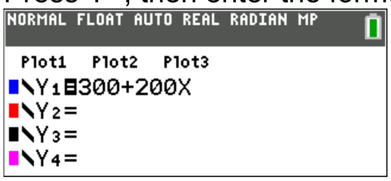
KEY to Practice Questions for MA 15300 Test 1 (Sections 1.1-1.5 and 2.1, 2.2, and 2.5-2.6)

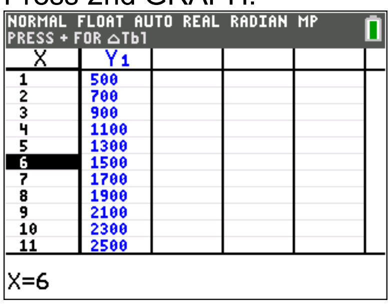
TIP: Open the bookmark panel by selecting the Bookmarks icon along the side margin for easier navigation.

1) True or False: Given that the point (5, 8) is on the graph of *f* is enough information to find the value of *f*(5).
   True since *f*(5) = 8.

2) Tuition cost *T* (in dollars) for part-time students at a college is given by *T* = 300 + 200*C*, where *C* represents the number of credits taken.
   a) Find the tuition cost for six credits.
      Substitute *C* = 6 in the formula *T* = 300 + 200*C*: *T* = 300 + 200·6 = **1500**.
      You can also use a calculator like the TI-84 Plus CE (or similar).
      Press Y=, then enter the formula, then press 2nd WINDOW for tblset and use the settings shown.



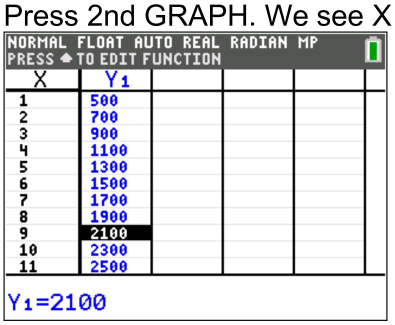
Press 2nd GRAPH.



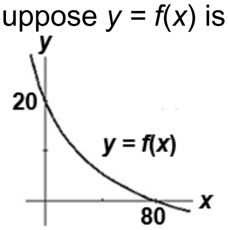
   b) How many credits were taken if the tuition was $2,100? Substitute *T* = 2100 and solve for *C*.

$$T = 300 + 200C$$
$$2100 = 300 + 200C$$
$$1800 = 200C$$
$$C = \frac{1800}{200} = \mathbf{9}$$

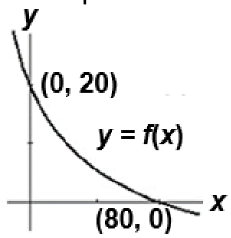
      $2,100 is the cost of taking **9** credits. You can also use a TI-84 Plus CE.
      Press 2nd GRAPH and find the value of X for which Y = 2100.
      In this case, since *C* is an integer, we do not have to change the settings for tblset.
      Press 2nd GRAPH. We see X = **9**.



3) Suppose *y* = *f*(*x*) is given by the graph shown:



Use the graph to find complete the blanks.
It can be helpful to label the intercepts with ordered pairs (x, y).



Replace the blanks with a variable and write equivalent statements.
Use the inverse property $y = f(x) \leftrightarrow x = f^{-1}(y)$.

a) $f(0) =$ _____
$f(0) = y$ can be written as follows: "if $x = 0$, then find y."
Here, when $x = 0$ we have $y = 20$, so $f(0) =$ **20**.

b) $f^{-1}($____$) = 0$. Replace the blank with y to write $f^{-1}(y) = 0$.
To find y, rewrite it using the inverse property $y = f(x) \leftrightarrow x = f^{-1}(y)$. So $f^{-1}(y) = 0$ means $f(0) = y$.
So from part a) we have $f(0) = 20$. Therefore, $f^{-1}($**20**$) = 0$

c) $f^{-1}(0) =$ _____
To find x when $f^{-1}(0) = x$, rewrite it using the inverse property $y = f(x) \leftrightarrow x = f^{-1}(y)$.
$f^{-1}(0) = x$ means $f(x) = 0$. As in part a), $f(x) = 0$ can be written as follows: "if $y = 0$, then find x."
Here, when $y = 0$ we have $x = 80$, so $f(80) = 0$ and, by the inverse property, $f^{-1}(0) =$ **80**.

d) $f($____$) = 0$
From part c) we have $f($**80**$) = 0$.
If we did not have part c), we could just rewrite $f(x) = 0$ as "if $y = 0$, then find x."

4) The table gives the amount of garbage, G, in tons, produced in a country in year t, so $G = f(t)$ since 1950.
For convenience, we have added a first column called **Years**.

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35
2000	50	40
2010	60	45

a) Find $f(40)$ and interpret.
To find $f(40) = G$, we locate $t = 40$ in the first column and report $G =$ **35** in the second column.

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35
2000	50	40
2010	60	45

Since $t = 40$ indicates 40 years after 1950, the interpretation of $f(40) = 35$ in real world terms is:
In 1990, the country produced 35 tons of trash.

b) Solve $f(t)$ =40 for t and interpret.
To solve $f(t) = 40$, we locate $G = 40$ in the second column and report t = **50** in the first column.

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35
2000	50	40
2010	60	45

Since t = 50 indicates 50 years after 1950, we the interpretation of $f(50) = 40$ in real world terms is: In 2000, the country produced 40 tons of trash.

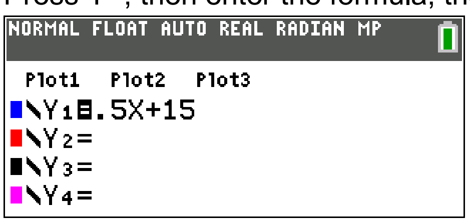
c) Find the average rate of change of the function from 30 to 40. **Report units in your answer**.
We can find this in the table as follows

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35

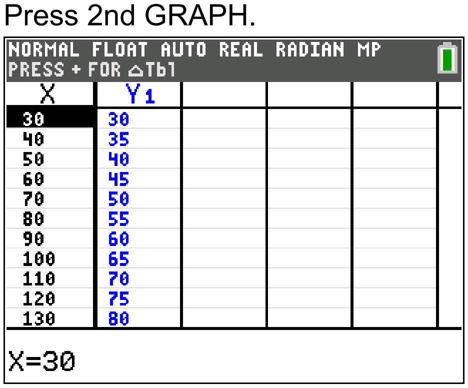
Find the change in t, or $\Delta t = t_2 - t_1$. We have Δt = 40 – 30 = 10 years.
Find the change in G, or $\Delta G = G_2 - G_1$. We have ΔG= 35 – 30 = 5 tons
The average rate of change of f from 1980 to 1990 is $\frac{\Delta G}{\Delta t} = \frac{5\ tons}{10\ years}$ = **0.5 tons of trash per year**.

d) Find a formula for $f(t)$ assuming the garbage increases at a steady rate.
We have from part c) that the slope, or average rate of change, is 0.5 tons per year. So
$G = 0.5t + b$
Substitute a point, such as $t = 40$, $G = 35$ (or any point in the table.)
$35 = 0.5(40) + b$
$35 = 20 + b$
$15 = b$
The formula is $\mathbf{G = 0.5t + 15}$.

You can use a calculator like the TI-84 Plus CE (or similar) to check your work.
Press Y=, then enter the formula, then press 2nd WINDOW for tblset and use the settings shown.



Press 2nd GRAPH.



The table on the calculator using our formula matches the original table given in the problem. Sweet!

e) Interpret the slope of your formula in practical terms. Don't write RISE over RUN.
Each year the country produces an additional 0.5 ton of trash
or **Every 10 years the country produces 5 tons of trash**
or **Every 2 years the country produces 1 ton of trash**
or any statement with equivalent proportions.

f) Interpret the *y*-intercept of your formula in practical terms.
In 1950 they had 15 tons of trash.

g) Predict the amount of garbage in the year 2050, assuming this trend continues.
You can use "table walking to keep adding rows in the table. For each row let's increase years by 10.

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35
2000	50	40
2010	60	45
2020	70	
2030	80	
2040	90	
2050	100	

If *t* is increased by 10 years in each row, then *G* must be increased by 5 tons.

Year	*t*, years since 1950	*G*, tons of garbage
1980	30	30
1990	40	35
2000	50	40
2010	60	45
2020	70	50
2030	80	55
2040	90	60
2050	100	65

We could have also increased *t* and *G* simultaneously row by row.
We predict that in 2050 they will have **65** tons of trash.
If we enter the formula in a calculator, we could also scroll the table as shown in part 4d).

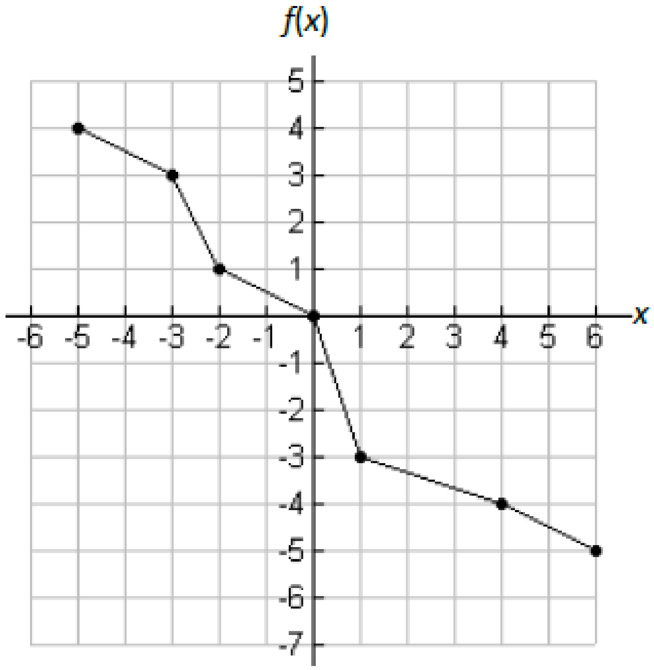
5) In 2006, the population of a town was 15,423 and growing by 200 people per year. Find a formula for *P*, the town's population, in terms of *t*, the number of years since 2006.
P* = 15,423 + 200*t

6) The average rate of change over an interval is the slope of the segment connecting the interval's endpoints. We can use a grid to find the slope.

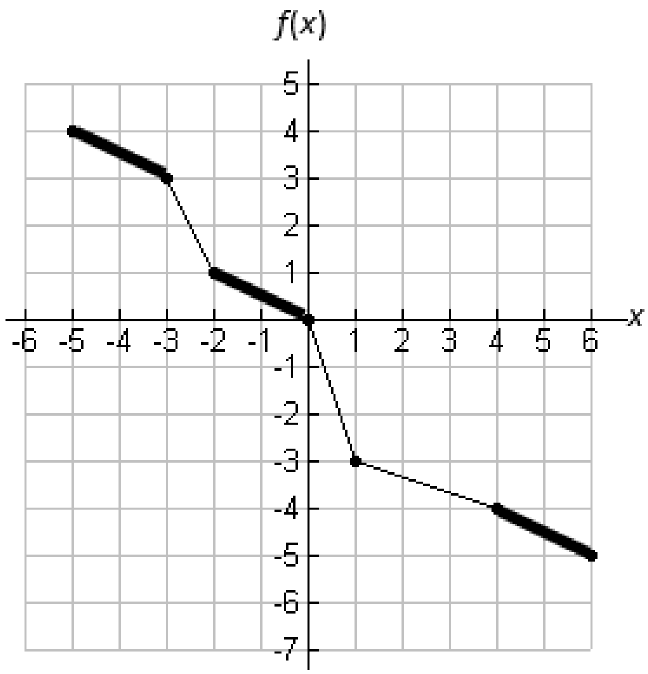
For *positive* slope, remember to "*run right to rise*" and
for *negative* slope, we "*run right to fall*."

Count each grid mark as you move right and move up or down.

In the graph we are given, the horizontal grid marks and vertical grid marks are the same units.
Our goal is to find intervals where the slope is the same.



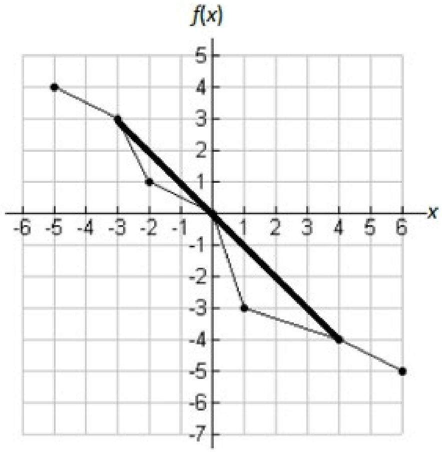
Each of the slopes shown below have a slope of −0.5 since we *run* to the right 2 and *fall* 1,
i.e., RUN = 2 and RISE = −1 so the slope = $\frac{RISE}{RUN} = \frac{-1}{2}$
(Note that when we "fall" we have a negative value of RISE.)



So we have the following:
The average rate of change from x = **−5** to x = **−3** is the same value as
the average rate of change x = **−2** to x = **0**.
or the average rate of change x = **4** to x = **6**.

This is not the only answer.
A different and yet correct answer is shown below.
Each of the slopes shown below have a slope of −1 since we *run* to the right 3 and *fall* 3,
i.e., RUN = 3 and RISE = −3 on the first segment, so the slope = $\frac{RISE}{RUN} = \frac{-3}{3}$ = −1.

On the second segment we *run* to the right 4 and *fall* 4,
i.e., RUN= 4 and RISE = −4, so the slope = $\frac{RISE}{RUN} = \frac{-4}{4}$ = −1.
Note: If the graph had curvature, the segment representing the average rate of change often intersects the graph only at the endpoints of the interval.



Based on the above:
The average rate of change from x = **−3** to x = **0** is the same value as the average rate of change x = **0** to x = **4**.

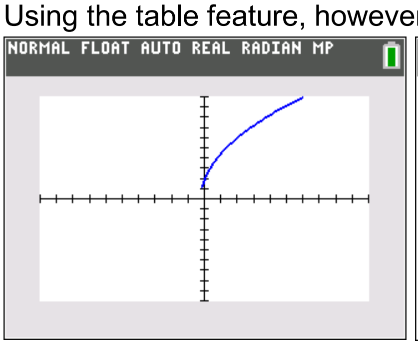
7) If $f(x) = \frac{4x}{x^2+4}$, then find $f(-1)$
Replace x with −1 and evaluate.
$f(-1) = \frac{4(-1)}{(-1)(-1)+4} = \frac{-4}{5}$ = **− 0.8**.

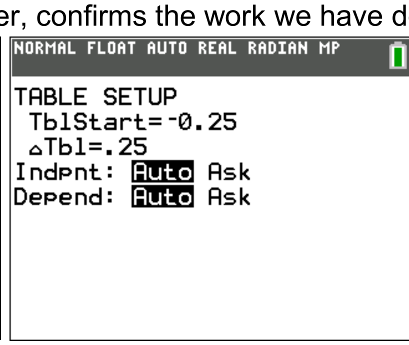
8) $f(x) = \sqrt{16x+4}$.
a) Evaluate $f(0)$.
Replace x with 0 and evaluate:
$f(0) = \sqrt{16 \cdot 0 + 4} = \sqrt{4}$ = **2**.

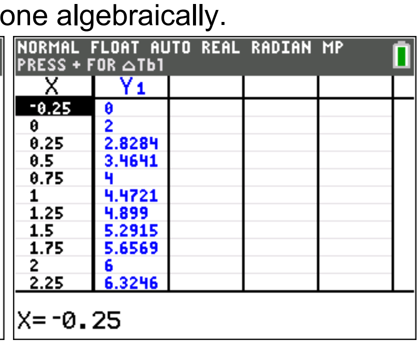
b) Solve the equation $f(x) = 0$. Show work using algebra.

$\sqrt{16x+4}$	=	0	Square both sides.
$16x + 4$	=	0	Subtract 4 from both sides.
$16x$	=	−4	Divide both sides by 16.
x	=	$\frac{-4}{16}$ = **−0.25**	

If you were to check this graphically with a calculator, you may see something strange.
Due to low resolution, it appears the graph does not have a horizontal intercept, even though it does.
Using the table feature, however, confirms the work we have done algebraically.

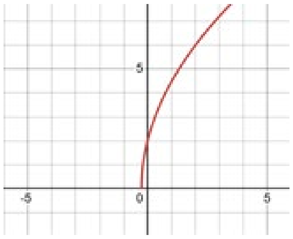




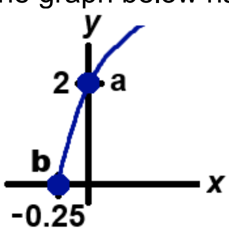


Using parts of the Rule of Four (formula, graph, table) we can make connections to what parts a) and b) represent on the graph.

Compare to the graph of the function using Desmos where the resolution is higher.



c) Sketch a graph of the function and label the points that correspond to what you found in a) and b).
The graph below has a y-intercept, from part **a**, of (0, 2) and an x-intercept, from part **b**, of (−0.25, 0).



d) Report the domain and range.
From the graph, we report the domain as

x ≥ −0.25 using inequality notation

[−0.25, ∞) using interval notation

We can also use the formula to find the domain.
Since only values of x are allowed which make the radicand 16x + 4 nonnegative,
we can solve the inequality 16x + 4 ≥ 0. However, this is just like solving the equation 16x + 4 = 0 in part b), only with ≥ instead of =.

$16x + 4 \geq 0$ Subtract 4 from both sides.

$16x \geq -4$ Divide both sides by 16.

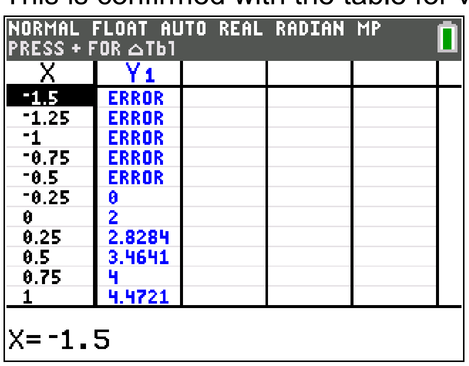
$x \geq -0.25$

We report the domain of $f(x)$ as:

***x* ≥ −0.25** using inequality notation

[−0.25, ∞) using interval notation

This is confirmed with the table for values of x less than the first point in its domain.


NORMAL FLOAT AUTO REAL RADIAN MP
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X	Y1
-1.5	ERROR
-1.25	ERROR
-1	ERROR
-0.75	ERROR
-0.5	ERROR
-0.25	0
0	2
0.25	2.8284
0.5	3.4641
0.75	4
1	4.4721

X=-1.5

To find the range, notice the outputs of a radical are never negative. They are zero or larger.
This is confirmed with the graph.
We report the range as:

***y* ≥ 0** using inequality notation

[0, ∞) using interval notation

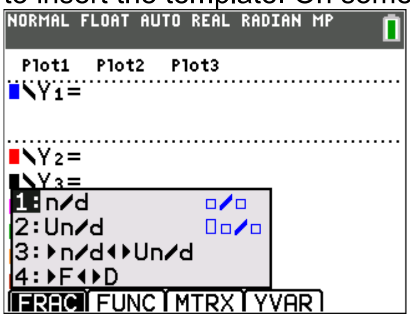
9) Find the domain and range.
Similar to 8d) we can use both the formula as well as the graph to find the answer.

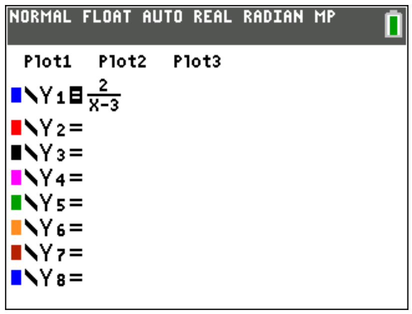
a) $f(x) = \frac{2}{x-3}$

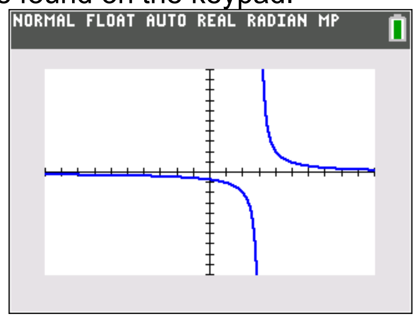
For the domain:
When the formula contains a fraction, remember the rule that division by zero is not allowed. Set the denominator x – 3 = 0, solve for *x*, and remember to **exclude** the solution value(s).
The domain is all reals but 3. We report the domain of *f*(*x*) as:

all reals but 3 or
***x* ≠ 3** or
(−∞, 3) U (3, ∞) using interval notation

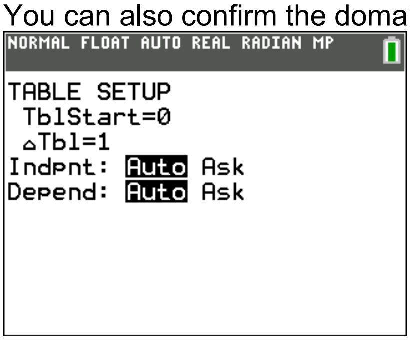
Using a grapher, the stacked fraction palette can be helpful. Press ALPHA, then Y=, then **1:n/d** to insert the template. On some calculator models it can also be found on the keypad.

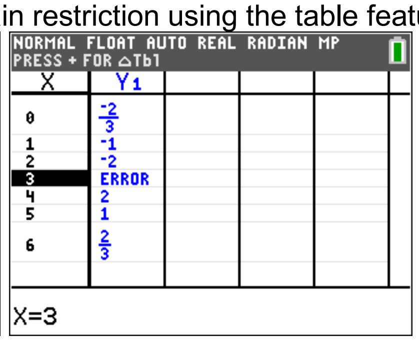






You can also confirm the domain restriction using the table feature.





For the range:
Outputs of a fraction can only be zero if its numerator is zero. Since *f*(*x*) has a numerator which is 2 ≠ 0, the function can never be zero. We report the range of *f*(*x*) as:

all reals but 0 or
***y* ≠ 0** or
(−∞, 0) U (0, ∞) using interval notation

b) $g(x) = \sqrt{x+3}$

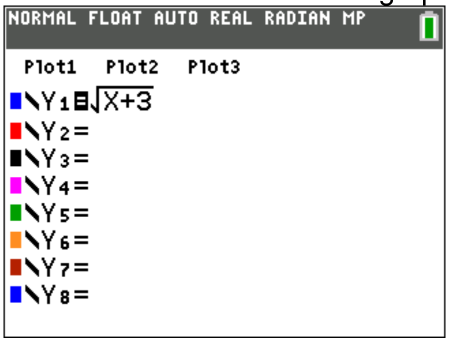
See Question 8d for a similar example.
For the domain:
When a formula contains the square root of an input (or any *even* root), remember the rule that we can not take the even root of a negative value and have the output be a real number.
The only values of *x* are allowed which make the radicand *x* + 3 nonnegative,
we can solve the inequality *x* + 3 ≥ 0. However, this is just like solving the equation *x* + 3 = 0, only with ≥ instead of =.

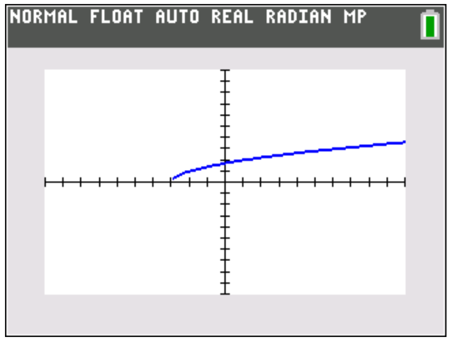
x + 3	≥	0	Subtract 3 from both sides.
x	≥	**−3**	

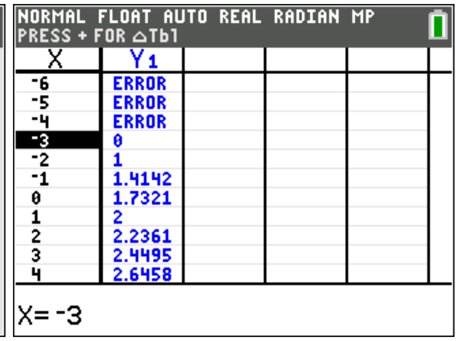
We report the domain of *f*(*x*) as:

***x* ≥ −3** using inequality notation
[−3, ∞) using interval notation

We can confirm this with the graph and the table.







To find the range, notice the outputs of a radical are never negative. They are zero or larger.
This is confirmed with the graph. We report the range as:

$y \geq 0$ using inequality notation
[0, ∞) using interval notation

c) $h(x) = \sqrt{3-x}$

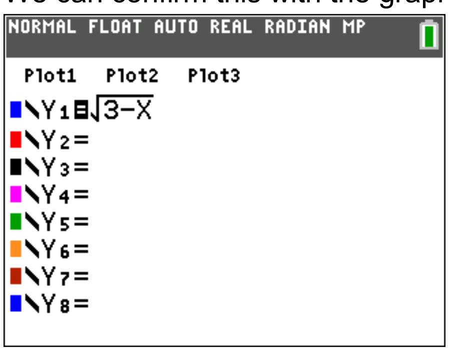
Using algebra, similar to part 9b, solve the inequality below:

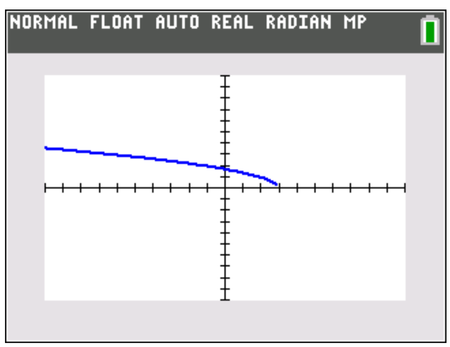
$3 - x \geq 0$ Add x to both sides.
$3 \geq x$

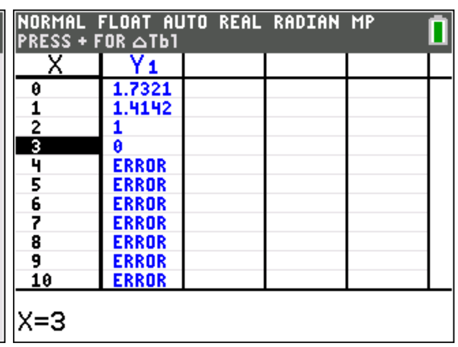
Be careful when a negative sign is in front of the x.
If you subtracted 3 from both sides and divided by −1, you would need to flip the inequality.
We report the domain of $f(x)$ as:

$x \leq 3$ using inequality notation
[3, ∞) using interval notation

We can confirm this with the graph and the table.







To find the range, notice the outputs of a radical are never negative. They are zero or larger.
This is confirmed with the graph.
We report the range as:

$y \geq 0$ using inequality notation
[0, ∞) using interval notation

d) $p(x) = \sqrt{x-3}$

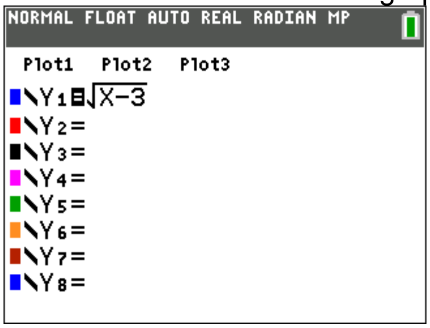
For the domain:
Using algebra, similar to part 9b and 9c, solve the inequality below:

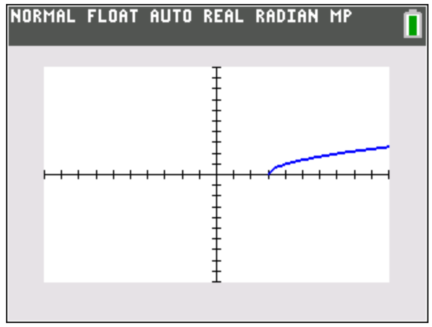
$x - 3 \geq 0$ Add 3 to both sides.
$x \geq 3$

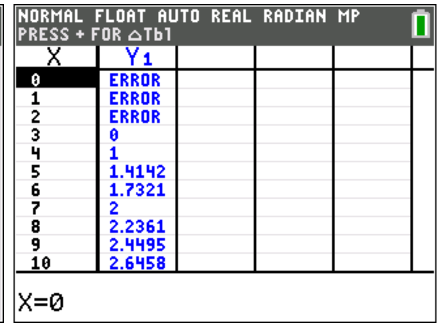
We report the domain of $f(x)$ as:

$x \geq 3$ using inequality notation
[3, ∞) using interval notation

We can confirm this with the graph and the table.







Fun fact: the graphs of $p(x) = \sqrt{x-3}$ and $h(x) = \sqrt{3-x}$ in part c are actually mirror images of each other about the vertical line *x*= 3. So their domains reflect this when you examine their tables.

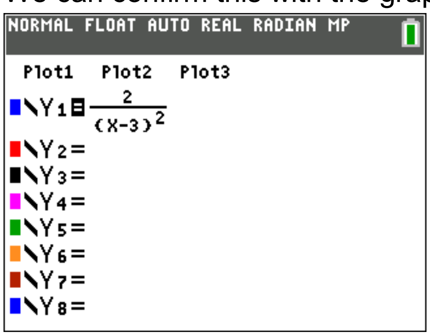
We report the range as:
***y* ≥ 0** using inequality notation
[0, ∞) using interval notation

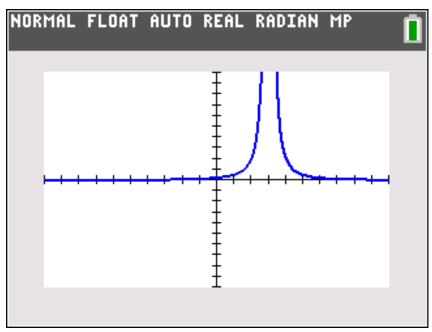
e) $q(x) = \frac{2}{(x-3)^2}$

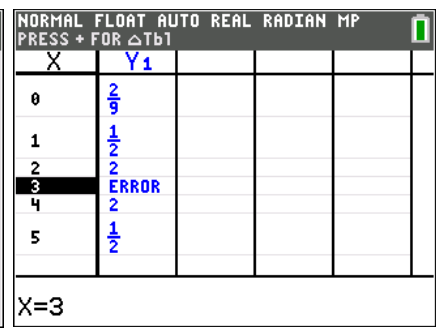
See Question 9a for a similar example. Remember the rule that division by zero is not allowed.
Set the denominator x – 3 = 0, solve for *x*, and remember to **exclude** the solution value(s).
We report the domain of *f*(*x*) as:

all reals but 3 or
***x* ≠ 3** or
(−∞, 3) U (3, ∞) using interval notation

We can confirm this with the graph and the table.







For the range:
Outputs of a fraction can only be zero if its numerator is zero. Since *f*(*x*) has a numerator which is 2 ≠ 0, the function can never be zero. In addition, outputs of any real number that is squared can never be negative. So, using the graph and the formula, we report the range as all positive numbers or:

***y* > 0** using inequality notation
(0, ∞) using interval notation

10) The entire graph of *g*(*x*) is shown.

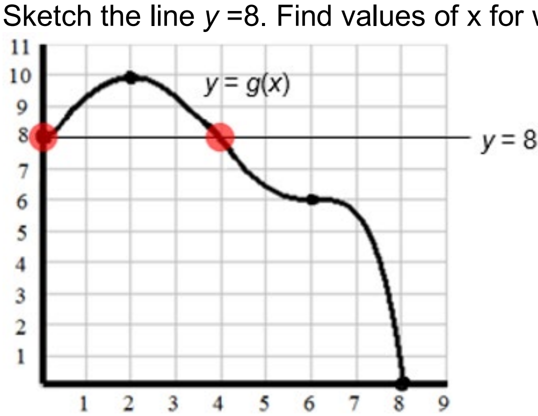
a) What is the domain of *g*?
The domain consists of all *x*-values for which the graph actually exists.
Move left to right across the graph, noting the first *x*-value, the last *x*-value, and any *x*-values of points in between.

0 ≤ *x* ≤ 8 using inequality notation
[0, 8] using interval notation

b) What is the range of *g*?
The range consists of all possible *y*-values. Move bottom to top, note the lowest *y*-value, the highest *y*-value, and any *y*-values of points in between.

0 ≤ *y* ≤ 10 using inequality notation
[0, 10] using interval notation

i) Report all values of *x* which solve the equation *g*(*x*) = 8.
Sketch the line *y* =8. Find values of x for which the graph of *g* **intersects** the line *y* = 8

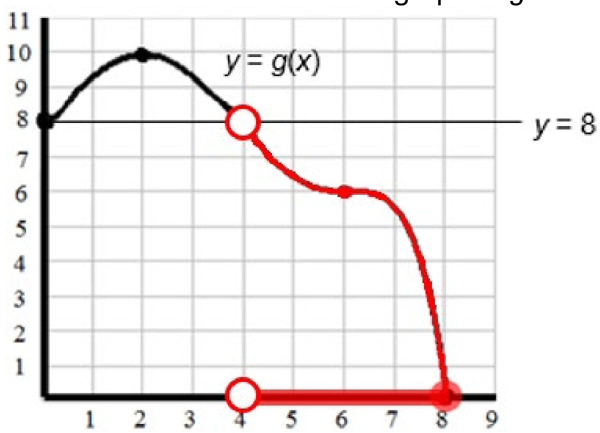


The solution is *x* = **0, 4**.

c) Solve $g(x) \geq 8$.
Find values of x for which the graph of g is **above** the line $y = 8$ or **intersects** the line $y = 8$.

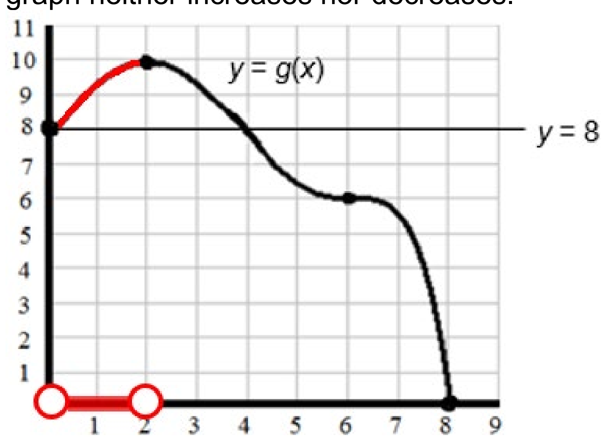
$0 \leq x \leq 4$ using inequality notation
[0, 4] using interval notation

d) Solve $g(x) < 8$.
Find values of x for which the graph of g is **below** the line.



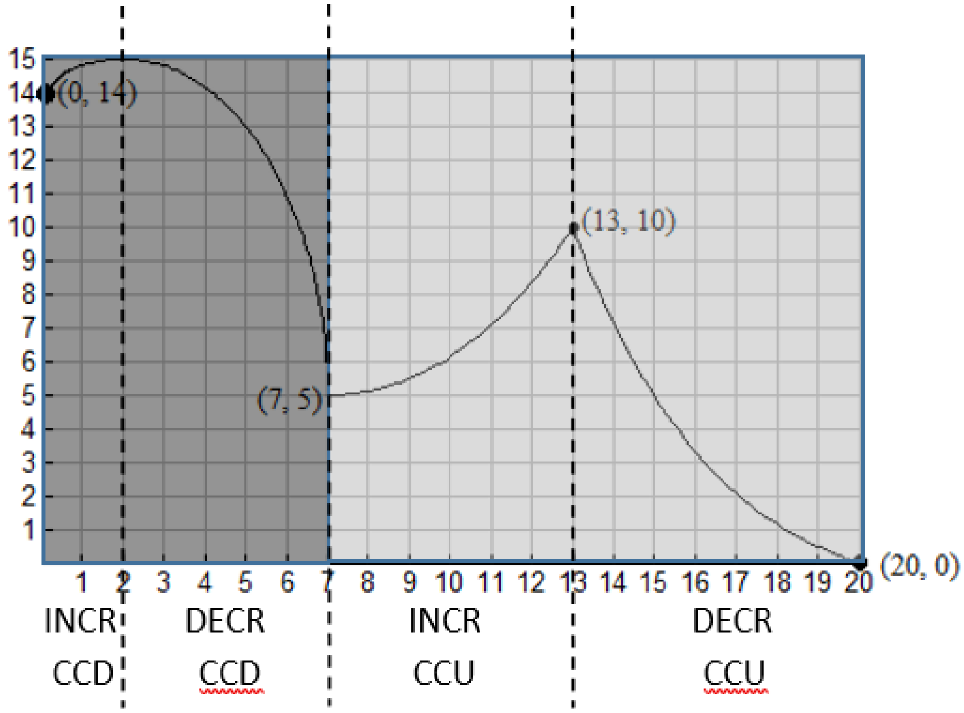
$4 < x \leq 8$ using inequality notation
(4, 8] using interval notation

e) For what values of x is the function increasing? ____ < x < ____
A graph is increasing wherever it rises as you move from left to right, so look for the intervals of x-values where the graph is climbing. Notice we exclude the endpoints because at the endpoint, the graph neither increases nor decreases.



The graph increases when **$0 < x < 2$**.

11) The entire graph of $y = h(x)$ is shown. Dashed vertical lines and use of light and dark shading are used to identify increasing and decreasing sections and concavity.



a) The function is concave down in the dark shaded region, or on the interval $\mathbf{0 < x < 7}$.

b) The function is increasing and concave down in the part of the dark shaded region that starts at the point (**0**, 14) and ends at the point (**2**, 15), so the open interval of x is $\mathbf{0 < x < 2}$.

c) The function is increasing and concave up in the part of the light shaded region that starts at the point (**7**, 5) and ends at the point (**13,** 10), so the open interval of x is $\mathbf{7 < x < 13}$.

d) The function is decreasing and concave down in part of the dark shaded region that starts at the point (**2**,15) and ends at the point (**7**, 5), so the open interval of x is $\mathbf{2 < x < 7}$**.**

e) The function is decreasing and concave up in the part of the light shaded region that starts at the point (**13**, 10) and ends at the point (**20**, 0), so the open interval of x is $\mathbf{13 < x < 20}$.

12) You need to rent a car and compare the charges of three different companies.

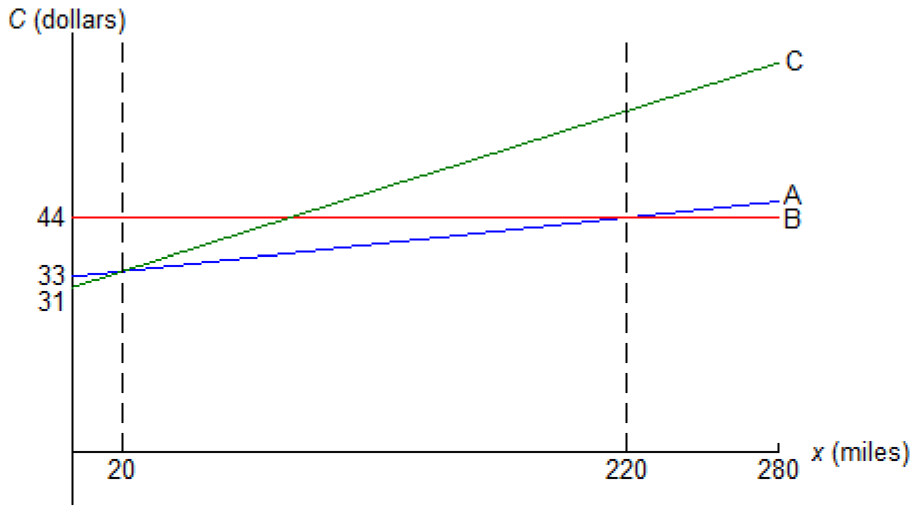
- Company A charges 5 cents per mile plus 33 dollars per day.
- Company B charges 44 dollars per day with no mileage charge.
- Company C charges 15 cents per mile plus 31 dollars per day.

a) Find formulas for the cost of driving cars rented from companies A, B, and C, in terms of x, the distance driven in miles in one day.

$\mathbf{y_A = 33 + 0.05x}$
$\mathbf{y_B = 44}$
$\mathbf{y_C = 31 + 0.15x}$

b) Graph the costs for each company for $0 \le x \le 500$. Put all three graphs on the same set of axes. Use this graph to find under what circumstances each agency is the cheapest.

Graphs A, B, and C below compare the three cost functions to show which option is the cheapest. Vertical dashed lines mark the *x*-values where the lowest graph changes, indicating when a different option becomes the least expensive. The intersection of graphs B and C is not relevant, since it does not change which graph is lower at that point.



To find the intervals when each line is below the other two, first find the intersection points. Remember we only need to find two intersection points. Solving algebraically, we have:

y_C	=	y_A	
$31 + 0.15x$	=	$33 + 0.05x$	Subtract 31 from both sides.
$0.15x$	=	$2 + 0.05x$	Subtract $0.05x$ from both sides.
$0.10x$	=	2	Divide both sides by 0.10.
x	=	20	

y_A	=	y_B	
$33 + 0.05x$	=	44	Subtract 33 from both sides.
$0.15x$	=	11	Divide both sides by 0.15.
x	=	220	

You can also use a grapher and an intersection feature to find the intersection points.

We conclude:
Company A (the blue graph) is cheapest **when you drive between 20 and 220 miles**.
Company B (the red graph) is cheapest **when you drive more than 220 miles**.
Company C (the green graph) is cheapest **when you drive less than 20 miles**.

13) $\boldsymbol{d = 122 - 43t}$

14) In a college meal plan you pay a membership fee; then all your meals are at a fixed price per meal. Suppose 30 meals cost \$265 and 60 meals cost \$460

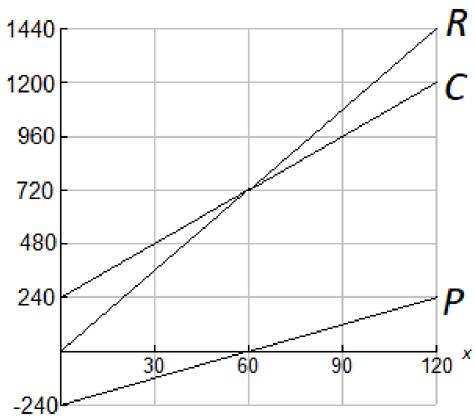
a) Write a formula for the cost of a meal plan, *C*, in terms of the number of meals, *n*.
The change in *C*, $\Delta C = 460 - 265 = 195$ dollars**.**
The change in *n*, $\Delta n = 60 - 30 = 30$ meals.

The average rate of change is $\frac{\Delta C}{\Delta n} = \frac{195\ dollars}{30\ meals}$ = 6.5 dollars per meal. So
$C = 6.5t + b$

Substitute a point, such as $n = 30$, $C = 265$ (or the other point, your choice.)
$C = 6.5n + b$
$265 = 6.5 \cdot 30 + b$
$265 = 195 + b$
$70 = b$
The formula is $\mathbf{C = 6.5n + 70}$.

b) What is the price per meal? **$6.50**
c) What is the membership fee? **$70**

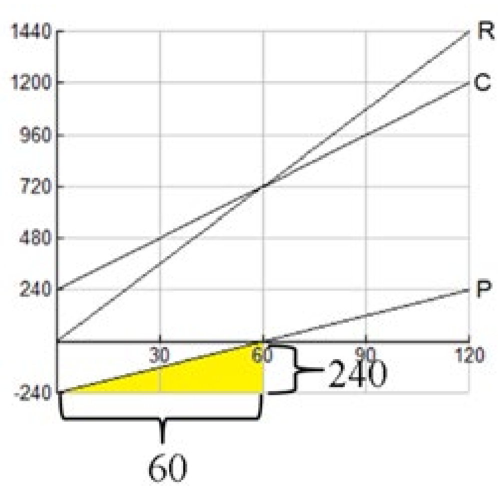
15) The revenue $R(x)$, cost $C(x)$, and profit $P(x)$ for a product are graphed in the figure below, where x is the quantity produced and sold. Note: $P(x) = R(x) – C(x)$.



a) Determine the number of items that must be sold to break even, i.e., revenue is equal to costs.
This is the x-value of the intersection point of R and C, or, equivalently, the x-value of x-intercept of the profit P. The break-even quantity is **60** units sold.

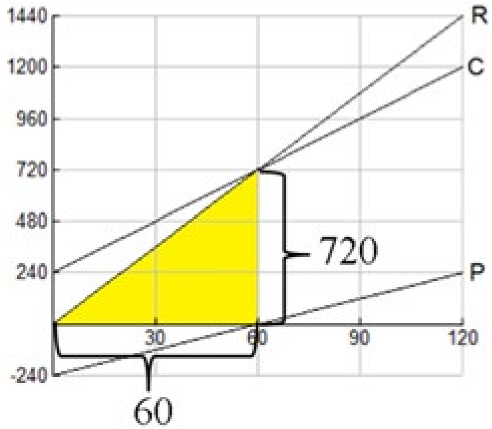
b) Find the formulas of the three functions revenue $R(x)$, cost $C(x)$, and profit $P(x)$.
Similar to previous questions, we can use the grid to find the slopes.
In the graph we are given, the horizontal grid marks and vertical grid marks are **NOT** the same units. Horizontal grid marks are 30 apart, and vertical grid marks are 240 apart.

i) For the profit $P(x)$, we can draw a slope triangle, as shown below, to find the slope of the line.
$\frac{\Delta P}{\Delta x} = \frac{\$240}{60\ items}$ = $4 per item.



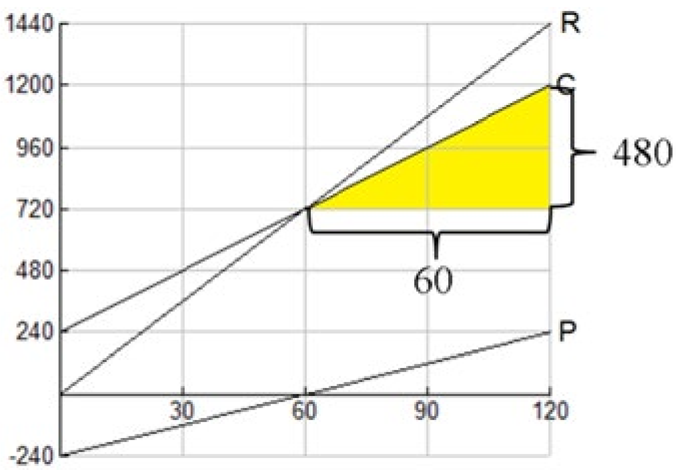
The vertical intercept is (0, –240). So for $P(x)$, the formula is $P(x) = \mathbf{4x – 240}$.

ii) For the revenue $R(x)$, from the slope triangle below, we have $\frac{\Delta R}{\Delta x} = \frac{\$720}{60\ items}$ = \$12 per item.



The vertical intercept is (0,0). So for $R(x)$, the formula is $R(x)$ = **12*x***.

iii) For the cost $C(x)$, from the slope triangle below, we have $\frac{\Delta C}{\Delta x} = \frac{\$480}{60\ items}$ = \$8 per item.



The vertical intercept is (0, 240). So for $C(x)$, the formula is $C(x)$ = **8*x* + 240**.

To double check that this is correct, we can make sure that $P(x) = R(x) - C(x)$.

$R(x) = 12x$
$\underline{C(x) = 8x + 240}$
$P(x) = 4x - 240$

16) Given $f(x) = \frac{x}{x+8}$ and $g(x) = 4x - 3$, find $f(g(x))$ and simplify.

We have $f(\blacksquare) = \frac{(\blacksquare)}{(\blacksquare)+8}$

Replace the contents of (■) with $g(x) = 4x - 3$ and simplify.

$$f(g(x)) = \frac{(4x-3)}{(4x-3)+8} = \frac{4x-3}{4x+5}$$

Choice **C**.

17) Given $f(x) = \frac{x}{\sqrt{x+2}}$ and $g(x) = x^2 - 1$, find $f(g(x))$ and simplify. Select **one**.

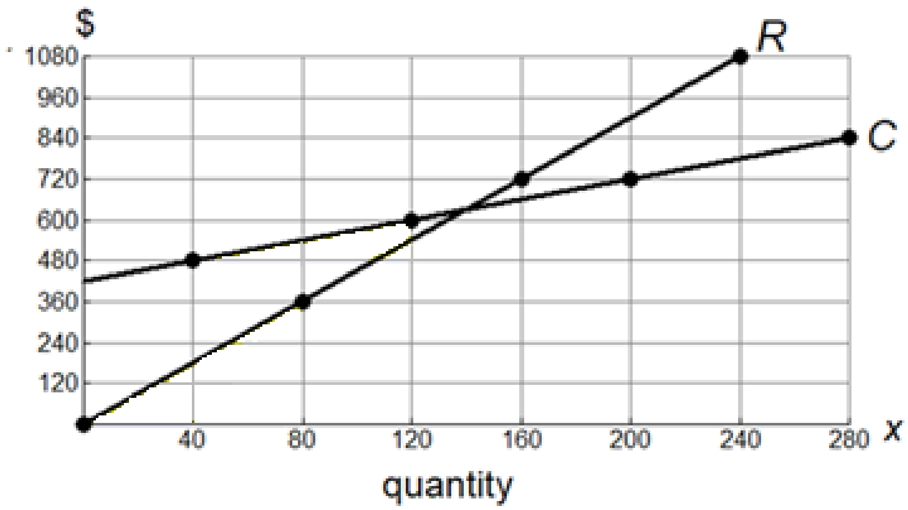
We have $f(\blacksquare) = \frac{(\blacksquare)}{\sqrt{(\blacksquare)+2}}$

Replace the contents of (■) with $g(x) = x^2 - 1$ and simplify.

$$f(g(x)) = \frac{(x^2-1)}{\sqrt{(x^2-1)+2}} = \frac{x^2-1}{\sqrt{x^2+1}}$$

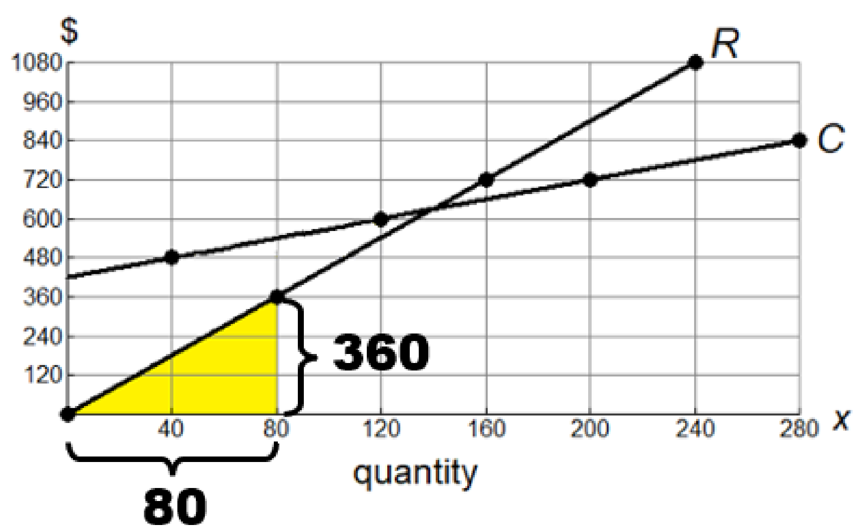
Choice **E**.

18) The graph below shows the revenue $R(x)$ and cost $C(x)$ functions for x units produced and sold.



Similar to previous questions, we can use the grid to find the slopes.
In the graph we are given, the horizontal grid marks and vertical grid marks are **NOT** the same units.
Horizontal grid marks are 40 apart, and vertical grid marks are 120 apart.

a) Find and interpret the slope of *R.* Then give its formula.
For the revenue $R(x)$, from the slope triangle below, we have $\frac{\Delta R}{\Delta x} = \frac{\$360}{80\ items}$ = \$4.50 per item.
Interpretation: **We earn \$4.50 in revenue for every item sold.**

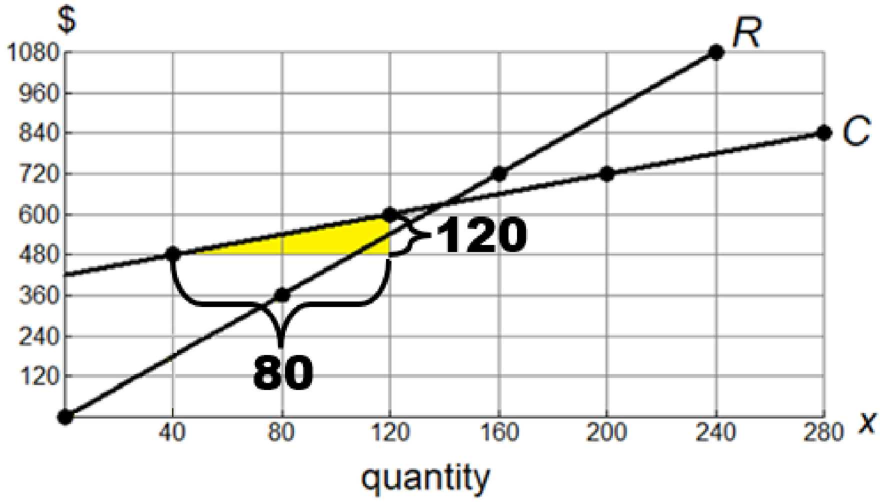


The vertical intercept is (0,0). So for $R(x)$, the formula is $R(x) = \mathbf{4.5x}$.

b) Find and interpret the slope of *C.* Then give its formula.

For the cost $C(x)$, from the slope triangle below, we have $\frac{\Delta C}{\Delta x} = \frac{\$120}{80\ items}$ = \$1.50 per item.

Interpretation: **It costs \$1.50 to produce each item.**



To find the formula for *C*, substitute a point, say $x = 40$, C = 480:

$C = 1.5x + b$

$480 = 1.5 \cdot 40 + b$

$480 = 60 + b$

$420 = b$

Note: Check that the value of 420 is consistent with the vertical intercept of *C* on the grid.
We see that 420 is between the two grid marks, 360 and 480.
The formula is C = $\mathbf{1.5x + 420}$

c) Find and interpret the slope of the profit function *P.* Then give its formula. Hint: $P = R - C$.

We can find the profit *P* by subtraction, i.e., $P = R - C$.

$P = R \quad - \quad C$

$P = 4.5x - (1.5x + 420)$

$P = 3x - 420$

The slope of the profit function is \$3 per item.

Interpretation of the slope: **The company nets \$3 in profit for each item sold.**

d) Write and solve an equation to find the break-even quantity.

Method 1: Solve $P = 0$

$3x - 420$	$= 0$	Add 420 to both sides.
$3x$	$= 420$	Divide both sides by 3.
x	$=$ **140**	

Method 2: Solve $R = C$

$4.5x$	$= 1.5x + 420$	Subtract $1.5x$ from both sides.
$3x$	$= 420$	Divide both sides by 3.
x	$=$ **140**	

Note: Check that the value of 140 is consistent with the intersection point on the grid.
We see that 140 is between the two grid marks, 120 and 160.

19) Scoop Dogg runs an Ice Cream Parlor in a small town in Alaska.
At a price of p = \$0.20 per scoop, q = 950 scoops per day are sold.
At a price of p = \$1.00 per scoop, q = 850 scoops per day are sold. Assume linearity.

a) Find a formula which gives q (quantity, in scoops) as a function of p (price, in dollars).

We might find it helpful to represent the above in a table.

***p*, dollars**	***q*, scoops**
\$0.20	950
\$1.00	850

Use the table to find the slope, $\frac{\Delta q}{\Delta p}$.

Find the change in p or $\Delta p = p_2 - p_1$. We have Δp= \$1.00 – \$0.20 = \$0.80.
Find the change in q, or $\Delta q = q_2 - q_1$. We have Δq= 850 – 950 = –100 scoops
The average rate of change is $\frac{\Delta q}{\Delta p} = \frac{-100}{0.8} = -125$ scoops per dollar increase in price.
In other words, for every \$1 increase in price, 125 less scoops per day are sold or
for every \$0.80 increase in price, 100 less scoops per day are sold or
for every \$1 decrease in price, 125 less scoops per day are sold.
We have multiple ways to find the formula.

Method 1:

$$q = -125p + b$$
$$850 = -125\cdot 1 + b$$
$$850 + 125 = b$$
$$975 = b$$

So $q = f(p) =$ $\mathbf{-125p + 975}$

Method 2:

$q = -125(p - 0) + 850$
$q = -125p + 125 + 850$
$q = -125p + 975$
So $q = f(p) =$ $\mathbf{-125p + 975}$.

b) Find $f(0)$ and $f^{-1}(0)$. Include units.

i) To find $f(0) = q$, we replace $q = -125p + 975$ with $p = 0$.
$q = -125\cdot 0 + 975 =$ **975 scoops per day**
$f(0) =$ **975**

ii) To find $f^{-1}(0)$, recall that $f(\mathbf{p}) = \mathbf{q} \Leftrightarrow f^{-1}(\mathbf{q}) = \mathbf{p}$.
In particular, $f^{-1}(\mathbf{0}) = \mathbf{p} \Leftrightarrow f(\mathbf{p}) = \mathbf{0}$.

The result of $f^{-1}(0)$ will be the value of p, which is the number of **dollars** per scoop that is charged.
To find $f^{-1}(0) = p$, we replace $q = -125p + 975$ with $q = 0$. and solve for p.

$-125p + 975 = 0$ Add $125p$ to both sides.
$975 = 125p$ Divide both sides by 125.
$p = \frac{975}{125} = 7.8$

Thus $p = f^{-1}(0) =$ **7.8**.

c) Interpret the meaning of your answer to part **bi**. in practical, real world terms.
From part **(b)**, we have $q = f(0) = 975$ scoops of ice cream.
Interpretation:
If the price were free, they would give out *f*(0) scoops (or 975 scoops) of ice cream per day.

d) Interpret the meaning of your answer to part **bii**. in practical, real world terms.
From part **(b)**, we have $p = f^{-1}(0) = 7.8$ days.
Interpretation:
If the price were as high as $f^{-1}(0)$ dollars (or \$7.80) per scoop of ice cream, none would be sold.

e) Use part b to complete the blanks:
i) f^{-1}(____) = 0
ii) f (____) = 0

Since $f^{-1}(0) = 7.8$, we have $f(\mathbf{7.8}) = 0$.
Since $f(0) = 975$ we have $f^{-1}(\mathbf{975}) = 0$.

f) To find the formula for $p = f^{-1}(q)$, we solve the equation $q = -125p + 975$ in general for p.
From parts **b** and **e**, we should expect $f^{-1}(0) = 975$ and $f^{-1}(7.8) = 0$.

q	$= -125p + 975$	Add $125p$ to both sides.
$q + 125p$	$= 975$	Subtract q from both sides.
$125p$	$= 975 - q$	Divide both sides by 125.
p	$= \frac{975-q}{125}$	

Thus $p = f^{-1}(q) = \mathbf{\frac{975-q}{125}}$ **.**

There are any equivalent ways to write the above formula.
We could divide both terms of the numerator by 125 to obtain
$f^{-1}(q) = = \frac{975-q}{125} = \frac{975}{125} - \frac{q}{125} = 7.8 - \frac{q}{125} = \mathbf{7.8 - 0.008}\boldsymbol{q}$.

Note: This formula confirms the earlier result in parts **b** and **e** that $f^{-1}(0) = 7.8$ and $f^{-1}(975) = 0$.

Would you like more practice over specific topics?
See the Flash Cards for Sections 1.1-1.5, 2.1, 2.2, and 2.5-2.6 as well as the Just for Practice sets.
Find these in your Brightspace course in the module **Flash Cards and Just for Practice Sets**.