KEY to Practice Questions for MA 15300 Test 2 (Sections 4.1-4.5 and 5.1-5.3)

TIP: Open the bookmark panel by selecting the Bookmarks icon along the side margin for easier navigation.
Note: The actual test will be shorter.

1) Answer the following for $f(x)$ and $g(x)$. Give only **exact** numerical answers

a) $f(x) = 80(1.139)^x$

The initial value is __80__. The growth factor is __1.139__.

The function grows by a percent rate of __13.9__%.

b) $g(x) = 500(0.25)^x$

The initial value is __500__. The growth factor is __0.25__.

The function decays by a percent rate of __0.75__% (since it keeps 25%).

2) A typical cup of coffee contains 75 milligrams of caffeine.
Each hour 81% of the amount of caffeine in the body is metabolized and eliminated.
Suppose a person drinks a cup of coffee. Suppose that after it the person does not consume caffeine, and they had no caffeine in their body before.

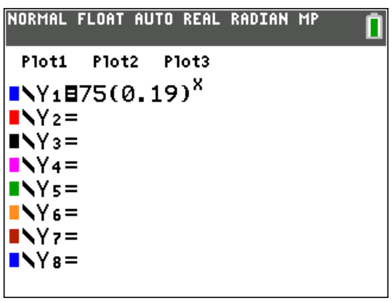
a) Write a formula $A(t)$ for the amount of caffeine in the body in mg of caffeine as a function of t, the number of hours since the coffee was consumed.

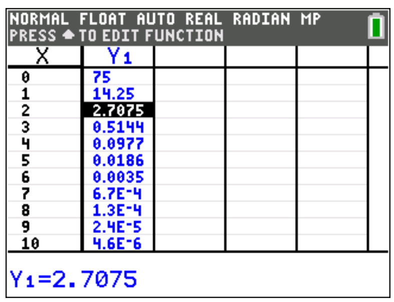
Answer: Since 81% leaves, 19% is kept. $A(t) = \mathbf{75(0.19)^t}$

b) How much caffeine is in the body after 2 hours? Report to the nearest 0.1 mg.

Answer: Calculate $A(2) = 75(0.19)^2 \approx$ **2.7 mg**

You could also scroll a table.





c) After how many hours is there less than 1 milligram left in the body?

Report your answer **to the nearest 0.1 hr**. SHOW WORK.

Answer: Solve $A(t) = 75(0.19)^t = 1$

Method 1: Solve an equation with logarithms.

$$75(0.19)^t = 1$$
$$(0.19)^t = \frac{1}{75}$$
$$\ln (0.19)^t = \ln \frac{1}{75}$$
$$t \ln (0.19) = \ln \frac{1}{75}$$
$$t = \frac{\ln(1/75)}{\ln(0.19)} \approx 2.5997513 \text{ or about } \mathbf{2.6} \text{ hours}$$

Method 2: Solve using a table.

Using a table, you would show work as follows.

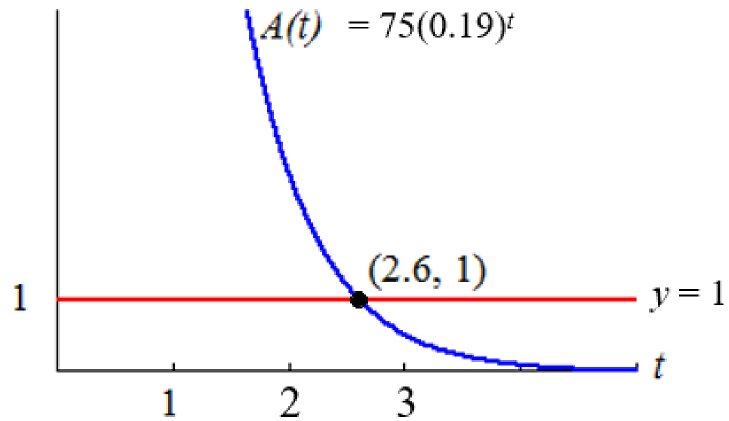
Enter your formula **75(0.19)^x** in Y= and scroll.

The above table shows the solution is between 2 and 3.

Press 2nd WINDOW and set your TblStart to 2 and your ΔTbl (step size) to 0.1.
A documented solution would include the table below with 5 entries as shown.

t, years	*B*, dollars
2.4	1.3934
2.5	1.1802
2.6	**0.9996**
2.7	0.8466
2.8	0.7171

Method 3: Solve graphically.
Enter the function y = 75(0.19)^x and y = 1 in a grapher and use 2nd CALC to find the point of intersection. A documented solution would include both graphs, labeled, and the intersection point.



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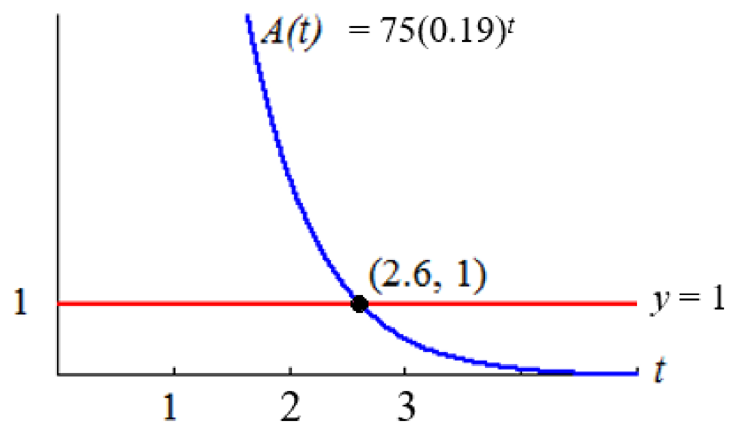


3) The balance *B* of an account climbs exponentially according to the following table.

t, years	*B*, dollars
2	1024
7	3125

a) Write a formula for the exponential function. SHOW WORK

We have $y = ab^t$. We expect $b > 1$ since the function is increasing.
Substitute the points. Find the ratio of the outputs

$$\frac{ab^7}{ab^2} = \frac{3125}{1024}$$

$$b^5 = \frac{3125}{1024}$$

Raise both sides to the $\frac{1}{5}$th power (or take fifths roots of both sides).

$$b = \left(\frac{3125}{1024}\right)^{1/5}$$

$b = 1.25$

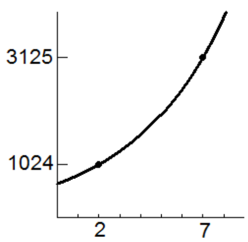
We have $y = a(1.25)^t$. Substitute either of these points in the formula and solve for a.

$1024 = a(1.25)^2$

$$a = \frac{1024}{1.25^2} = 655.36$$

Therefore the formula is y **= 655.36(1.25)**^{t}.

Check the table and graph align with the given values, (2, 1024) and (7, 3125).



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PRESS + FOR ΔTbl

X	Y1
0	655.36
1	819.2
2	1024
3	1280
4	1600
5	2000
6	2500
7	3125
8	3906.3
9	4882.8
10	6103.5

X=0

b) Find the amount in the account at year $t = 4$.
Using the above table, the amount when $t = 4$ is A = **$1600**.
You can also calculate $655.36(1.25)^4$.

c) How many years does it take until the balance first exceeds $5929? Report your answer **to the nearest to the nearest 0.01 months**. SHOW WORK

Method 1: Solve using a table
The above table in part a) shows the solution is between 9 and 10.
Press 2nd WINDOW and set your TblStart to 9 and your DTbl (step size) to 0.1.

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PRESS + FOR ΔTbl

X	Y1
9	4882.8
9.1	4993
9.2	5105.7
9.3	5220.9
9.4	5338.7
9.5	5459.2
9.6	5582.3
9.7	5708.3
9.8	5837.1
9.9	5968.8
10	6103.5

X=9

The above table shows the solution is between 9.8 and 9.9.
Press 2nd WINDOW and set your TblStart to 9.8 and your DTbl (step size) to 0.01.

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS ◆ TO EDIT FUNCTION

X	Y_1
9.8	5837.1
9.81	5850.2
9.82	5863.2
9.83	5876.3
9.84	5889.4
9.85	5902.6
9.86	5915.8
9.87	5929
9.88	5942.2
9.89	5955.5
9.9	5968.8

Y_1=5929.0042139483

The answer is **9.87** years.

Method 2: Solve an equation with logarithms

$$655.36(1.25)^t = 5929$$

$$(1.25)^t = \frac{5929}{655.36}$$

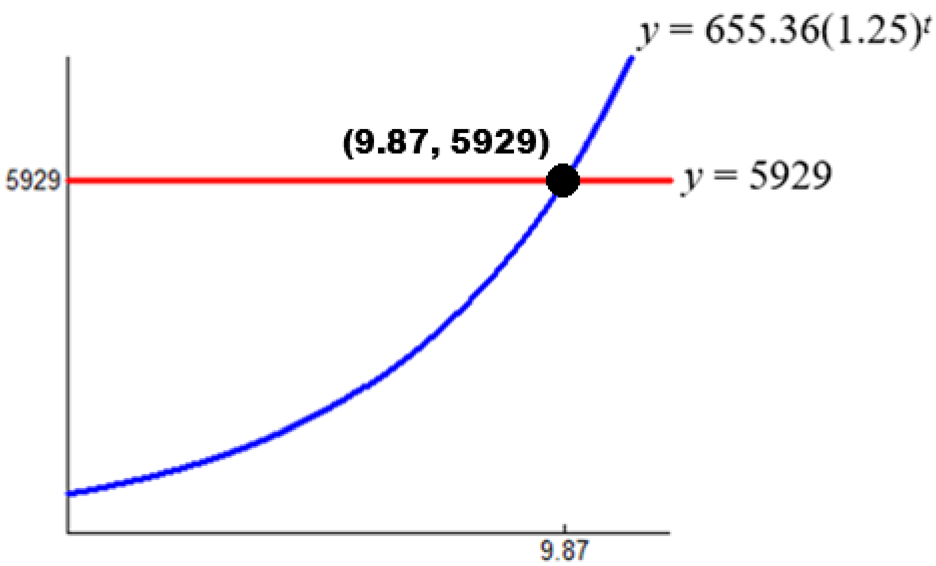
$$\ln (1.25)^t = \ln \frac{5929}{655.36}$$

$$t \ln (1.25) = \ln \frac{5929}{655.36}$$

$$t = \frac{\ln (5929/655.36)}{\ln(1.25)}$$ ≈ 9.869996815 or about **9.87** hours

Method 3: Solve graphically.

A documented solution would include both graphs, labeled, and the intersection point.



d) What was the initial amount of the investment? Report to nearest penny.
Substitute $t = 0$ in the formula $B = 655.36(1.25)^t$. Recall $1.25^0 = 1$.
In general, the initial amount for any function of the form $y = ab^t$. is the value of a.
For this problem, the initial amount is $**655.36.**

e) What **annual interest rate** does the account pay?
The growth factor $b = 1.25 = 1 + r$
$= 1 + 0.25$
Thus the rate $r = 0.25$, or as a percent: **25%**.

4) An amount of $4000 increases to $4600 in one month.

a) By what percent does the amount increase? **Do not round off.** SHOW WORK.
Solve $4000b = 4600$ to first get the growth factor b.

$655.36(1.25)^t = 5929$

$b \;=\; \frac{4600}{4000} = 1.15$

The growth factor is $b = 1.15$, and since $b = 1 + r$, the rate $r = 0.15$, or as a percent: **15%**.

b) By what **<u>total</u>** percent does the amount of $4000 increase over the length of the whole 2 month period? (Assume the trend continues the next month.) **Do not round off.** SHOW WORK.

Method 1: After 4000 grows to 4600 in the first month, then 4600 grows to $4600 \cdot 1.15 = 5290$ in the second month. From the initial value of 4000, the growth factor b over the whole 2 month period is such that $4000b = 5290$. Solve for b by dividing both sides by 4000. We have $b = \frac{5290}{4000} = 1.3225$.
Since $r = 1 + b$, then we have the rate $r = 0.3225$, or as a percent: **32.25%**.
Method 2. Calculate $b = (1.15)^2 = 1.3225$. Then use b to find r as in Method 1.
Why does this work? Since $4000(1.15)^2 = 5290$, that is the same as

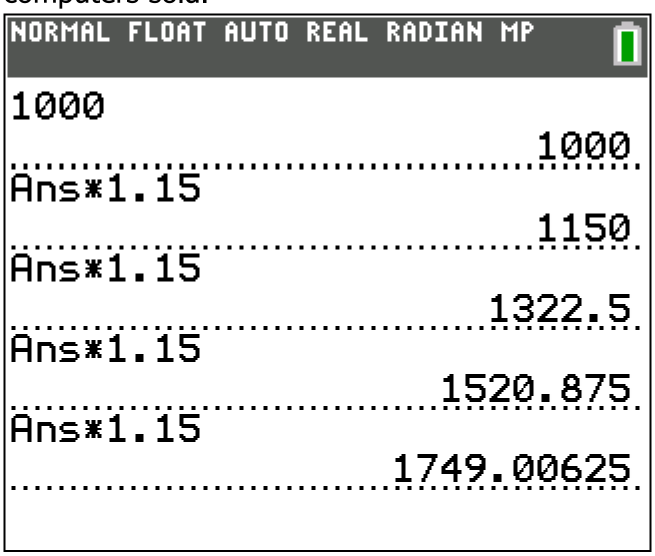
$4000(1.15)^2 = 4600 \cdot 1.15$

$4000(1.15)^2 = 4000 \cdot b$

So dividing both sides of the equation by 4000 gives you b.

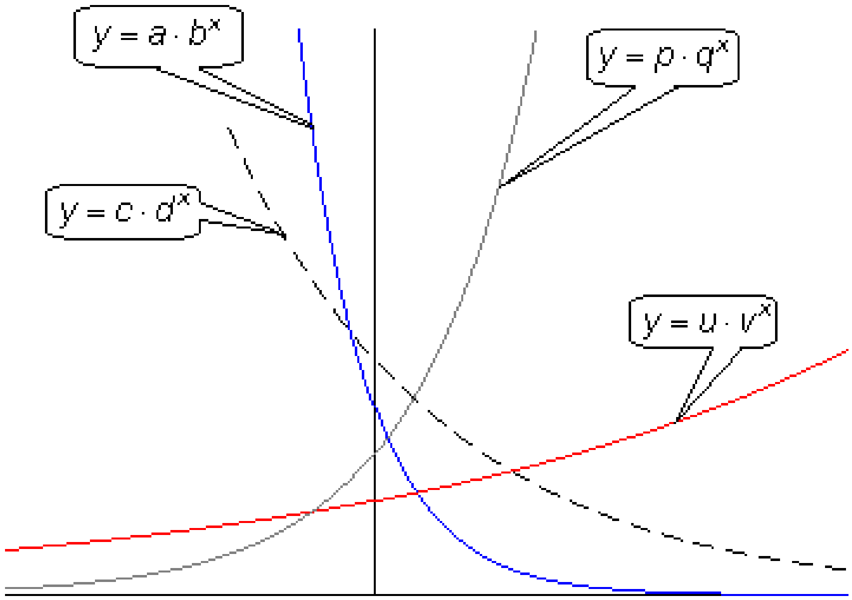
5) Sales of computers increase by 15% **<u>per month</u>** over a 4-month period.
Let $g(t)$ be the number of computers sold, where t is in months.

a) If 1000 computers were sold at month $t = 0$, determine the number sold in month $t = 1$ and month $t = 4$. Round each to **<u>whole number</u>** of values.
Repeated multiplication of 1.15 using the home screen of a calculator as shown below gives us at $t = 1$ month a value of $g(1) =$ **1150** computers sold and at $t = 4$ months a value of $g(4) =$ **1749** computers sold.

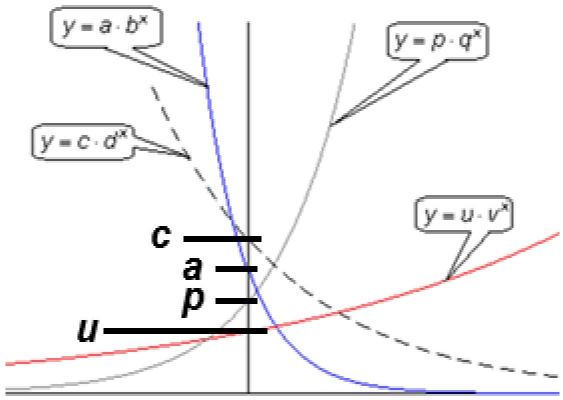


b) By what **<u>total</u>** percent did sales of computers increase over the length of the whole 4 month period? Round the answer correct to **<u>one</u>** decimal place.
Similar to Question 4b, solve $1000b = 1749$ which gives $b = 1.749$ as a growth factor over the whole 4 month period. Since $b = 1 + r = 1 + 0.749$, the sales grew by **74.9%.**
We could have also found $b = (1.15)^4 \approx 1.749$.

6) Consider the exponential functions whose graph is shown.

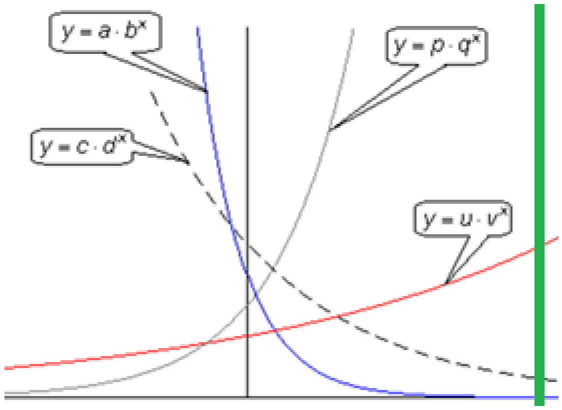


a) Which of *a, c, p*, and *u* is the **largest** ? If $x = 0$ in $y=c \cdot d^x$, then $y=c \cdot d^0 = c \cdot 1 = c$ is the *y*-coordinate of the *y*-intercept of the graph. Similarly, the values *a, c, p*, and *u* are where the graph crosses the vertical axis. The largest vertical intercept is at ***c***.



b) Which of *a, c, p*, and *u* is the **smallest** ?
See part a. The smallest vertical intercept is at ***u.***

c) Which of *b, d, q*, and *v* is the **largest** ?
The values *b, d, q*, and *v* are the growth factors. Visualize or sketch a vertical line past all places where the curves intersect each other as shown below. The y-values of the where each curve intersects the green line can be used to compare their rates of growth. The curve intersecting the green line above all others is $y = pq^x$. Then $y = uv^x$ would be next highest, followed by the graph of $y = uv^x$, followed the graph of $y = ab^x$. Of *b, d, q*, and *v* , the **largest is *q*.**



d) Which of *b, d, q*, and *v* is the **smallest** ? See part c.
Notice the graph of $y = ab^x$ decreases the fastest as x→∞. Of *b, d, q*, and *v* , the **smallest is *b*.**

Useful Formulas: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$.

For 7 and 8 please show work. The correct answer with no calculations or work still earns no points.

7) Suppose you invest $600 at an annual rate of 6.25%. Report your answer rounded correct to the nearest $0.01

a) How much will you have in 21 years if it is compounded **monthly**?
Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ with $P = 600$, $r = 0.0625$, $t = 21$, and $n = 12$.
We have $A = 600\left(1 + \frac{0.0625}{12}\right)^{12 \cdot 21}$ ≈ **$2221.69.**

b) How much will you have in 21 years if it is compounded **continuously**?
Use $A = Pe^{rt}$ with $P = 600$, $r = 0.0625$, and $t = 21$. We have $A = 600e^{0.0625 \cdot 21}$ ≈ **$2229.27.**

8) You need $4734 thirteen years from now. What is the minimum amount of money you need to deposit into a bank account that pays 6.25% annual interest, compounded **continuously**?
Report your answer rounded correct to the nearest $0.01.

Use $A = Pe^{rt}$ with $A = 4734$, $r = 0.0625$, and $t = 13$
We have $Pe^{0.0625 \cdot 13} \approx 4734$. Divide both sides by $e^{0.0625 \cdot 13}$. Do not round off until the end.

$$\frac{Pe^{0.0625\times13}}{e^{0.0625\times13}} = \frac{4734}{e^{0.0625\times13}}$$

$P = \frac{4734}{e^{0.0625\times13}}$ ≈ $2100.69976 ≈ $**2100.70**

Check: $2100.70e^{0.0625 \cdot 13} \approx 4734.000527$.

9) Suppose $Q = 5000e^{0.0742t}$ gives the balance in an account, where t is in years.

a) Describe how the interest is compounded by completing the blanks.
The account pays ___**7.42**__ % compounded continuously.

b) Find the percent annual growth rate. Report to the nearest **0.1** %.
$Q = 5000e^{0.0742t} = 5000(e^{0.0742})^t = 5000^{bt}$ when $b = e^{0.0742} \approx 1.077$ so $r = 0.077 =$ **7.7%**.

c) Approximate the amount of time t it will take for the account to first exceed $6200.
Write an equation and solve algebraically using logarithms. Report to the nearest **0.1** years

$5000e^{0.0742t}$ = 6200

$e^{0.0742t}$ = $\frac{6200}{5000}$ = 1.24

$\ln e^{0.0742t}$ = ln 1.24

$0.0742t$ = ln 1.24

t = $\frac{\ln(1.24)}{0.742}$ ≈ 2.899075197 or about **2.9** hours

You can check your answer with a table or a graph.

d) Approximate the amount of time *t* it will take for the account to triple.

Write an equation and solve algebraically using logarithms. Report to the nearest **0.1** years.

$5000e^{0.0742t}$ = 5000·3

$e^{0.0742t}$ = $\frac{5000\cdot 3}{5000}$ = 3

$\ln e^{0.0742t}$ = ln 3

$0.0742t$ = ln 3

t = $\frac{\ln(3)}{0.742}$ ≈ 14.80609553 or about **14.8** hours

You can check your answer with a table or a graph.

10) Find the logarithm

a) $\ln e^{5x-1}$ = **5*x*−1** by the inverse property.
b) $\log 10^{7x}$= **7*x*** by the inverse property.
c) $\log_5\sqrt{5} = \log_5 5^{1/2}$= **1/2** by the inverse property.
d) $\log_{\sqrt{5}}\sqrt{5} = \log_{\sqrt{5}}(\sqrt{5})^1$ = **1** by the inverse property.
e) $\ln\sqrt{e^{7x}} = \ln(e^{7x})^{1/2} = \ln e^{7x/2}$ by Laws of Exponents. So $\ln e^{7x/2}$ = **7*x*/2** by the inverse property.
f) $\ln\frac{1}{\sqrt{e^{7x}}} = \ln(e^{7x})^{-1/2} = \ln e^{-7x/2}$ by Laws of Exponents. So $\ln e^{-7x/2}$ = **−7*x*/2** by the inverse property.
g) $\log_5\sqrt[7]{5^x} = \log_5 5^{x/7}$= ***x*/7** by the inverse property.
h) $\log_5\frac{1}{25} = \log_5\frac{1}{5^2} = \log_5 5^{-2}$= **−2** by the inverse property.
i) $\log 100^{11x} = \log(10^2)^{11x} = \log 10^{22x}$ by Laws of Exponents. So $\log 10^{22x}$ = **22*x*** by the inverse property.

11) Simplify, and report an exact answer.

a) $e^{\ln\sqrt{5x}} = \sqrt{5x}$ by the inverse property.
b) $100^{2\log x^5} = (10^2)^{2\log x^5} = 10^{4\log x^5} = 10^{5\cdot 4\log x} = 10^{20\log x} = 10^{\log x^{20}}$ = ***x*****^20^**.

12) $\log\frac{x^{20}}{y^2}$ = log x^{20} – log y^2 = 20log x −2log y. This is Choice **H**.

13) Solve the equations.
Report both an exact solution (involving a logarithm) and an approximate solution to 3 decimal places.

a) $3e^{5x-10} = 60$

We have $3e^{5x-10} = 60$ Divide both sides by 3.

$e^{5x-10} = 20$ Since the base is *e*, take natural logarithms of both sides.

$\ln e^{5x-10} = \ln 20$ Use the inverse property $\ln e^{Q} = Q$

$5x - 10 = \ln 20$ Add 5 to both sides and divide by 3.

$5x = \ln 20 + 10$

$x = \frac{ln\ 20 + 10}{5} \approx 2.599$

Check: If x ≈ 2.599 and $3e^{5x-10} = 60$, then $3e^{5 \cdot 2.599-10} \approx 60$.

EXACT: $x = \frac{\mathbf{ln\ 20 + 10}}{\mathbf{5}}$

APPROXIMATE: $x \approx$ **2.599**

b) $7 \cdot 10^{x} + 10 = 70$

We have $7 \cdot 10^{x} + 10 = 70$ Subtract 10 from both sides.

$7 \cdot 10^{x} = 60$ Divide both sides by 7. (Don't round off)

$10^{x} = \frac{60}{7}$ Since the base is 10, take common logarithms of both sides.

$\log 10^{x} = \log \frac{60}{7}$ Use the inverse property

$x = \log \frac{60}{7} \approx 0.933$

Check: If $x \approx 0.933$ and $7 \cdot 10^{x} + 10 = 70$, then $7 \cdot 10^{0.933} + 10 \approx 70$.

EXACT: $x = \mathbf{\log \frac{60}{7}}$

APPROXIMATE: $x \approx$ **0.933**

c) $26(0.5)^{x/7} + 40 = 48$

We have $26 \cdot (0.5)^{x/7} + 40 = 48$ Subtract 40 from both sides.

$26 \cdot (0.5)^{x/7} = 8$ Divide both sides by 26. (Don't round off)

$(0.5)^{x/7} = \frac{8}{26}$ Take common or natural logarithms of both sides.

$\ln (0.5)^{x/7} = \ln \frac{8}{26}$ Use Bob Barker property.

$\frac{x}{7} ln\ (0.5) = \ln \frac{8}{26}$ Multiply both sides by 7.

$x ln\ (0.5) = 7 ln \frac{8}{26}$ Divide both sides by ln(0.5).

$x = \frac{7 ln \frac{8}{26}}{ln(0.5)} \approx 11.903$

Check: If $x \approx 11.903$ and $26 \cdot (0.5)^{x/7} + 40 = 48$, then $26 \cdot (0.5)^{11.903/7} + 40 \approx 48$..

EXACT: $x = \frac{\mathbf{7 ln \frac{8}{26}}}{\mathbf{ln(0.5)}}$

APPROXIMATE: $x \approx$ **11.903**

14) Solve the equations. Report both an exact solution and an approximate solution to 3 decimal places.

a) $5\ln(3x) = 20$

$5\ln(3x)$	=	20	Divide both sides by 5.
$\ln(3x)$	=	4	Make both sides a power of *e*.
$e^{\ln(3x)}$	=	e^4	Use inverse property
$3x$	=	e^4	Divide both sides by 3
x	=	$\frac{e^4}{3}$	

Check: If $x \approx 18.199$ and $5\ln(3x) = 20$, then $5\ln(3\cdot 18.199) \approx 20$

EXACT: $x = \frac{e^4}{3}$ or $x = \frac{1}{3}e^4$

APPROXIMATE: $x \approx 18.199$

EXACT: $x = 10^{3/5}$

APPROXIMATE: $x \approx 3.981$

b) $5\log x + 7 = 10$

$5\log x + 7$	$= 10$	Subtract 7 from both sides.
$5\log x$	$= 3$	Divide both sides by 5.
$\log x$	$= \frac{3}{5}$	Make both sides a power of 10.
$10^{\log x}$	$= 10^{3/5}$	Use inverse property.
x	$= 10^{3/5} \approx 3.981$	

Check: If $x \approx 3.9815$ and $5\log x + 7 = 10$, then $\log(3.981) + 7 \approx 10$

15) A function $Q = f(x)$ gives the amount, in mg of drug in a patient's body x hours after they take a single dose. The function Q decays exponentially.
To see a graph of Q as a function of x, open this Desmos graph.

a) Complete the entry in the first row in the table.

x, hours	*Q*, mg
0	**1920** = 960·2
8	960
16	480
24	240
32	120

b) Assuming the pattern holds, complete the last row of the table shown below, shaded.

x, hours	*Q*, mg
8	960
16	480
24	240
32	120
40 = 32+8	**60** = 120 ·0.5

c) Report the half-life, in hours.
The half life is the Δt, which is **8 hours**.

d) Find a formula for this function.
$Q = \mathbf{1920(0.5)^{t/8}}$ or $Q = \mathbf{1920(0.917)^{t}}$ since $b = (0.5)^{1/8} \approx 0.91$

e) What was the original amount of medication taken? __**1920**__ mg

f) Complete, reporting each to the nearest 0.1 percent.
Every hour the patient loses __**8.3**__ % and keeps __**91.7**__% of the drug.
Since $b = (0.5)^{1/8} \approx 0.91$, 0.91 = 91.7% is kept and 100 – 91.7 = 8.3 percent decays.

g) Find, to the nearest 0.01 hour, the time it takes for the amount of drug to first fall below 1000 mg.
Show work. $t \approx$ __**7.53**______ hours

Method 1: Solve using logarithms. We can use common or natural logarithms. We will use natural.

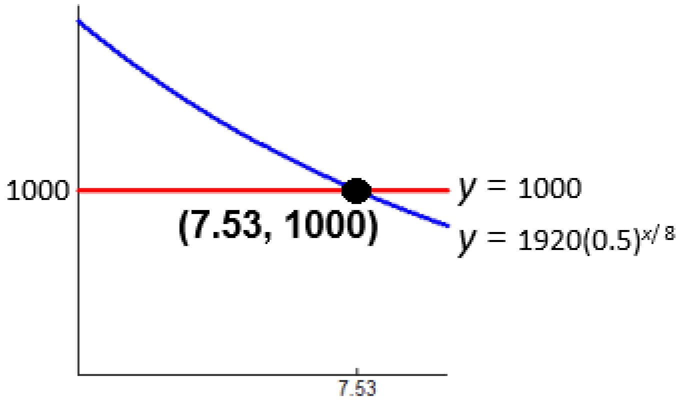
Solve $1920(0.5)^{t/8}$	$= 1000$	Divide both sides by 1920
$(0.5)^{t/8}$	$= 1000 / 1920$	Take natural logs of both sides.
$\ln(0.5)^{t/8}$	$= \ln(1000 / 1920)$	Use the Bob Barker Property.
$\frac{t}{8}\ln(0.5)$	$= \ln(1000 / 1920)$	Multiply both sides by 8.
$\ln(0.5)$	$= 8\ln(1000 / 1920)$	Divide both sides by ln(0.5)
t	$= \dfrac{8\ln\frac{1000}{1920}}{\ln(0.5)} \approx \mathbf{7.53}$	

Method 2: Solve using a table.
See the solution in Question 3 part c for how to solve to two decimal places using the table.
A documented solution would include the table below with 5 entries as shown.

x	*y*
7.51	1001.6
7.52	1001.8
7.53	**999.9**
7.54	999.03
7.55	998.17

Method 3: Solve graphically.

Enter the function y = **1920(0.5)^(x/8)** and y = 1000 in a grapher and use 2nd CALC to find the point of intersection. A documented solution would include both graphs, labeled, and the intersection point.

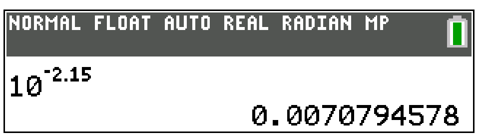


16) The relationship of pH to the hydrogen ion concentration C is pH = −log C.
If the pH is 2.15 what is the hydrogen ion concentration? Report to three decimal places.

Since the pH is 2.15 and pH = −log C we have

2.15	$= -\log C$	Multiply both sides by −1
$\log C$	$= -2.15$	Make both sides a power of 10.
$10^{\log C}$	$= 10^{-2.15}$	Use the inverse property.

$C = 10^{-2.15} \approx 0.007$ which is **Choice C**.



17) A small town of 2000 people increases to 3340 people in five years, growing ***exponentially.***

a) By what total percent did the population increase in this five year time span?
This part of the question is similar to Question 4b and Question 5b.
The five year growth factor is $\frac{3340}{2000} = 1.67$. So the growth rate over 5 years is $r = 0.67$ or **67%**.

b) By what percent did the population grow each year? Round the answer correct to **one** decimal place.

Method 1:
We will use b for the annual growth factor. Then $2000b^5 = 3340$. Solve for b.
Then $b^5 = \frac{3340}{2000} = 1.67$ and $b = 1.67^{1/5} \approx 1.108$. So the growth rate over 1 year is $r = .108$ or **10.8%**.
Method 2:
Since every 5 years the population grows by 1.67 and the initial population is 2000, we have
$P = 2000(1.67)^{x/5} \approx 2000(1.67^{1/5})^x$. So $b = 1.67^{1/5} \approx 1.108$ and $r = .108$ or **10.8%**.

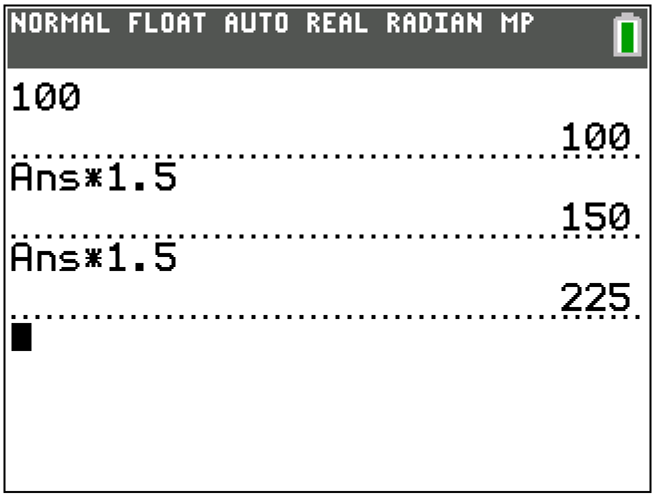
c) Write a formula for the population $P = f(x)$ of the town in year x.
From Question 17 part b we have $P = 2000(1.67)^{x/5}$ or $P = 2000(1.108)^x$.

18) Sales of an item increase by 50% every 9 years.
Assume sales $f(t)$ continue to grow exponentially, where t is in years.

a) If 100 items were sold at year $t = 0$, complete the shaded cells of the table to determine the number sold in year $t = 9$ and year $t = 18$.Report **whole number** of values.
Assume sales $f(t)$ continue to grow exponentially, where t is in years.

Repeated multiplication of 1.5 using the home screen of a calculator as shown below gives us at $t = 9$ years a value of **150** items sold and at $t = 18$ years a value of **225** items sold.

t	$f(t)$
0	100
9	**150** = $100 \cdot 1.5$
18	**225** = $150 \cdot 1.5$



b) At what effective percent rate does it increase **per year**? Round to the nearest **0.1** percent.
The formula for f(t) is $y = 100b^t$. Substitute a point, say $t = 9$, $y = 150$. Then solve for b.

$$100b^9 = 150$$
$$b^9 = \frac{150}{100} = 1.5$$
$$b = 1.5^{1/9} \approx 1.046$$

Note that we didn't need the initial value 100 to find the value of b. We just needed the growth factor per 9-month period. The growth factor per year is 1.046, so the growth rate is **4.6%.**

c) Write a formula for $f(t)$.

$$\mathbf{f(t) = 100(1.5)^{t/9}} \text{ or } \mathbf{f(t) = 100(1.046)^{t}}.$$

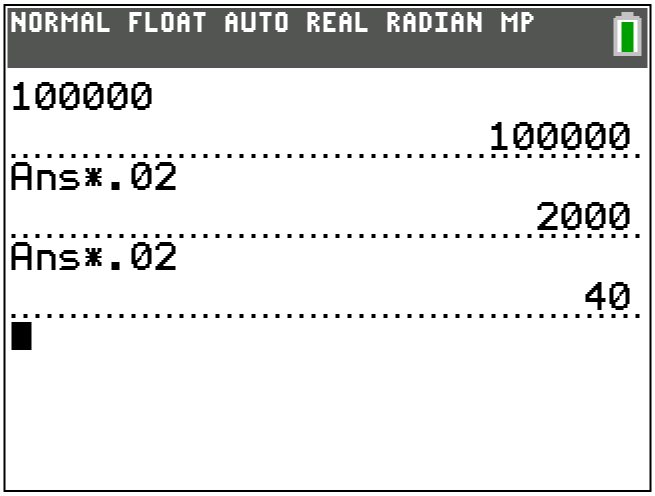
19) Sales of an item decrease by 98% every 6 years. Assume sales $f(t)$ continue to decay exponentially, where t is in years.

a) If 100,000 items were sold at year $t = 0$, complete the shaded cells of the table to determine the number sold in year $t = 6$ and year $t = 12$.Report **whole number** of values.

If we lose 98%, we keep only 2% = 0.02 every 6 years.

Repeated multiplication of 0.02 using the home screen of a calculator as shown below gives us at $t = 6$ years a value of **2000** items sold and at $t = 12$ years a value of **40** items sold.

t	$f(t)$
0	100,000
6	**2000** = 100000·0.02
12	**40** = 2000·0.02



b) At what effective percent rate does it decrease **<u>per year</u>**? Round to the nearest **0.1** percent.
The formula for $f(t)$ is $y = 100000b^t$. Substitute a point, say $t = 6$, $y = 2000$. Then solve for b.

$$100000b^6 = 2000$$
$$b^6 = \frac{2000}{100{,}000} = 0.02$$
$$b = 0.02^{1/6} \approx 0.521$$

Note that we didn't need the initial value 100,000 to find the value of b. We just needed the decay factor per 6-month period. The decay factor per year is 0.521, so we keep 52.1%, and thus lose 100% − 52.1% = **47.9%** per year.

c) Write a formula for $f(t)$.
$f(t) = 100000(0.02)^{t/6}$ or **$f(t) = 100000(0.521)^t$.**

20) If a function decays according to the formula $Q = 400(0.5)^{t/53}$, where t is in minutes.

a) Report the half-life, in minutes.

In 53 seconds, $P = 400(0.5)^{53/53} = 400 \times 0.5^1 = 200$
so the time it takes to decay to half its original amount is **53** minutes.

b) By what percent does it decay each minute?

The function $Q = 400(0.5)^{t/53}$ can be written $Q = 400(0.5^{1/53})^t$ and by laws of exponents and the fact that $\frac{t}{53} = \frac{1}{53}t$.

Use a calculator to find $(0.5)^{1/53} \approx 0.987007$ so $Q \approx 400(0.987007)^t$.

Each second we keep about 98.7%, so we lose 100% − 98.7% = **1.3% per minute**.

21) A function increases at a rate of 17.76% per day.

a) Write a formula for the amount Q at day t, where Q_0 is the initial amount. Do not round any values.

$Q = Q_0 \cdot (\ \mathbf{1.1776}\ )^t$

b) Report the doubling time.

i) Solve analytically and report your **exact** answer involving natural or common logarithms.

Set $Q_0 \cdot (1.1776)^t$	$= 2Q_0$	Divide both sides by Q_0
$(1.1776)^t$	$= 2$	Take natural logs of both sides.
$\ln(1.1776)^t$	$= \ln(2)$	Use the Bob Barker Property.
$t \cdot \ln(1.1776)$	$= \ln(2)$	Divide both sides by ln(1.1776)
t	$= \dfrac{\ln 2}{\ln 1.1776}$	

ii) Report an **approximate** answer of the doubling time accurate to 0.01 days

$t = \dfrac{\ln 2}{\ln 1.1776}$ ≈ **4.24 days**

c) Report the tripling time.

i) Solve analytically and report your **exact** answer involving natural or common logarithms.

Set $Q_0 \cdot (1.1776)^t$	$= 3Q_0$	Divide both sides by Q_0
$(1.1776)^t$	$=3$	Take natural logs of both sides.
$\ln(1.1776)^t$	$= \ln(3)$	Use the Bob Barker Property.
$t \cdot \ln(1.1776)$	$= \ln(3)$	Divide both sides by ln(1.1776)
t	$= \dfrac{\ln 3}{\ln 1.1776}$	

ii) Report an **approximate** answer of the tripling time accurate to 0.01 days

$t = \dfrac{\ln 3}{\ln 1.1776}$ ≈ **6.72 days**

Would you like more practice over specific topics?
See the Flash Cards for Sections 4.1-4.5 and 5.1-5.3 as well as the Just for Practice sets.
Find these in your Brightspace course in the module **Flash Cards and Just for Practice Sets**.