Enter in a grapher to check.)

c) Use the graph. Notice A' to see $h(x)$ crosses the x-axis at **−8**.
Check with the formula.
If $x = -8$, $h(x) = 0.5(x + 6)^3 + 4$
$= 0.5(-8 + 6)^3 + 4$
$= 0.5(-2)^3 + 4$
$= 0.5(-8) + 4 = 0$.
You can also use the table to check.

d) Use the formula. It crosses the y-axis when x = 0.
$h(0) = 0.5(0 + 6)^3 + 4 =$ **112**. You can also use the table.

2) The graph of $y = 0.5x^3$ is shown (dashed), along with the graph of $g(x)$ on the set of axes in Figure 2.

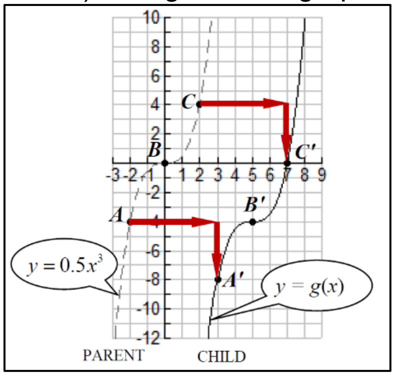


Figure 2: Translation of $y = 0.5x^3$ to $y = g(x)$

a) **Horizontal shift 5 right** and **vertical shift 4 down**.
Notice B' is $(5, -4)$ and B is $(0, 0)$.

b) $g(x) = \mathbf{0.5(x - 5)^3 - 4}$ (Enter in a grapher to check.)

c) Use the graph. Notice *C'* to see $g(x)$ crosses the x-axis at **7**.

Check with the formula.

If $x = 7$, $g(x) = 0.5(x - 5)^3 - 4$

$= 0.5(7 + 5)^3 - 4$

$= 0.5(2)^3 - 4$

$= 0.5 \cdot (8) - 4 = 0.$

You can also use the table to check.

d) Use the formula. It crosses the y-axis when $x = 0$.
$g(0) = 0.5(0 - 5)^3 - 4 =$ **−66.5**. You can also use the table.

3) Suppose $y = f(x)$ is given by the graph below.

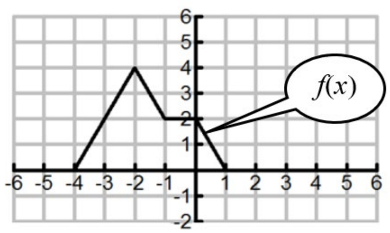


Figure 3: Graph of $y = f(x)$

Describe each transformation and write a formula for each function in terms of $f(x)$.

a) The graph of $a(x)$ is a horizontal shift of the graph of $y = f(x)$ to the right 6 so $a(x) = \mathbf{f(x - 6)}$.

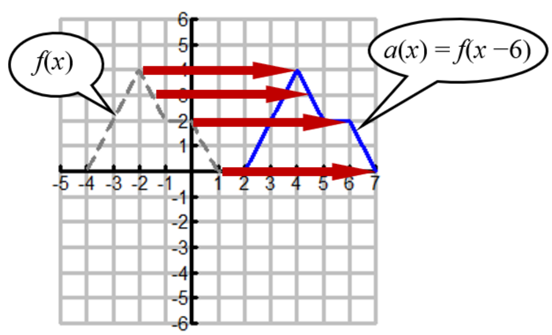
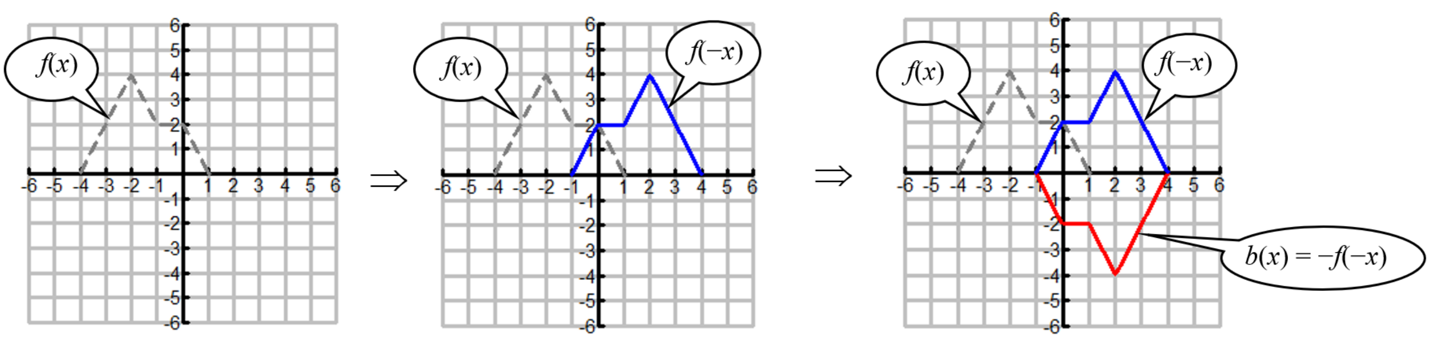


Figure 3a: Graph of $y = f(x)$ and $y = a(x)$.

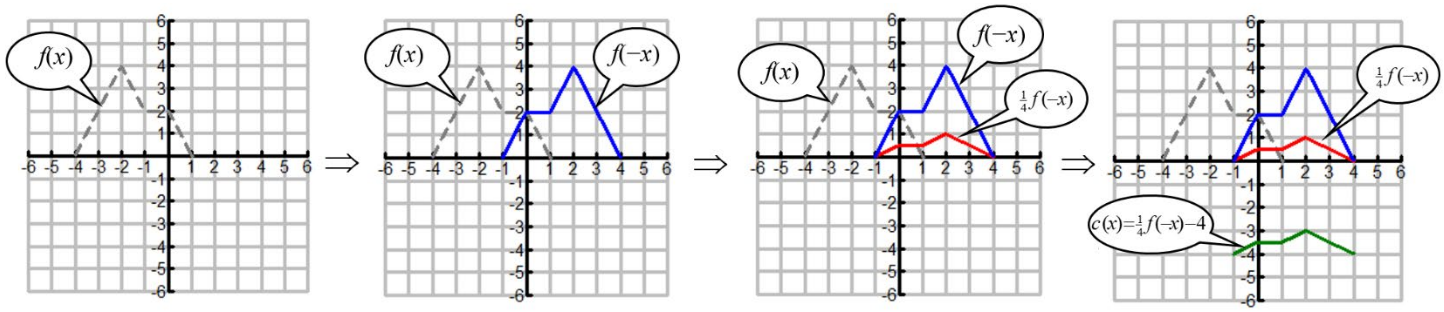
b) The graph of $y = b(x)$ is a horizontal and vertical reflection of the graph of $y = f(x)$ so $b(x) = \mathbf{-f(-x)}$.



$f(-x)$ is a horizontal reflection of $f(x)$. $-f(-x)$ is a vertical reflection of $f(-x)$.

Figure 3b: Graphs of $y = f(x)$ and $y = f(-x)$ and $y = b(x) = \mathbf{-f(-x)}$.

c) The graph of $y = c(x)$ is a horizontal reflection, followed by a vertical compression by a factor of $\frac{1}{4}$, followed by a vertical shift down 4 units, so $c(x) = \mathbf{\frac{1}{4} f(-x) - 4}$.



$y = f(-x)$ is a horizontal reflection of the graph of $f(x)$.

$y = \frac{1}{4} f(-x)$ is a vertical shrink of the graph of $f(-x)$ by a factor of $\frac{1}{4}$.

$y = \frac{1}{4} f(-x) - 4$ is a vertical shift of $\frac{1}{4} f(-x)$ down 4.

Figure 3c: Graphs of $y = f(x)$ and $y = f(-x)$ and $y = \frac{1}{4}f(-x)$ and $y = c(x) = \mathbf{\frac{1}{4} f(-x) - 4}$.

4) Suppose the point $P(3,–2)$ is a point on the graph of $y = f(x)$.

a) Suppose $f(x)$ is **even:**

i) Report the coordinates of another point Q, which corresponds to $P(3,–2)$.
Outputs of opposites are the same, so we have Q (**−3**, **−2**).

ii) Plot the point Q on the grid provided.

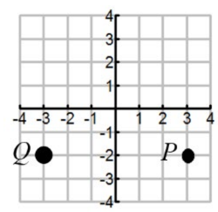


Figure 4a

b) Suppose $f(x)$ is **odd:**

i) Report the coordinates of another point R, which corresponds to $P(3,–2)$.
Outputs of opposites are opposite, so we have R (**−3**, **2**).

ii) Plot the point R on the grid provided.

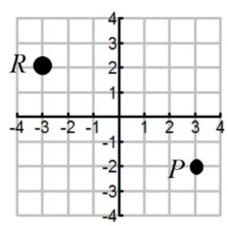


Figure 4b

5) A ballet dancer jumps in the air. The height, $h(t)$, in feet, of the dancer at time, t in seconds since the start of the jump, is given by $h(t) = -16t^2 + 12t$. No work need be shown. **Do not round off any calculations.**

a) To factor $h(t) = -16t^2 + 12t$, remove a greatest common factor of $-4t$: $h(t) = -16t^2 + 12t = \mathbf{-4t(4t - 3)}$.
Alternatively: $h(t) = \mathbf{4t(-4t + 3)}$ is also correct. So also is $h(t) = \mathbf{-16t(t - 0.75)}$ or $h(t) = \mathbf{16t(-t + 0.75)}$.

b) To find the zeros of the function, set each factor equal to 0. Thus the zeros are $t = \mathbf{0}$ and $t = \mathbf{0.75}$.

c) The vertex of the function can be found on the axis of symmetry. First, plot the zeros. The vertex is midway between them. The x-coordinate of the vertex is $\frac{1}{2} \times 0.75$ = 0.375. Find the y-coordinate of the vertex by substituting t = 0.375 in the formula or use a table with TblStart = 0 and ∆Tbl = 0.375. We have y = 2.25. So the vertex is (**0.375, 2.25**).

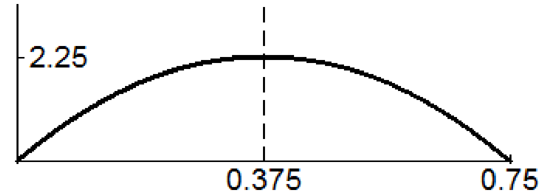


Figure 5c

d) The equation of the axis of symmetry is ***x* = 0.375**. The equation ***t* = 0.375** is also correct. Reporting simply 0.375 is not correct since the axis of symmetry is an equation of a vertical line.
e) How much time in seconds is the dancer in the air? **0.75 sec**
f) What is the maximum height of the jump? **2.25 feet**
g) When does the maximum height of the jump occur? **0.375 sec**
h) To write the formula in vertex form, use the fact that the parabola is a translation of $y = at^2$. We are given the formula in standard form $y = at^2 + bt + c = -16t^2 + 12t$ so we know $a = -16$. If we shift $y = -16t^2$ to the right 0.375 and up 2.25, as shown in Figure 5h, we have the translated function $y = -16(t - 0.375)^2 + 2.25$. We can check with a grapher that this produces the same graph and table as $h(t) = -16t^2 + 12t$.

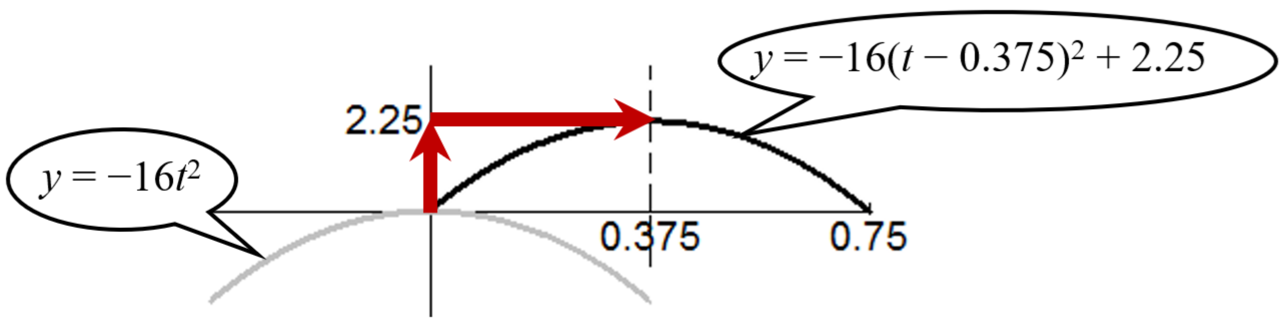


Figure 5h

6) Write formulas for the parabolas You may use vertex form, factored form, or standard form, whichever is most efficient. **SHOW ALL WORK.**

a)

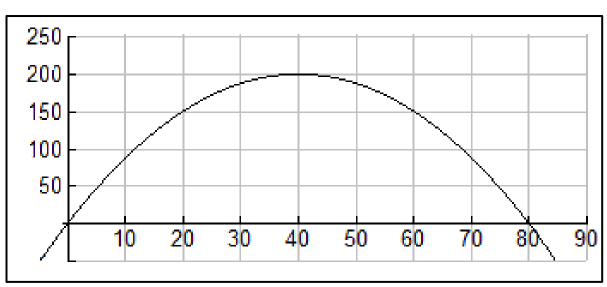


Figure 6a: Parabola for part 6a.

You can use factored form since zeros are at 0 and 80 or vertex form since the vertex is at (40, 200).

Method 1: Use factored form: $y = a(x)(x - 80)$

Substitute a point $x = 40$, $y = 200$ ⇒

$$\begin{aligned} y &= a(x)(x-80) \\ 200 &= a(40)(40-80) \\ 200 &= -1600a \quad \text{Divide both sides by } -1600. \\ a &= \frac{200}{-1600} \\ a &= -0.125 \end{aligned}$$

The formula in factored form is y = $\mathbf{-0.125(x)(x-80)}$.

Method 2: Alternatively, we could use vertex form: $y = a(x - 40)^2 + 200$

Substitute a point $x = 80$, $y = 0$ ⇒

$$\begin{aligned} y &= a(x-40)^2 + 200 \\ 0 &= a(80-40)^2 + 200 \\ 0 &= 1600a + 200 \\ -200 &= 1600a \quad \text{Divide both sides by 1600.} \\ a &= \frac{-200}{1600} \\ a &= -0.125 \end{aligned}$$

The formula in vertex form is y = $\mathbf{-0.125(x-40)^2 + 200}$.

These two formulas are equivalent and either one is correct.

b)

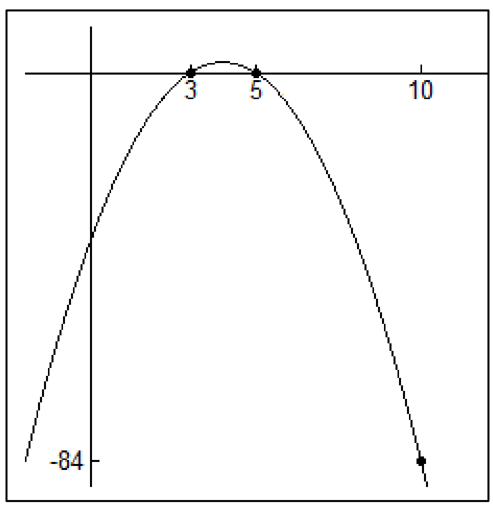


Figure 6b: Parabola for part 6b.

We can only use factored form $y = a(x - 3)(x - 5)$ since we are not given the vertex.

Substitute a point $x = 10$, $y = -84$ ⇒

$$\begin{aligned} y &= a(x-3)(x-5) \\ -84 &= a(10-3)(10-5) \\ -84 &= a(7)(5) \\ -84 &= 35a \quad \text{Divide both sides by 35.} \\ a &= \frac{35}{-84} \\ a &= -2.4 \end{aligned}$$

The formula in factored form is y = $\mathbf{-2.4(x-3)(x-5)}$.

c)

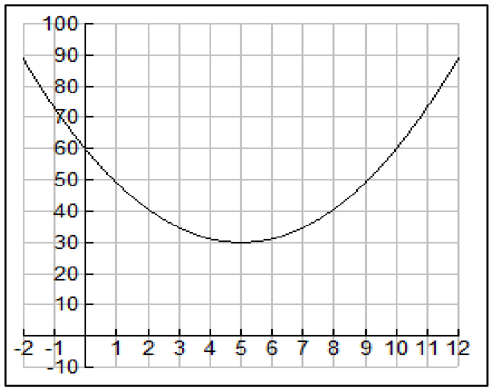


Figure 6c: Parabola for part 6c

We can only use vertex form $y = a(x - 5)^2 + 30$ since there are no zeros.
We can choose a point (0, 60) or (10,60), or others.
Let's substitute $x = 0$, $y = 60$ ⇒

$$
\begin{aligned}
y &= a(x-5)^2 + 30 \\
60 &= a(0-5)^2 + 30 \\
60 &= 25a + 30 && \text{Subtract 30 from both sides.} \\
30 &= 25a && \text{Divide both sides by 25.} \\
a &= \frac{30}{25} \\
a &= 1.2
\end{aligned}
$$

The formula in vertex form is $y = \mathbf{1.2(x - 5)^2 + 30}$.

7) The graph of $y = f(x)$ is shown. It is not a parabola.
Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$.

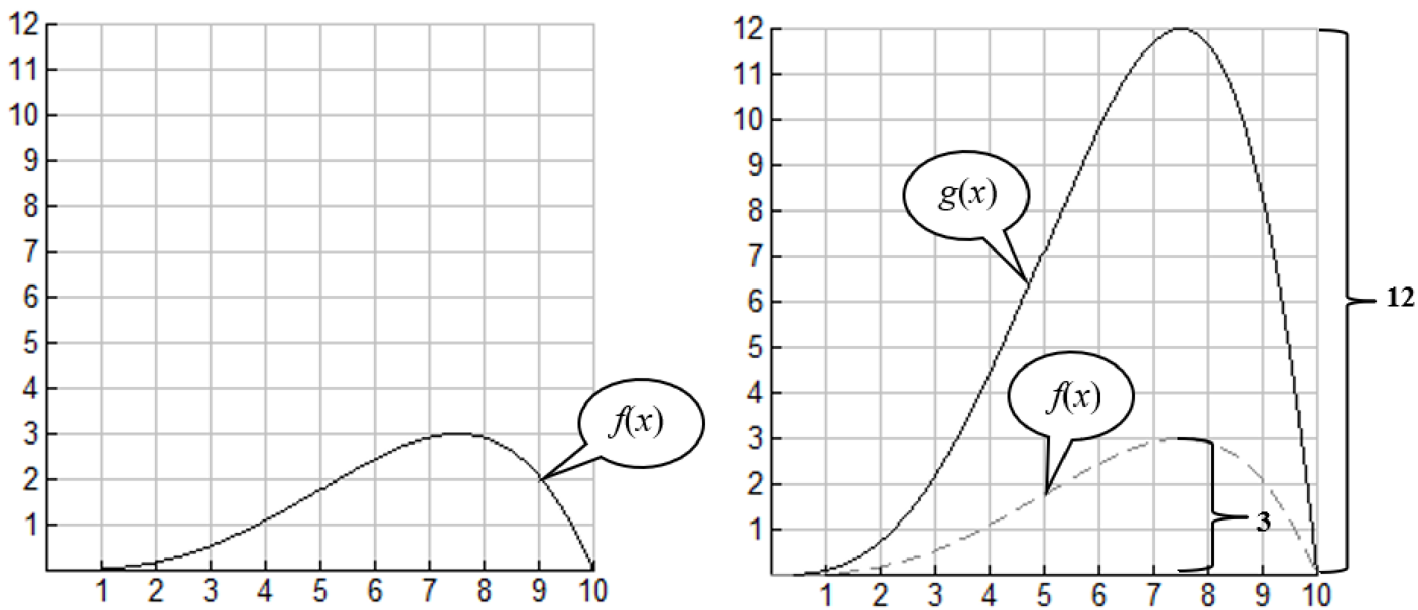
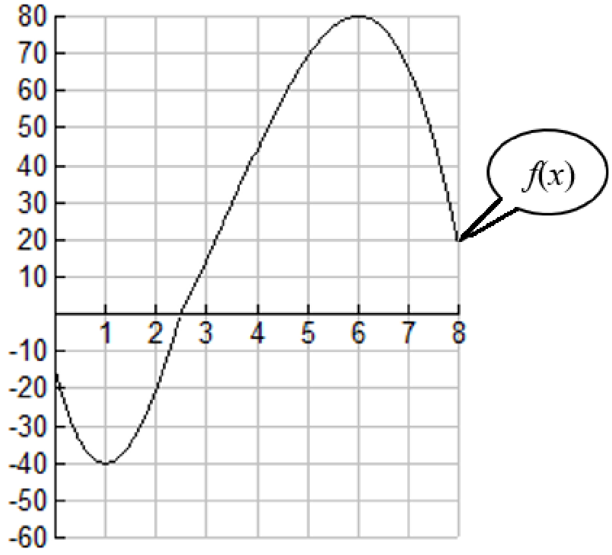


Figure 7: Graph of $y = f(x)$ and $y = g(x)$.

The outputs of $g(x)$ are *larger* than those for $f(x)$ so it is a *vertical stretch*. Compare maximum points.
The graph of $g(x)$ is a vertical stretch of the graph of $f(x)$ by a factor of k, where $3k = 12$.
Thus $k = 4$ and $g(x) = \mathbf{4f(x)}$.

8) The graph of $y = f(x)$ is shown. Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$.



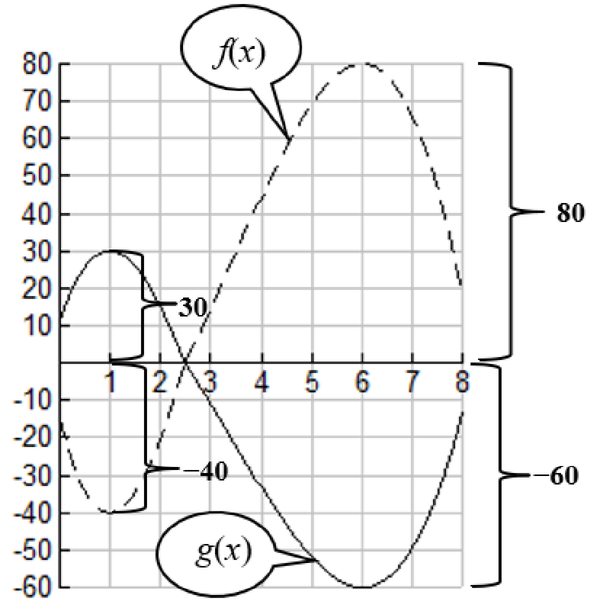


Figure 8b: Graph of $y = f(x)$ and $y = g(\text{x})$.

The outputs of $g(x)$ are *smaller* than those for $f(x)$ so it is a *vertical shrink*. Compare maximum points. The graph of $g(x)$ is a vertical compression of the graph of $f(x)$ by a factor of k, where $80k = -60$. You could also compare minimum points: $-40k = 30$. In either case, $k = -0.75$ and $g(x) =$ **$-0.75\, f(x)$.**

9) The graphs below are power functions of the form $y = kx^p$.

a)

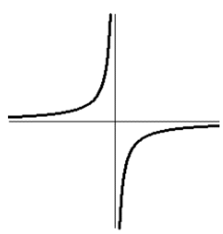


Figure 9a

i) The leading coefficient, k, is **negative**.
ii) The power, p, is **odd** (like ±1, ±3,…).
iii) The symmetry of the graph is **odd**.

b)

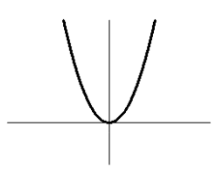


Figure 9b

i) The leading coefficient, k, is **positive**.
ii) The power, p, is **even** (like ±2, ±4,…).
iii) The symmetry of the graph is **even**.

c)

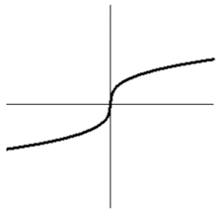


Figure 9c

i) The leading coefficient, *k*, is **positive**.
ii) The power *p* is **fractional** (like $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{5}, \ldots$).
iii) The symmetry of the graph is **odd**.

d)

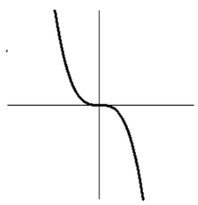


Figure 9d

i) The leading coefficient, *k*, is **negative**.
ii) The power, *p*, is **odd** (like ±1, ±3,…).
iii) The symmetry of the graph is **odd**.

e)

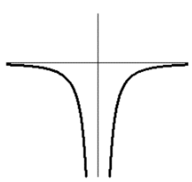


Figure 9e

i) The leading coefficient, *k*, is **negative**.
ii) The power *p* is **even** (like ±2, ±4,…).
iii) The symmetry of the graph is **even**.

f)

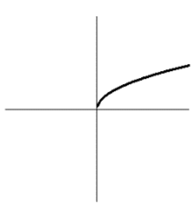


Figure 9f

i) The leading coefficient, *k*, is **positive**.
ii) The power *p* is **fractional** (like $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{5}, \ldots$).
iii) The symmetry of the graph is **neither even nor odd**.

10) Find the formula for the power function $y = kx^p$ given by each table. Show work.

a) Table 10a

x	***y***
1	2
16	128

Substitute x = 1, y = 2 in the formula $y = kx^p$.

x = 1, y = 2 ⇒ $y = kx^p$

$2 = k(1)^p$

Since $(1)^p = 1$ for any p, we have $2 = k$.

Now substitute x = 16, y = 128 in the formula $y = 2x^p$.

x = 16, y = 128 ⇒ $y = 2x^p$

$128 = 2(16)^p$ Divide both sides by 2.

$64 = (16)^p$ Take common or natural logs of both sides.

$\log 64 = \log(16)^p$ Use a property of logs.

$\log 64 = p\log(16)$ Divide both sides by log 16.

$p = \frac{\log 64}{\log 16} = 1.5$

The formula for the power function $y = \mathbf{2x^{1.5}}$. Check with a grapher that your table matches.

b) Table 10b

x	***y***
81	900
625	1500

Substitute x = 81, y = 900 in the formula $y = kx^p$.

x = 81, y = 900 ⇒ $900 = k{\cdot}81^p$ (Equation 1)

Substitute x = 625, y = 1500 in the formula $y = kx^p$.

x = 625, y = 1500 ⇒ $1500 = k{\cdot}625^p$ (Equation 2)

Divide Equation 1 by Equation 2 to eliminate k.

$$\frac{900}{1500} = \frac{k{\cdot}81^p}{k{\cdot}625^p}$$

$$\frac{900}{1500} = \frac{81^p}{625^p}$$ Use the cancellation property $\frac{k}{k} = 1$.

$$\frac{900}{1500} = \left(\frac{81}{625}\right)^p$$ Use a property of exponents.

$$0.6 = \left(\frac{81}{625}\right)^p$$

Solve for p by taking logarithms of both sides (common or natural).

$$\log 0.6 = \log\left(\frac{81}{625}\right)^p$$

$$log\ 0.6 \quad = \quad plog\left(\frac{81}{625}\right)$$ Divide both sides by log 16.

$$p \quad = \frac{log\ 0.6}{log\ (81/625)} = 0.25$$

Now substitute x = 81, y = 900 in the formula $y = kx^{0.25}$. (You could also use x = 625, y = 1500)

x = 81, y = 900 ⇒ $y = kx^{0.25}$

$900 = k(81)^{0.25}$

$900 = 3k$ Divide both sides by 3.

$p = \frac{log\ 64}{log\ 16} = 1.5$

The formula for the power function $y = \mathbf{30x^{0.25}}$. Check with a grapher that your table matches.

11) Consider the polynomial $f(x) = 80 + 70x - 30x^3 - 5x^7$.

a) The leading term kx^p is $\mathbf{-5x^7}$.

b) The leading coefficient k is **–5**.

c) The degree p of $f(x)$ is **7**.

d) The long run behavior of $f(x)$ is "**up down**" or, resembling the graph that looks like .

12) $g(x) = -20(x-50)^4(x+200)^2 .= -20(x^4$ + terms of lower degree) (x^2 + terms of lower degree)^2

$= -20x^6$ + terms of lower degree.

a) The leading term kx^p is $\mathbf{-20x^6}$**.**

b) The leading coefficient k is **–20**.

c) The degree p of $g(x)$ is **6**.

d) Report the long run behavior of $f(x)$ is "**down down**" or .

13) Use the graph to write each polynomial in factored form.

a) $p(x) = x^3 - 31x + 30 = \mathbf{(x+6)(x-1)(x-5)}$ since all of the zeros are single zeros.

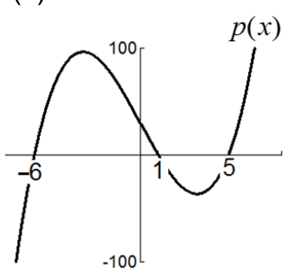


Figure 13a

Check that both $x^3 - 31x + 30$ and $(x+6)(x-1)(x-5)$ have the same leading term.

b) $q(x) = x^4 + 3x^3 - 4x = \mathbf{x(x+2)^2(x-1)}$ since –2 is a double zero and 0 and 1 are single zeros.

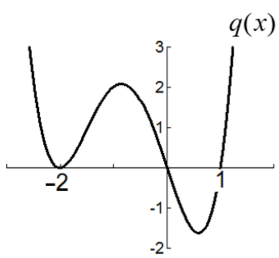


Figure 13b

Check that both $x^4 + 3x^3 - 4x$ and $x(x+2)^2(x-1)$ have the same leading term.

14) Suppose the polynomial *f* graphed in figure shows its entire long run behavior and has leading term ax^n, that is, $f(x) = ax^n$ + remaining terms of lower degree.

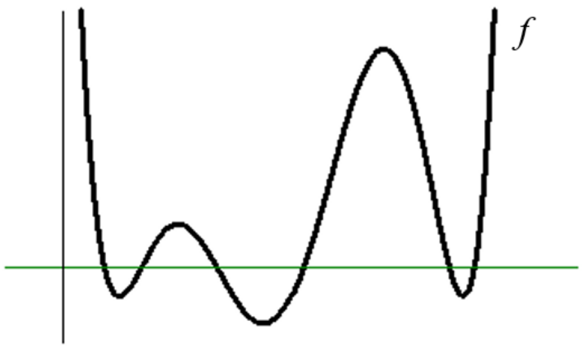


Figure 14

a) The degree, *n*, of the leading term is **even** since both arms go the same way, either both up or both down. In this case both arms are up.
b) The leading coefficient, *a,* is **positive** since both arms are up.
c) Report the minimum possible value of *n:* $n \geq$ **6** since there are 6 zeros (and 6 linear factors).

15) Write a possible formula for each polynomial function.

a) Consider the polynomial shown. Report long run and short run behavior.

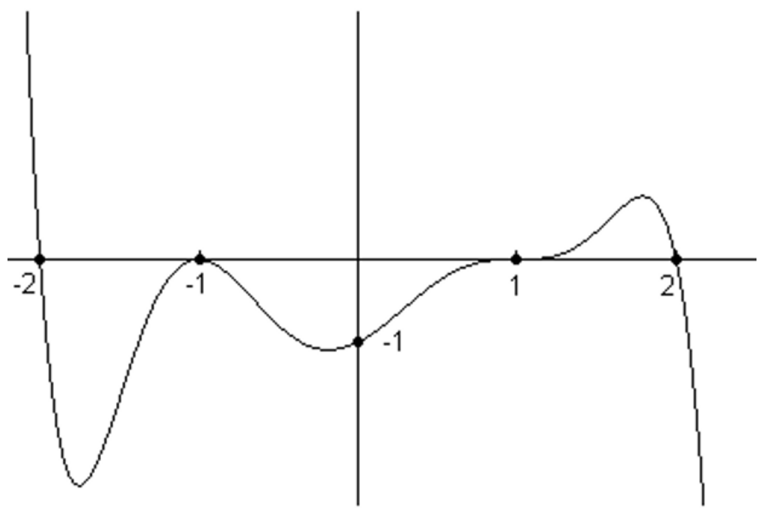


Figure 15a

The long run behavior is “up down” since the arms look like .
The short run behavior:

- −2 is a single zero since, very close to −2, the graph looks like a line.
- −1 is a double zero, since very close to −1, the graph bounces.
- 1 is a triple zero, since very close to 1, the graph looks like a chair.
- 2 is a single zero since, very close to 2, the graph looks like a line.
- The vertical intercept is (0, −1).

Based on the short run behavior, we have $y = a(x+2)(x+1)^2(x-1)^3(x-2)$, a degree 7 polynomial.

The arms look like  which is consistent with a degree 7 polynomial (arms not doing the same thing.)

Substitute $x = 0$, $y = -1$ in the formula $y = a(x+2)(x+1)^2(x-1)^3(x-2)$.

$x = 0, y = -1 \Rightarrow \quad y = a(x+2)(x+1)^2(x-1)^3(x-2)$

$-1 = a(0+2)(0+1)^2(0-1)^3(0-2)$

$-1 = a(2)(1)^2(-1)(-2)$

$-1 = 4a$ Divide both sides by 4.

$a = \frac{-1}{4} = -0.25$

The formula for the polynomial function $\mathbf{y = -0.25(x+2)(x+1)^2(x-1)^3(x-2)}$.
Check with a grapher.

b) Consider the polynomial shown. Report long run behavior, zeros, and short run behavior.

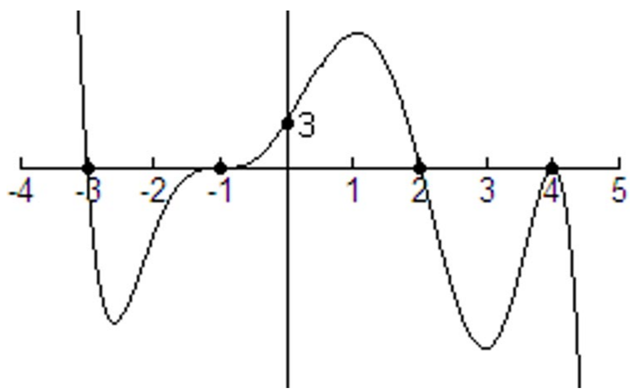


Figure 15b

The long run behavior is “up down” since the arms look like .
The short run behavior:

- −3 is a single zero since, very close to −2, the graph looks like a line.
- −1 is a triple zero, since very close to −1, the graph looks like a chair.
- 2 is a single zero, since very close to 2, the graph looks like a line.
- 4 is a double zero since, very close to 4, the graph bounces.
- The vertical intercept is (0, 3).

Based on the short run behavior, we have $y = a(x+3)(x+1)^3(x-2)(x-4)^2$, a degree 7 polynomial.

The arms look like  which is consistent with a degree 7 polynomial (arms not doing the same thing.)

Substitute $x = 0$, $y = 3$ in the formula $y = a(x+3)(x+1)^3(x-2)(x-4)^2$.

$x = 0, y = 3 \Rightarrow \quad y = a(x+3)(x+1)^3(x-2)(x-4)^2$

$3 = a(0+3)(0+1)^3(0-2)(0-4)^2$

$3 = a(3)(1)^3(-2)(16)$ Divide both sides by 3.

$1 = -32a$ Divide both sides by −32.

$a = \frac{1}{-32} = -0.3125$

The formula for the polynomial function $\mathbf{y = -0.3125(x+3)(x+1)^3(x-2)(x-4)^2}$.
Check with a grapher.

16) A model rocket is launched from the roof of a building with height h_0. Its height above ground (in meters) t seconds later is given by $h = f(t) = -5t^2 + 40t + 20$.

a) The value of h_0, the initial height of the rocket, is **20 meters**.

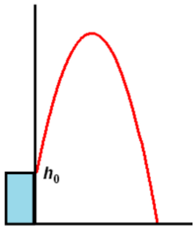


Figure 16a is not necessarily to scale.

b) The rocket will hit the ground, to two decimal places, in **8.47 seconds**.
Use the table to find the viewing window.

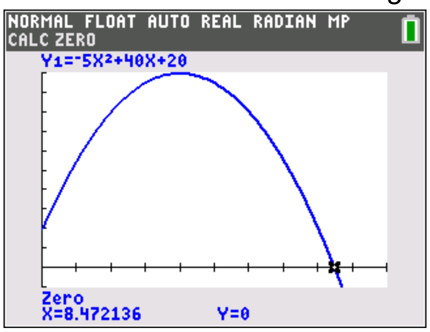


Figure 16b

c) The **exact** maximum height of the rocket is. **100 meters**.

X	Y1	
0	20	
1	55	
2	80	
3	95	
4	100	← vertex
5	95	
6	80	
7	55	
8	20	
9	-25	
10	-80	

Table 16c

d) The rocket reach its maximum height in **4 seconds**.

e) The length of time the rocket will be 15 feet or higher, to two decimal places, is **8.12 seconds**.

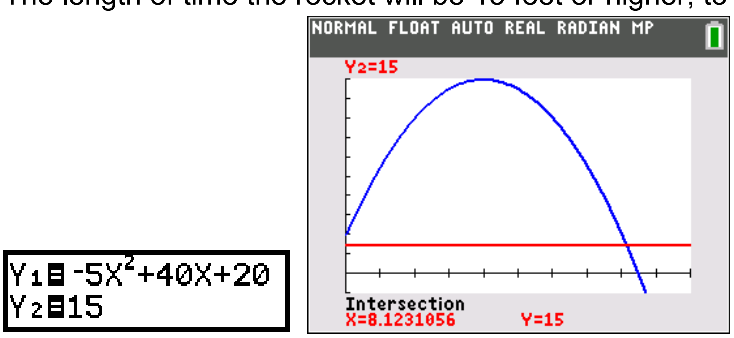


Figure 16e The formulas and graphs to solve $-5t^2 + 40t + 20 = 15$.

f) The domain (restricted according to the ***context of the problem situation***) is **$0 \le t \le 8.47$**.

g) The range (restricted according to the ***context of the problem situation***) is **$0 \le f(t) \le 100$**.

For more practice:
See the Flash Cards for Sections 2.4, 6.1, 6.2, 3.1, 3.2, and 11.1-11.3 as well as the Just for Practice sets. Find these in your Brightspace course in the module **Flash Cards and Just for Practice Sets**.