KEY to Practice Questions to help you prepare for MA 15300 Final Exam

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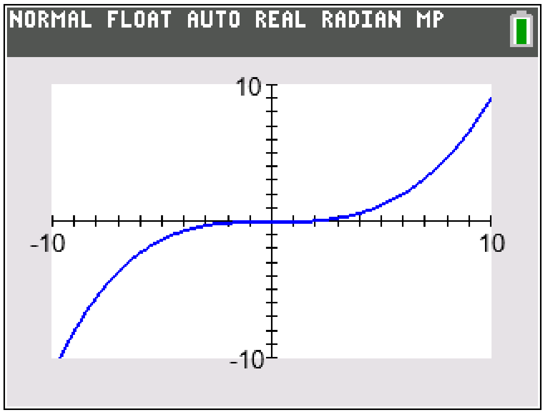
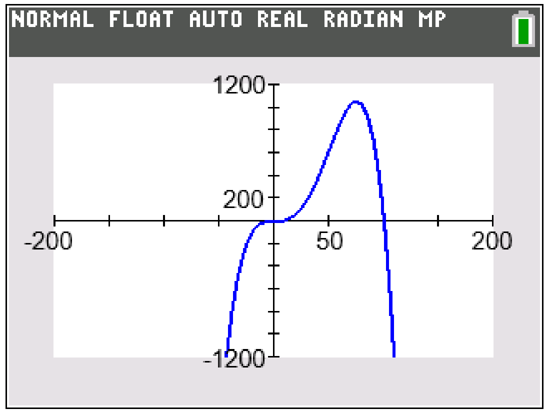
1) The long run behavior of $y = 0.01x^3 - 0.0001x^4$ is the same as its leading term, which is $-0.0001x^4$.
We thus expect a long run behavior that we can describe as both arms going down.
If Eustace were to expand his graphing calculator with the window $-200 \le x \le 200$ and $-1200 \le y \le 1200$, he would see the full long run behavior.





The actual long run behavior is as follows:
"As $x \to -\infty$, then $y \to -\infty$.
As $x \to \infty$, then $y \to -\infty$."   This is Choice **C**.
Fun fact. It turns out this polynomial can be factored: $0.01x^3 - 0.0001x^4 = 0.01x^3(1 - 0.01x)$ which shows, if we set each factor equal to 0, that the polynomial has two zeros, 0 and 100. So it must turn around and cross the x-axis, after which, it descends forever.

2) We have $C(t) = \frac{P(t)}{R(t)} = \frac{360+9t}{12000+12t}$. Therefore $C(0) = \frac{360+9(0)}{12000+12(0)} = \frac{360}{12000} = 0.3$ or 3%. Choice **B**.

3) As t gets larger and larger, the function $C(t) = \frac{360+9t}{12,000+12t}$ approaches the ratio of the leading terms, namely $\frac{9t}{12t} = 0.75$. Eventually 75% of the reservoir's total volume would consist of pollutants. This can be confirmed with a graph of the function or a view of its table for large values of t. Choice **E**.

4) $Q = 20(0.4)^t = 20(1 - 0.6)^t$, so 60% of the drug is lost per hour. Choice **E**.

5) The growth factor of $y = ab^t$ is b. For $Q = 20(0.4)^t$, $b = 0.4$. Choice **A**.

6) There are zeros at 0, 2, and 7. Therefore:

$$x = 1,\ y = -1 \Longrightarrow \begin{aligned} -1 &= k(1)(1-2)(1-7) \\ -1 &= k(-1)(-6) \\ k &= -\tfrac{1}{6} \end{aligned}$$

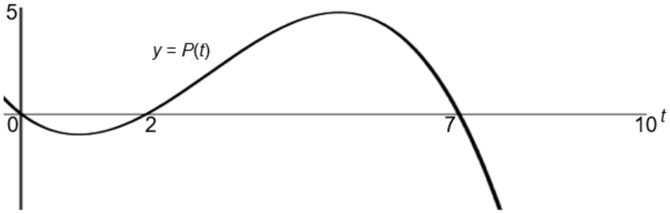
The minimum value of $P(t)$ in the first ten seconds must be $P(10) = -40^\circ\text{C}$. This can be found using a graph or table or by evaluating for $t = 10$. $P(10) = -\frac{1}{6}(10)(10-2)(10-7) = -\frac{1}{6}(10)(8)(3) = -40$.

$P(t) = -\frac{1}{6}t(t-2)(t-7)$. Choice **D**.

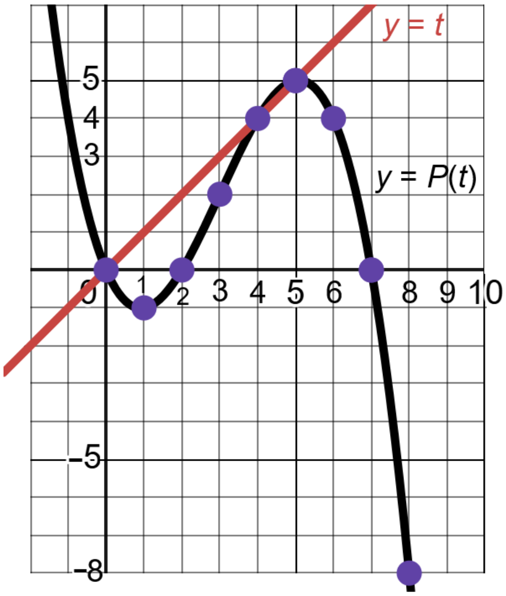
7) $Q(t) = -\frac{1}{6}t^3$ + *remaining terms of lower degree*

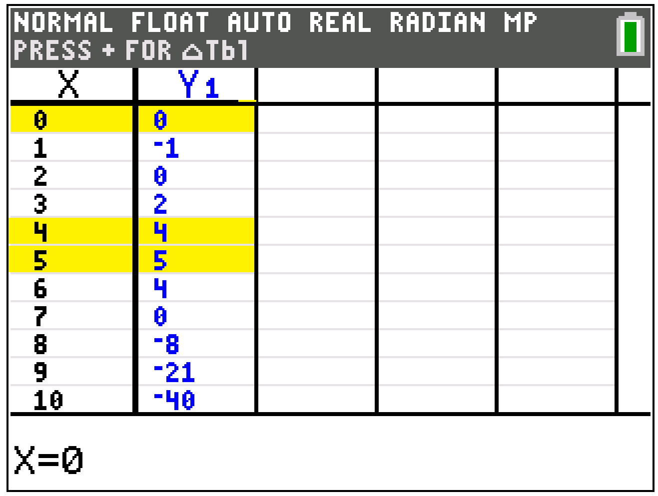
Therefore, $P(t)$ and $Q(t)$ look nearly indistinguishable for large values of t. Note that the $-\frac{1}{6}$ is not optional. You can check this by graphing $P(x)$, $Q(x)$ and $y = x^3$ in a large window. Choice **C**.

8) We can not simply eyeball the given graph since the scales are different on each axis.



Using a graph or table on a graphing calculator, we see that $P(t) = t$ at the values $t = 0$, 4, and 5.

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X	Y1
0	0
1	-1
2	0
3	2
4	4
5	5
6	4
7	0
8	-8
9	-21
10	-40

X=0

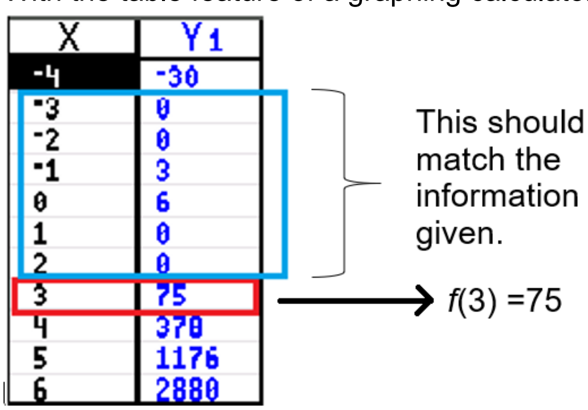
Choice **E**.

9) Because the polynomial has single zeros at –3, 1, and 2 and a double zero at –2 we can write $y = k(x-2)(x-1)(x+3)(x+2)^2$ Now substitute the point (0,6):

$$x = 0,\ y = 6 \Rightarrow \quad 6 = k(0-2)(0-1)(0+3)(0+2)^2$$
$$6 = k(-2)(-1)(3)(4)$$
$$6 = 24k$$
$$k = \frac{1}{4} = 0.25$$

Therefore $f(x) = 0.25(x-2)(x-1)(x+3)(x+2)^2$

With the table feature of a graphing calculator we can check that the formula is correct and find $f(3) = 75$.



X	Y1
-4	-30
-3	0
-2	0
-1	3
0	6
1	0
2	0
3	75
4	378
5	1176
6	2880

You could also substitute $x = 3$: $f(3) = 0.25(3-2)(3-1)(3+3)(3+2)^2 = 0.25(1)(2)(6)(5)^2 = 75$.

Choice **B**.

10) Utilize the fortunate coincidence that a value of $t = 1$ is given in the table.
For a power function $y = kt^p$, when $t = 1$, the value of $y = k \cdot 1^p = k \cdot 1 = k$.

***t*, days**	***E*(*t*), employees who relocated**
0	0
1	**This is *k* ⟹ 20**
16	160

So for $E(t) = kt^p$, the value of $k = 20$ and $E(t) = 30t^p$.
Use the other point to find p.

$t = 16$, $y = 160 \Rightarrow 160 = 20 \cdot 16^p$ Divide both sides by 20
$8 = 16^p$ Take logarithms of both sides (common or natural)
$\log 8 = \log 16^p$ Use the power property
$\log 8 = p \cdot \log 16$ Divide both sides by log 16
$p = \dfrac{\log 8}{\log 16}$ Calculate
$p = 0.75$

This means $E(t) = 20t^{0.75}$ and $E(7) = 20(7)^{0.75} \approx 86$. Choice **B**.

11) Since we do not have a value of $t = 1$ given in the table, we substitute values in the formula $y = k \cdot t^p$ and divide the equations to eliminate k.

$t = 12$, $y = 67 \Rightarrow 67 = k \cdot 12^p$
$t = 16$, $y = 96 \Rightarrow 96 = k \cdot 16^p$

It does not matter which equation we divide by which:

$$\frac{67}{96} = \frac{k \cdot 12^p}{k \cdot 16^p}$$

$$\frac{67}{96} = \frac{12^p}{16^p} = \left(\frac{12}{16}\right)^p$$

Take logarithms of both sides (common or natural):

$$\log \frac{67}{96} = \log \left(\frac{12}{16}\right)^p$$

Use the power property and solve for p by dividing both sides by $\log \left(\frac{12}{16}\right)$:

$$\log \frac{67}{96} = p \log \left(\frac{12}{16}\right)$$

$$p = \frac{\log \frac{67}{96}}{\log \left(\frac{12}{16}\right)} \approx 1.25018416689 \text{ or, to two decimal places, } 1.25.$$

Use one of the other points to find k.

$t = 12$ $y = 67 \Rightarrow 67 = k \cdot 12^{1.25}$ Divide both sides by $12^{1.25}$
$k = \dfrac{67}{12^{1.25}}$ Calculate
$k = 2.99984105967$ or, to the nearest whole number, 3.

This means $S(t) = 3t^{1.25}$ and $S(19) = 3(19)^{1.25} \approx 119.0044649$ or, to the nearest whole number, 119. .
Had we used the other point, $t = 16$, $y = 96$, to find k, we would have also had $k = 3$. Choice **D**.

12) We have $E(t) = 20t^{0.75}$ and $S(t) = 3t^{1.25}$ so we must create the function representing the percentage of sick employees, $\frac{S(t)}{E(t)} = \frac{3t^{1.25}}{20t^{0.75}} = \frac{3}{20}t^{1.25-0.75} = 0.15t^{0.5}$. We then solve $0.15t^{0.5} > 0.75$ for t.

The graph of $y = 0.15t^{0.5}$ is increasing for all t.

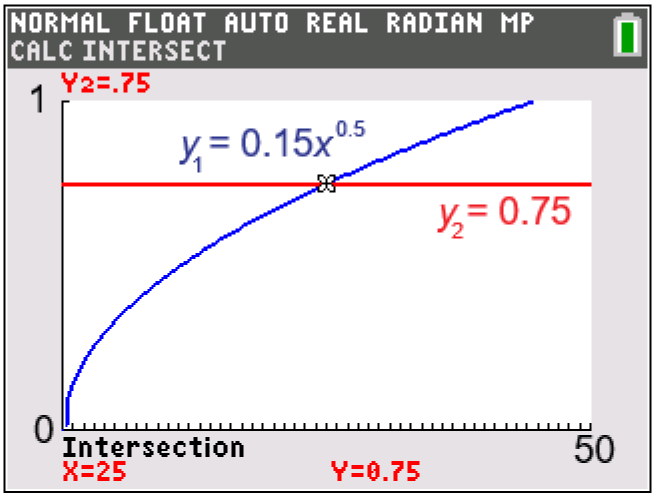
First find when y = 0.75 intersects $y = 0.15t^{0.5}$. We can solve graphically or use algebra.

Using algebra:

$0.15t^{0.5}$	=	0.75	Divide both sides by 0.15
$t^{0.5}$	=	$\frac{0.75}{0.15} = 5$	Square both sides
t	=	25	

To solve graphically, enter $Y1 = 0.15x^{0.5}$ and Y2 = 0.75 and find when they intersect.

You could also use the table feature. Both are shown below.



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X	Y1	Y2
18	0.6364	0.75
19	0.6538	0.75
20	0.6708	0.75
21	0.6874	0.75
22	0.7036	0.75
23	0.7194	0.75
24	0.7348	0.75
25	0.75	0.75
26	0.7649	0.75
27	0.7794	0.75
28	0.7937	0.75

X=25

The graph of $\frac{S(t)}{E(t)} = 0.15t^{0.5}$ first exceeds 0.75 at 25 days, which is when 75% of the relocated Pallidex employees at Rotting Hill have become sick. Choice **C**.

13) We have $e^{x\ln a} = e^{\ln a^x}$ by the power property of logarithms.

By the inverse property, since $e^{\ln Q} = Q$, then $e^{\ln a^x} = a^x$. Choice **E**.

14) The average rate of change is $\frac{\Delta V}{\Delta t}$. Find the change in time, Δt, and the change in volume, ΔV, over the intervals as shown below.

Δt	Time, t (min)	Volume, V (gal)	ΔV
	30	1075	
30 min	60	1150	75 gal
30 min	90	1225	75 gal
30 min	120	1300	75 gal

Taking ratios, we have that the volume climbs by 75 gallons every 30 minutes. The average rate of change is the rate at which the water fills the pool, $\frac{\Delta V}{\Delta t} = \frac{75 \text{ gal}}{30 \text{ min}}$ = 2.5 gallons per minute.

The answer is Choice **E**.

15) We are given the vertex (25, 275). In real world terms, this means that 25 lb of fertilizer produces a maximum yield of 275 pecks of peppers. Choice **B**.

16) Without applying any fertilizer at all, the vertical intercept of the graph indicates that if m = 0, then the orchard will produce 150 pecks of peppers. Choice **C**.
17) The range is $0 \le f(m) \le 275$. Note: You can also write [0, 275]. Choice **E**.
18) The function $f(m)$ is **increasing** for $0 \le m < 25$. Choice **C**.
Note: The function $f(m)$ is **decreasing** for $m > 25$.
19) The function $f(m)$ is never **concave up**. It is **concave down** for all values of m. Choice **E**.
20) $f(m) > 150$ for $0 < m < 50$.
Determine where the graph of $y = f(m)$ is above the line $y = 150$.
The yield is more than 150 pecks of peppers when the amount of fertilizer applied is more than 0 lb and less than 50 lb. Choice **D**
21) We first find a formula for $f(m)$. We can use vertex form or factored form followed by a shift.
Both are shown below but only one form is needed.

Vertex Form:

To find the vertex form, use a shift transformation of the graph of $y = ax^2$ (left 25 and up 275).
We have $y = a(x - 25)^2 + 275$. Substitute the point (0, 150).

$x = 0, y = 150 \Longrightarrow$ y	=	$a(x - 25)^2 + 275$	Substitute.
150	=	$a(0 - 25)^2 + 275$	Subtract 275 from both sides.
-125	=	$625a$	Divide both sides by 625.
a	=	$\frac{-125}{625} = -0.2$	

We have $y = -0.2(x - 25)^2 + 275$ or, equivalently, $P(t) = -0.2(x - 25)^2 + 275$.

Factored Form (plus a shift):

If we shift the parabola down 150, we have a parabola with the same shape as $f(x)$ but with zeros at 0 and 50 and a maximum of (25, 125) since $125 = 275 - 150$.

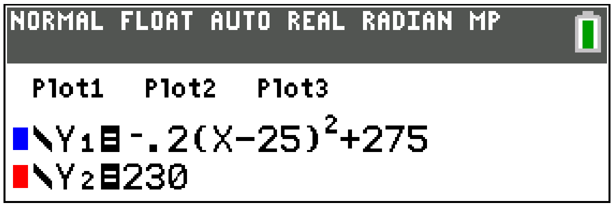
The factored form of the shifted parabola is $y = ax(x - 50)$, but from our work above*, $a = -0.2$.
Shifting this up 150, we have $f(x) = -0.2x(x - 50) + 150$ or $P(t) = -0.2t(t - 50) + 150$.
*If we had not solved for a already, you can also find a by substituting the point (25, 125).
However, the leading coefficient a is the same for expanded form, vertex form, and factored form, so if you already have one of these formulas, you have a. Here is how to find a with the factored form:
We have $y = ax(x - 50) + 150$. Substitute the point (25, 275).

$x = 25, y = 275 \Longrightarrow$ y	=	$ax(x - 50) + 150$	Substitute.
275	=	$a \cdot 25 \cdot (25 - 50) + 150$	Subtract 150 from both sides.
125	=	$a \cdot 25 \cdot (-25)$	
125	=	$-625a$	Divide both sides by –625.
a	=	$\frac{125}{-625} = -0.2$	

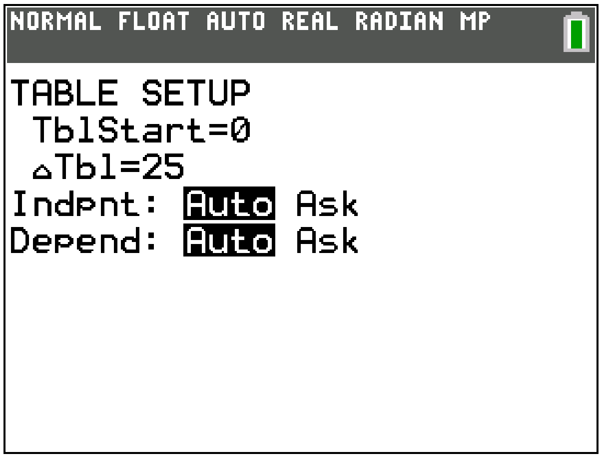
This gives us $P(t) = -0.2t(t - 50) + 150$.

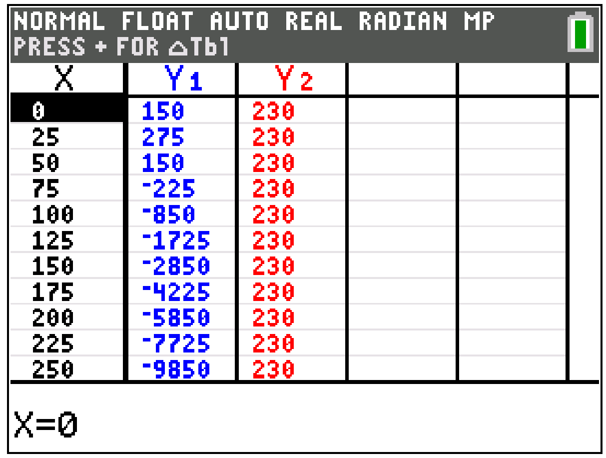
a) We now use the formula to approximate the solutions to $f(m) = 230$.
Enter both the function in your grapher and the line $y = 230$.

Since the question provided a graph, so we can use that to set a viewing window.
One possibility is $0 \le x \le 75$ by $0 \le x \le 300$.
TIP: Check your graph passes through the given points shown in the question.



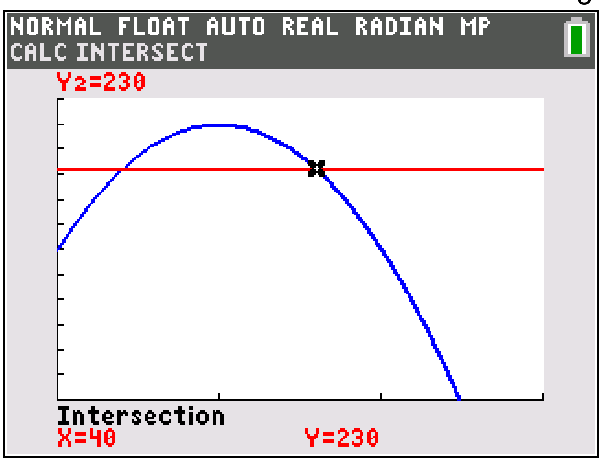
Press 2nd [WINDOW] for the TblSet screen, making TblStart = 0 and ΔTbl = 25.

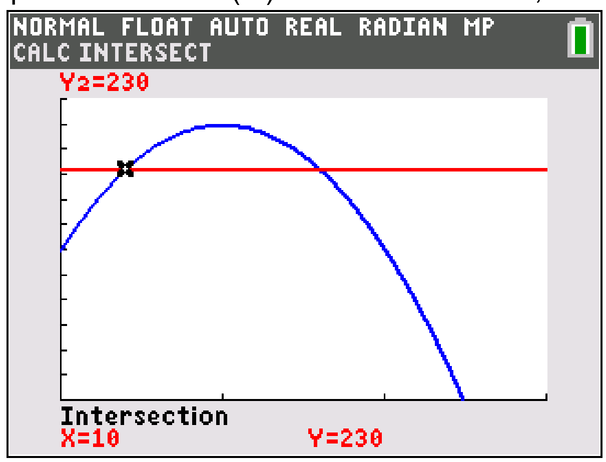




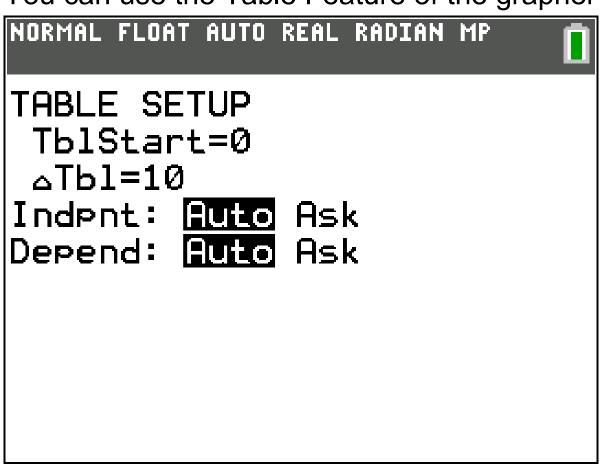
X	Y_1	Y_2
0	150	230
25	275	230
50	150	230
75	-225	230
100	-850	230
125	-1725	230
150	-2850	230
175	-4225	230
200	-5850	230
225	-7725	230
250	-9850	230

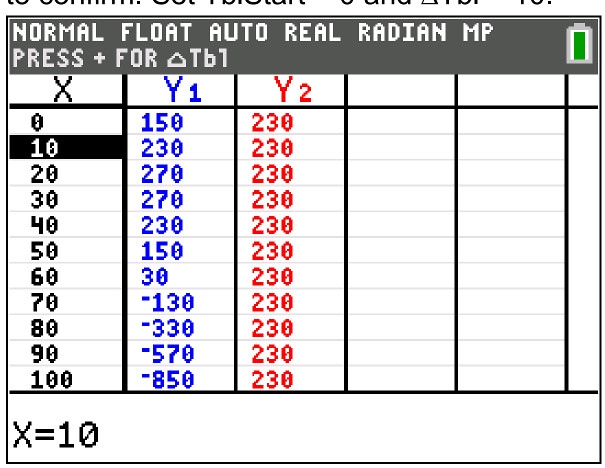
You can use the Intersection Feature of the grapher to find that *f*(*m*) = 230 when *m* = 10, 40.





You can use the Table Feature of the grapher to confirm. Set TblStart = 0 and ΔTbl = 10.

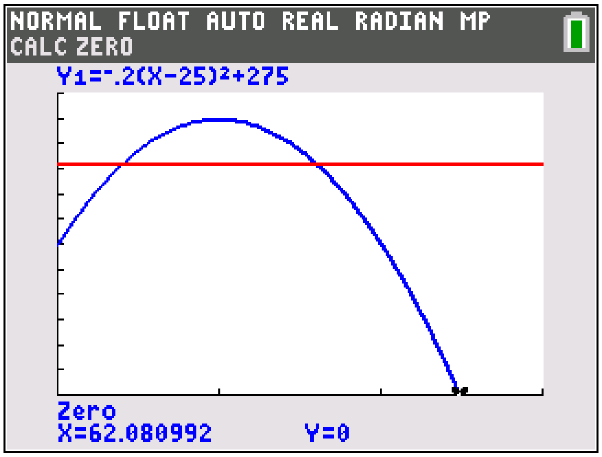




X	Y_1	Y_2
0	150	230
10	230	230
20	270	230
30	270	230
40	230	230
50	150	230
60	30	230
70	-130	230
80	-330	230
90	-570	230
100	-850	230

The solutions are *m* = 10 and *m* = 40 so we have Choice **D**.

b) With the formula we can use the calculator to find that the positive zero is $m \approx 62.080992$. To the nearest whole number, $m \approx 62$ so we have Choice **C**.



We could also solve the equation $P(t) = -0.2t(t - 50) + 150$ or $P(t) = -0.2(x - 25)^2 + 275$ by writing either in expanded form and then solving the resulting quadratic equation $-0.2t^2 + 10t + 150 = 0$, either by factoring or by using the quadratic formula. However, this approach is more time-consuming and is more likely to lead to algebraic or mechanical errors.

22) We substitute values in the formula $y = a \cdot b^t$ and divide the equations to eliminate a.

$t = 3,\ y = 13{,}122 \implies 13{,}122 = a \cdot b^3$

$t = 18,\ y = 76{,}787 \implies 76{,}787 = a \cdot b^{18}$

While it does not matter which equation we divide by which, there is an advantage dividing the term with the larger exponent by the term with the smaller exponent. Then the difference $18 - 3 = 15$ represents the distance between the input values, which is the power of the b once we eliminate the a's:

$$\frac{ab^{18}}{ab^3} = \frac{76787}{13122}$$

$$b^{15} = \frac{76787}{13122}$$

Raise both sides to the power of 1/15 to solve for b (or take fifteenth roots).

$b = \sqrt[15]{\frac{76787}{13122}} = \left(\frac{76787}{13122}\right)^{1/15} \approx 1.124999971$, or, to two decimal places, 1.125.

We have the formula $f(t) = a(1.125)^t$. The growth factor $1.125 = 1 + r$, where r is the growth rate. So $r = 0.125$ or 12.5%. Choice **C**.

23) We find the value of a in the formula $f(t) = a(1.125)^t$ by substituting one of the points. It does not matter which one we substitute.

$t = 3,\ y = 13{,}122 \implies 13{,}122 = a \cdot 1.125^3$

$a = \frac{13122}{1.125^3} = 9216.$

Alternatively,

$t = 18,\ y = 76{,}787 \implies 76{,}787 = a \cdot 1.125^{18}$

$a = \frac{76787}{1.125^{18}} \approx 9215.996399$, or, about 9216.

The initial amount when $t = 0$ is \$9,216. Choice **C**.

24) Solve $4{,}000e^{0.073t} = 12{,}000$

$4000e^{0.073t} = 12{,}000$ — Divide both sides by 4000 to get $e^{0.073t}$ all by itself

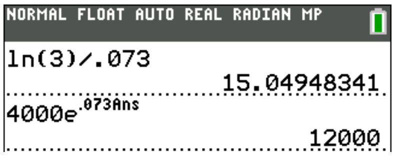
$e^{0.073t} = 3$ — Take natural logarithms of both sides.

$\ln e^{0.073t} = \ln 3$ — Use the inverse property: $\ln e^{0.073t} = 0.073t$.

$0.073t = \ln 3$ — Divide both sides by 0.073 to solve for t.

$t = \frac{\ln 3}{0.073} \approx 15.05$

TIP: Check by resubstituting:



Choice **D**.

25) Use $P(1+\frac{r}{n})^{n\cdot t}$ with P = 2200, r = 0.0382, and n = 4. The balance in year t is $2200(1+\frac{0.0382}{4})^{4t}$.

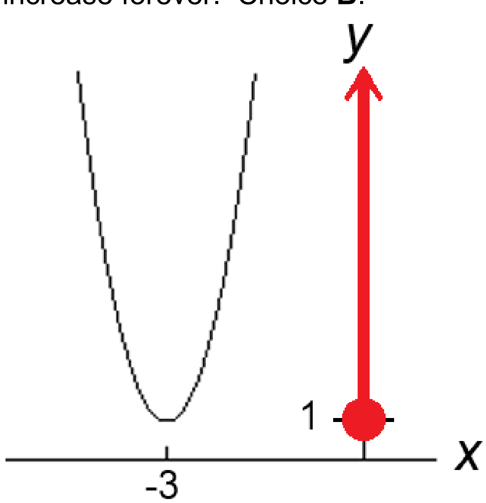
Remember that 3.82 per cent is $\frac{3.82}{100}$ = 0.0382 = 3.82%.

To divide 3.82 by 100, move the decimal point of 3.82 two places to the left.
For example: **03.**82% becomes 0**.03**82
The answer is Choice **C**.

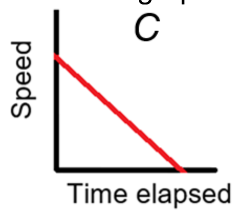
26) Since you are compounding continuously, use $Pe^{r\cdot t}$ with P = 2200, r = 0.0382. (See previous question.)
The balance in year t is $2200e^{0.0382t}$. Note: $2200e^{0.382t}$ grows at a continuous rate of 0.382 = 38.2 %.
Since the balance is none of the choices listed, the answer is Choice **E.**

To multiply 0.382 by 100, move the decimal point of 0.382 two places to the right.
For example: 0**.38**2 becomes **38.**2%

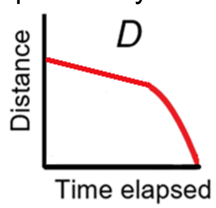
27) The function is a translation of $y = 5x^2$ up 1, so the range is all real numbers greater than equal to 0, which we can write as [1, ∞) or as $y \geq 1$. Notice on the graph below that the values of y begin at $y = 1$ and increase forever. Choice **B**.



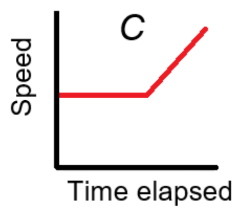
28) Each graph is of the train's *speed*, which must decrease to 0 to let off passengers, so Choice **C**. Note that graphs B and D do not represent a function. You cannot have two speeds at the same time.



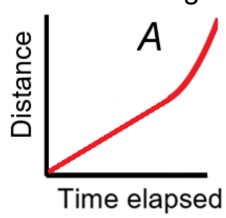
29) Each graph gives the *distance as you walk to class* on the vertical axis. Choice **D** is the only graph that fits. A steady (nonzero) speed is a slanted distance-time graph. The slope of a distance-time graph represents speed. Only A and D start off slanted linearly, both decreasing. This means the distance must be how far you are *away from* your classroom. As the clock chimes and you speed up, you continue to decrease, but faster and faster, so the graph must be concave down and hit the horizontal axis. Choice A indicates a sprint away from class, not towards the class.



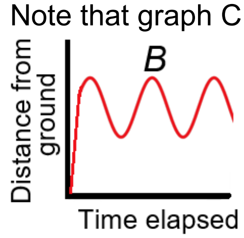
30) Each graph is of the rhino's *speed*, which, at a steady space, means the rhino speed does not change, ie., the graph is horizontal. When the rhino starts to run, the speed must increase, which must be Choice **C**.



31) Each graph is of the rhino's *distance*, which, at a steady space, means the rhino's distance-time graph is slanted linear. Only A and D begin linearly. When the rhino runs, its speed increases. The slope of a distance-time graph represents speed. So the graph must be Choice **A**.



32) Each graph is of your *distance from the ground*. The ferris wheel car climbs to its highest point, then descends, then climbs again.The radius of the wheel is fixed, so once you have boarded, the high points are all the same and the low points are all the same. So the graph must be Choice **B**. Note that graph C does not represent a function. You cannot be in two places at the same time.



33) Given $f(x) = \frac{4}{x^2}$ and $g(x) = \sqrt{x^2+4}$, then we must create $f(g(x))$ and simplify.

A. $f(g(x)) = \frac{4}{x^2+4}$ B. $f(g(x)) = \frac{4}{\sqrt{x^2+4}}$ C. $f(g(x)) = x^2+4$ D. $f(g(x)) = \frac{4}{x^2\sqrt{x^2+4}}$ E. $f(g(x)) = \frac{1}{x^2}$

The function $f(x) = \frac{4}{x^2}$ takes any input and returns 4 divided by the square of the input.

We can replace *x* by a placeholder, such as an empty box, i.e. $f(\square) = \frac{4}{(\square)^2}$

If *f* takes the function $g(x) = \sqrt{x^2+4}$ as an input, then we have the following:

$$f\big(g(x)\big) = \frac{4}{(\sqrt{x^2+4})} = = \frac{4}{x^2+4}$$

This is as simplified as possible. The answer is Choice **A**.

34) For the function $f(x) = \frac{\sqrt{x+1}}{2}$ we can replace *x* by a placeholder, such as an empty box, i.e. $f(\square) = \frac{\sqrt{\square+1}}{2}$

If *f* takes the function $g(x) = x^2+3$ as an input, then we have the following:

$f\left(\boxed{g(x)}\right) = \frac{\sqrt{\boxed{x^2+3}+1}}{2} = \frac{\sqrt{x^2+3+1}}{2} = \frac{\sqrt{x^2+4}}{2}$. This is as simplified as possible. The answer is Choice **B**.

35) Assuming *x*, *y*, and *w* are positive real numbers, which of the following is $\log \frac{x^3y^2}{\sqrt{w}}$?

$$\begin{aligned}\log_b\left(\frac{x^3y^2}{\sqrt{w}}\right) &= \log_b x^3 + \log_b y^2 - \log_b \sqrt{w}\\ &= \log_b x^3 + \log_b y^2 - \log_b w^{1/2}\\ &= 3\log_b x + 2\log_b y - \frac{1}{2}\log_b w\end{aligned}$$

The correct answer is Choice **C**.

36) Solving this graphically with most technology is very difficult.

$25^x = 3^{600}$

$\ln 25^x = \ln 3^{600}$

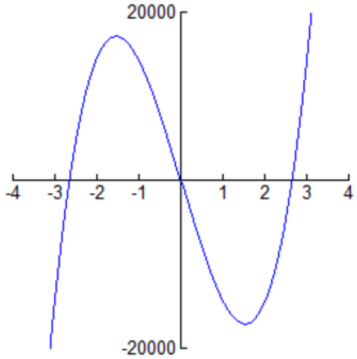
$x\ln 25 = 600\ln 3$

$x = \frac{600\ln 3}{\ln 25}$ 204.78

The correct answer is Choice **C**.

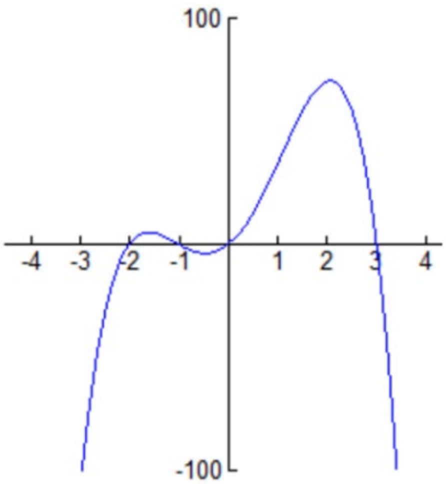
37) We find the zeros by solving $f(x) = 400x(6x^2-42) = 0$.
Set each factor equal to 0, and solve.
The first factor, 400x is equal to 0 when x = 0.
The remaining factor, $6x^2-42$ is equal to 0 when $6x^2 = 42$. Divide by 6 and take square roots.
We have $x^2 = 7$, so x = $\sqrt{7}$, and $-\sqrt{7}$. There are three zeros in total: 0, $\sqrt{7}$, and $-\sqrt{7}$.
We can check graphically.

This third degree polynomial crosses the x-axis three times. Notice $\pm\sqrt{7} \approx \pm 2.6$ so this checks out.



Choice **D**.

38) Similar to the previous question, we incorporate technology.
This fourth degree polynomial $f(x) = -3(x^4 - 7x^2 - 6x)$ crosses the x-axis four times.



We can factor out x from each term of $x^4 - 7x^2 - 6x$: $f(x) = -3x(x^3 - 7x - 6)$
However, try as you might $x^3 - 7x - 6$ cannot be factored.
The only way to find the zeros is with a graph or a table.
Since they are all integers, the zeros of $y = -3x(x^3 - 7x - 6)$ display readily in the table.

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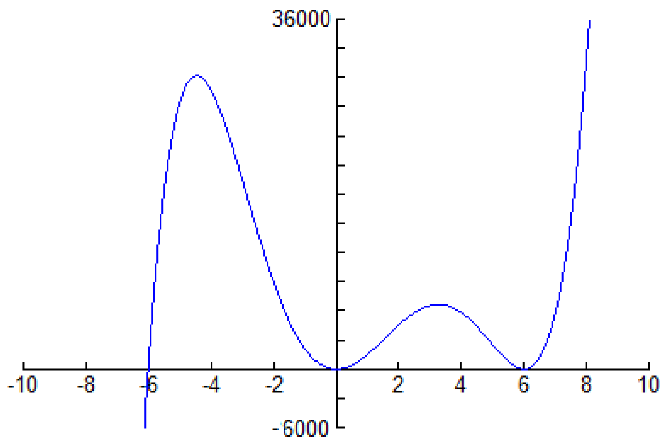
X	Y1			
-2	0			
-1	0			
0	0			
1	36			
2	72			
3	0			
4	-360			
5	-1260			
6	-3024			
7	-6048			
8	-10800			

X= -2

The zeros are −2, −1, 0, and 3. Choice **C**.

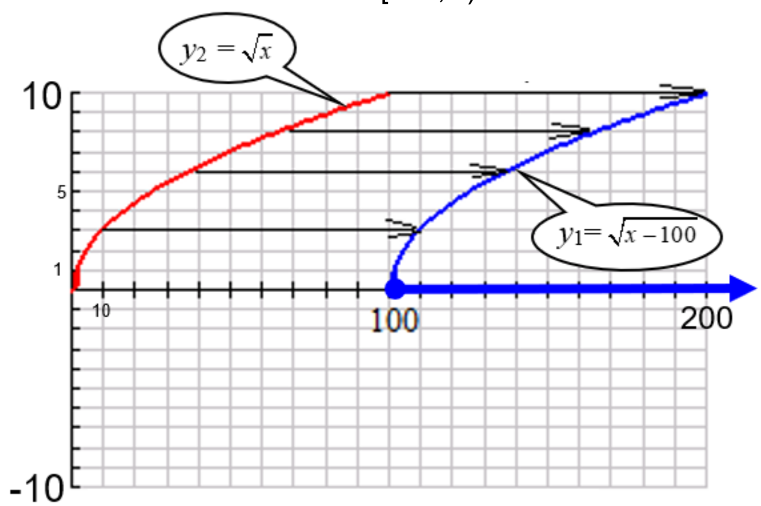
39) One way to solve $9x^2(x+6)(x-6)^2 \geq 0$ is to sketch a graph of the polynomial $f(x) = 9x^2(x+6)(x-6)^2$ and first solve $9x^2(x+6)(x-6)^2 = 0$. Then use the graph to determine when $f(x)$ is above the x-axis.

You can also use analytical reasoning from the formula. Since the factors $9x^2$ and $(x-6)^2$ are never negative, then $9x^2(x+6)(x-6)^2 \geq 0$ and $(x+6) \geq 0$ must have the same solution set, which is $x \geq -6$. Choice **C**.



40) We can find the domain of $f(x) = \sqrt{x-100}$ using the graph, the table, or reason from the formula. The graph of $f(x) = \sqrt{x-100}$ is a horizontal shift of the graph of the power function $y = \sqrt{x}$ right 100 units. The domain is $x \geq 100$. You can also write the domain $[100, \infty)$.

X	Y1
95	ERROR
96	ERROR
97	ERROR
98	ERROR
99	ERROR
100	0
101	1
102	1.4142
103	1.7321
104	2
105	2.2361



Reasoning from the formula:
The formula $f(x) = \sqrt{x-100}$ tells you that $f(x)$ is defined if the radicand $x - 100 \geq 0$.
When you solve this inequality, you have $x \geq 100$. Choice **B**.

41) $\ln\left(\frac{1}{\sqrt{e^x}}\right) = = \ln\left(\frac{1}{e^{x/2}}\right) = \ln\left(e^{-x/2}\right) = -\frac{x}{2}$ so Choice **C**.

42) The function $y = b^x$ is a special case of $f(x) = ab^x$, with $a = 1$. In general, the graph of $y = f(x) = ab^x$ increases for $b > 1$ and decreases for $0 < b < 1$ .The graph of $y = f(x) = ab^x$ has y-intercept $(0, a)$.

✓ I. It increases if $b > 1$

~~II. It decreases if $b < 0$~~

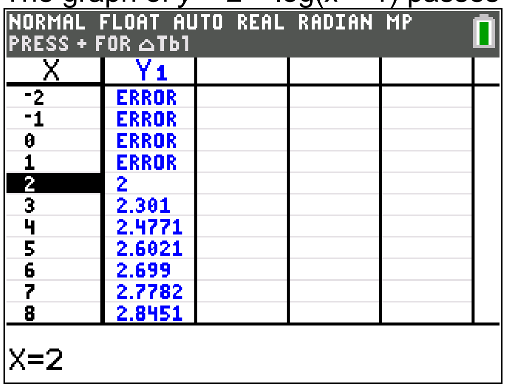
✓ III. It has y-intercept (0, 1) if $b > 0$ Choice **D.**

43) The graph of $y = 2 + \log(x - 1)$ is a horizontal shift 1 unit to the right and a vertical shift 2 units up of the graph of $y = \log(x)$.

- Since the graph of $y = \log(x)$ has a vertical asymptote of $x = 0$, the graph of $y = 2 + \log(x - 1)$ has a vertical asymptote of $x = 1$.

- Since the domain of $y = \log(x)$ is the set of all real numbers $x > 0$, the domain of $y = 2 + \log(x - 1)$ is the set of all real numbers $x > 1$. Therefore the graph of $y = 2 + \log(x - 1)$ does **not** cross the x-axis at 1 and it never touches the y-axis. See the table below.

- The graph of $y = 2 + \log(x - 1)$ passes through the point (2, 2):


NORMAL FLOAT AUTO REAL RADIAN MP
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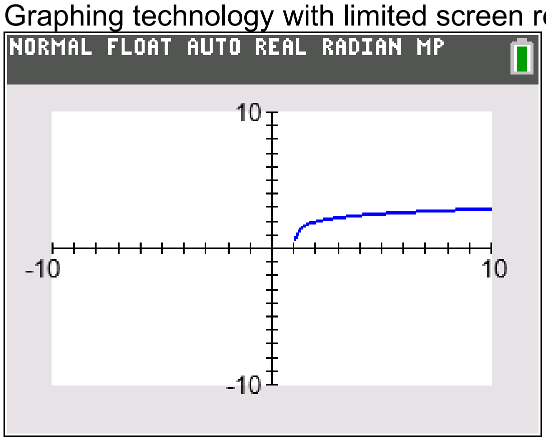
X	Y1
-2	ERROR
-1	ERROR
0	ERROR
1	ERROR
2	2
3	2.301
4	2.4771
5	2.6021
6	2.699
7	2.7782
8	2.8451

X=2

- The range of the function $y = 2 + \log(x - 1)$ is all real numbers.

Graphing technology with limited screen resolution can be deceiving:



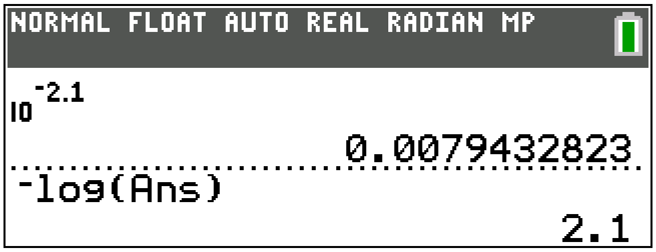
✓ I. increases for all values of x in its domain.
~~II. crosses the x-axis at 1~~
✓ III. never touches the y-axis
✓ IV. passes through the point (2, 2).

Therefore Items I, III, and IV are correct. Choice **E**.

44) Since pH = –log C and pH = 2.1, we must solve the logarithmic equation

pH	=	$-\log C$	Substitute 2.1 in for pH.
2.1	=	$-\log C$	Multiply both sides by −1.
−2.1	=	$\log C$	Make both sides a power of 10.
$10^{-2.1}$	=	$10^{\log C}$	Use the inverse property $10^{\log Q} = Q$.
$10^{-2.1}$	=	C	
C	=	$10^{-2.1} = \frac{1}{10^{2.1}} \approx 0.0008$ so Choice **B**.	

We can check our answer using the Last Answer feature.



The answer is Choice **D**.

45) To solve ln2x^3 = 5, exponentiate both sides to base *e*:

$e^{\ln 2x^3} = e^5$ Make both sides a power of *e*.

$2x^3 = e^5$ Use the inverse property.

The answer is Choice **D**.

46) We want the **exact** solution to the equation ln 2x^3 = 5.

$2x^3 = e^5$ From Question 57.

$x^3 = \frac{1}{2}e^5$ Divide both sides by 2.

$x^3 = \frac{e^5}{2}$ Take the cubed root of both sides

$x = \sqrt[3]{\frac{e^5}{2}}$

You can check by substitution: $\ln 2\left(\sqrt[3]{\frac{e^5}{2}}\right)^3 = \ln 2\left(\frac{e^5}{2}\right) = \ln e^5 = 5$. The answer is Choice **C**.

47) To solve the equation $20 = 3e^x + 5$, first subtract 5 from both sides:

This gives us $15 = 3e^x$. The answer is Choice **D**.

48) We want the **exact** solution to the equation20 = 3e^x + 5.

$15 = 3e^x$ From Question 47.

$e^x = 5$ Divide both sides by 3.

$\ln e^x = \ln 5$ Take natural logs of both sides.

$x = \ln 5$ Use the inverse property.

You can check by substitution: $3e^{\ln 5} + 5 = 3 \cdot 5 + 5 = 20$. The answer is Choice **E**.

49) Since the vertical asymptote is *x* = *a*, the **denominator** must have (*x* – *a*) as a factor.
Since the function has a single zero through the origin (0, 0), the **numerator** must be 0 when *x* = 0.

The short run behavior of the function near its vertical asymptote looks like
requiring the factor in the **denominator to be raised to an odd power**.

The equation $y = \frac{x}{x-a}$ is the only choice which meets these three criteria. Choice **C**.

50) As x →∞ or as x →−∞, $f(x) = \frac{2ax}{(x-a)^2} \approx \frac{2ax}{x^2} = \frac{2a}{x}$.

In other words, the graph of $y = \frac{2ax}{(x-a)^2}$ and the graph of $y = \frac{2a}{x}$ have the same long run behavior.

For large x, the graph of $y = \frac{2a}{x}$ has looks like  (if a > 0) or  (if a < 0)

In either case, as $x \to -\infty$ or as $x \to \infty$, the function approaches 0.
The horizontal asymptote is *y* = 0. Choice **D**.

51) To report all zeros, factor the numerator and denominator: $y = \frac{8x^2-8}{2x^2-4x} = \frac{8(x^2-1)}{2x(x-2)} = \frac{4(x-1)(x+1)}{x(x-2)}$.

The function has zeros when the numerator is zero (and the denominator is not)..
Set the numerator equal to 0. The zeros are –1 and 1. Choice **C**.

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X	Y1
-2	3/2
-1	0
0	ERROR
1	0
2	ERROR
3	32/3
4	15/2

X= -1

52) The function has vertical asymptotes when the denominator is zero (and the numerator is not).
The denominator $x(x-2) = 0$ when *x* = 0 and *x* =2. This matches the data in the above graphing calculator table where the y-value is ERROR. Choice **E**.

53) To find any *y*-intercepts, find the *y*-value when *x* = 0. However, the function is undefined at *x* = 0 so the graph never crosses the *y*-axis. See the above table. Choice **E**.

54) To find a horizontal asymptote, examine the long run behavior.

a) $f(x) = \frac{8x^2-8}{2x^2-4x} \to \frac{8x^2}{2x^2} = 4$ as $x \to \pm\infty$

For very large values of *x*, the function looks like the line *y* = 4. The line *y* = 4 is the horizontal asymptote.
You can scroll a table with a large Δ*x*, such as Δ*x* = 10,000, and TblStart = −50,000, to confirm this.

NORMAL FLOAT AUTO REAL RADIAN MP
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X	Y1
-50000	3.9998
-40000	3.9998
-30000	3.9997
-20000	3.9996
-10000	3.9992
0	ERROR
10000	4.0008
20000	4.0004
30000	4.0003
40000	4.0002
50000	4.0002

X= -50000

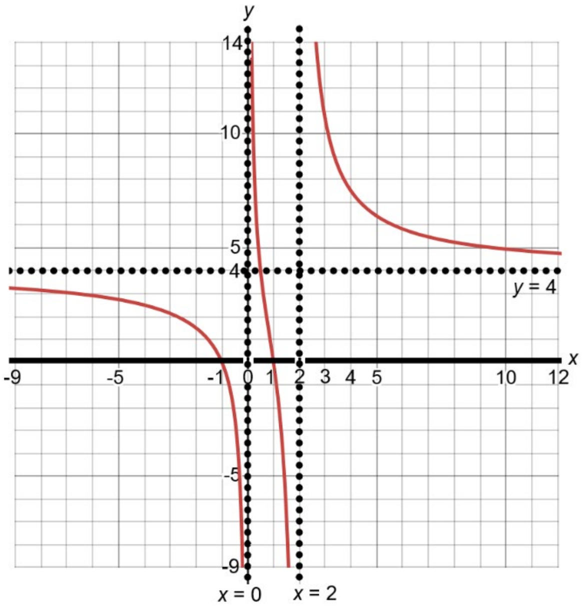
Choice **D**.

In addition to the table, we can also check using a graph. See next page.

b) To find all values of x where $f(x)$ intersects the line $y = 4$, solve the equation $f(x) = 4$.

$\frac{8x^2-8}{2x^2-4x}$	$= 4$	Set $f(x) = 4$
$8x^2 - 8$	$= 4(2x^2 - 4x)$	Clear fractions by multiplying both sides by $(2x^2 - 4x)$ and distribute.
$8x^2 - 8$	$= 8x^2 - 16x$	Subtract $8x^2$ from both sides.
-8	$= -16x$	Divide both sides by –16
x	$= 0.5$	

The rational function crosses its horizontal asymptote at only one place, the point (0.5, 4). Choice **B**.



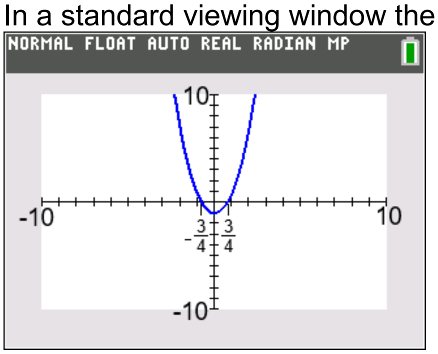
55) To find the zeros of $f(x) = \frac{63x^2}{36 - x^2} - 1$, set the equation equal to zero and solve.

This gives us $\frac{63x^2}{36 - x^2} - 1 = 0$. Then add 1 to both sides. This gives us $\frac{63x^2}{36 - x^2} = 1$. Choice **C**.

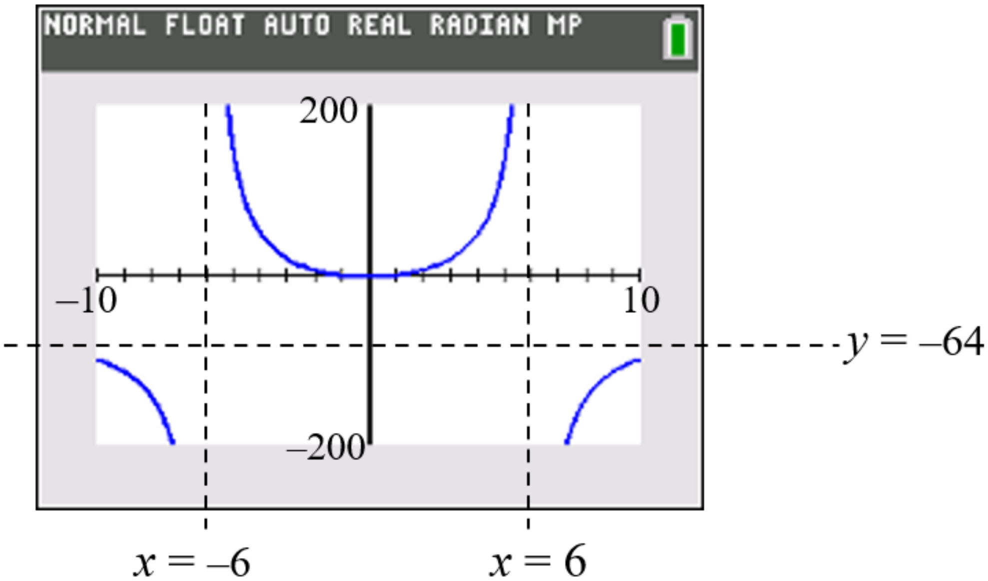
56) To solve the inequality, use the graph to report when $f(x)$ is below the x-axis. It is helpful to find zeros and vertical asymptotes since at these values of x the function can change signs.

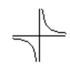
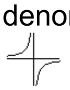
$\frac{63x^2}{36 - x^2} = 1$	From Question 55, i.e., add 1 to both sides.
$63x^2 = 36 - x^2$	Multiply both sides by $36 - x^2$ to clear fractions.
$64x^2 = 36$	Add x^2 to both sides to combine like terms.
$x^2 = \frac{36}{64}$	Divide both sides by 64.
$x^2 = \sqrt{\frac{36}{64}} = \pm\frac{6}{8} = \pm\frac{3}{4}$	Take square roots of both sides. Confirm graphically.

In a standard viewing window the graph of $f(x)$ looks like a parabola with zeros at ±0.75.



However, looks can be deceiving. We notice that if $36 - x^2 = 0$, i.e., when $x = -6$ and $x = 6$, the function has vertical asymptotes. Use the table feature of the calculator to help choose how to expand the viewing window to see near the vertical asymptotes. Scroll the table to see we need to change Ymin to -200, Ymax to 200.



The function $f(x) = \dfrac{63x^2}{36 - x^2} - 1$ can be written $f(x) = \dfrac{63x^2}{(6-x)(6+x)} - 1$ and linear factors in the denominator match the graphical behavior, namely the function looks like  near x = –6 and looks like  near x = 6.

We now report when $f(x)$ is below the x-axis, which is $x < -6,\ -\dfrac{3}{4} < x < \dfrac{3}{4},\ x > 6$.

We could also report this in interval notation: (–∞, –6) U (–0.75, 0.75) U (6, ∞).
The answer is Choice **E**.

57) As $x \to \pm\infty$, the function $y = \dfrac{63x^2}{36 - x^2} - 1 \to \dfrac{63x^2}{-x^2} - 1 \to -63 - 1 = -64$ so the horizontal asymptote is $y = -64$. See the above graph. The answer is Choice **E**.

58) $P = 1160 + 10t$ and $Q = 1000(1.0113)^t$
Set the equations equal to each other and solve using technology.
They intersect at t = 39 years. Choice **B**.

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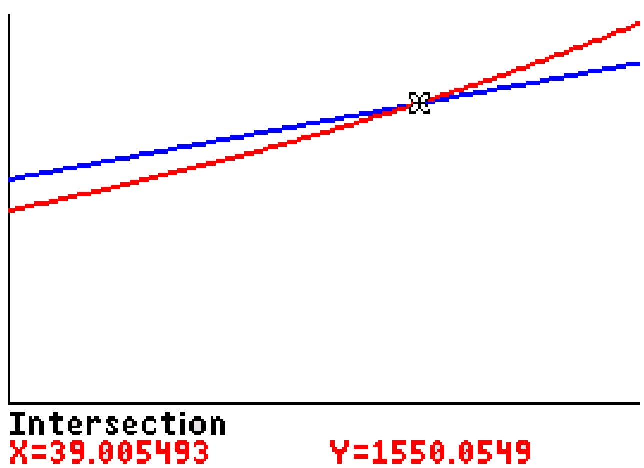
X	Y1	Y2
36	1520	1498.6
37	1530	1515.5
38	1540	1532.6
39	1550	1550
40	1560	1567.5
41	1570	1585.2
42	1580	1603.1
43	1590	1621.2
44	1600	1639.5
45	1610	1658.1
46	1620	1676.8

X=39

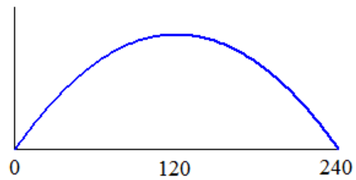


59) The path of the cannonball, in feet, is given by $h(x) = 0.96x - 0.004x^2$. How can you find the vertex? You *could* memorize the formula $x = \frac{b}{2a}$ to find the vertex of $y = ax^2 + bx$, but no need! Because the cannonball's path is perfectly symmetrical, it reaches its maximum height exactly halfway between launch and target (the zeros). Factor $h(x)$ to get the zeros:

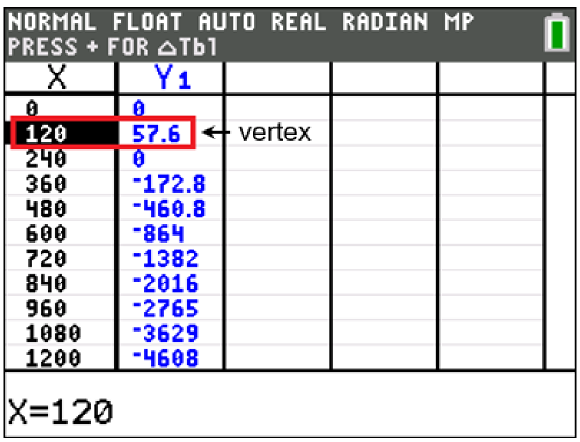
$$0.96x - 0.004x^2 = 0$$
$$x(0.96 - 0.004x) = 0$$

Set each factor equal to 0, and solve. The first factor, x is equal to 0 when x = 0. (No surprise.)
The remaining factor, $0.96 - 0.004x$ is equal to 0 when $0.96 = 0.004x$. Divide by 0.004.

We have $x = \frac{0.96}{0.004} = 240$. Thus $h(x)$ has zeros at 0 and 240 and is concave down since $a = -0.004$.



Use the symmetry in the table to find the vertex, which is halfway between the zeros at the point (120, 57.6) so the exact maximum height is 57.6 ft. Choice **D**. This handy tip is also used in the Seabreeze Park Problem # 64.
Unfortunately for Admiral Boom, since ships like the *HMS Endeavour* have a mainmast 150 ft high, he is going to have some explaining to do to the East India Trading Company!



X	Y1
0	0
120	57.6
240	0
360	-172.8
480	-460.8
600	-864
720	-1382
840	-2016
960	-2765
1080	-3629
1200	-4608

60) If a population with initial amount P_0 doubles every 12 years, it is modeled by $P(t) = P_0(2)^{\frac{t}{12}}$.

To find the tripling time, solve $P(t) = P_0(2)^{\frac{t}{12}} = 3P_0$.

Divide both sides by P_0 and take logarithms: $(2)^{\frac{t}{12}} = 3$ — Take the log of both sides.

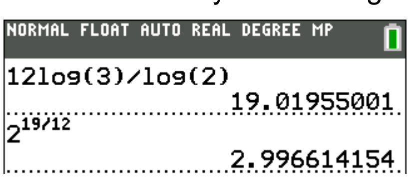
$\log(2)^{\frac{t}{12}} = \log 3$ — Use the power property

$\frac{t}{12}\log(2) = \log 3$ — Multiply both sides by 12

$t\log(2) = 12\log 3$ — Divide both sides by log 2

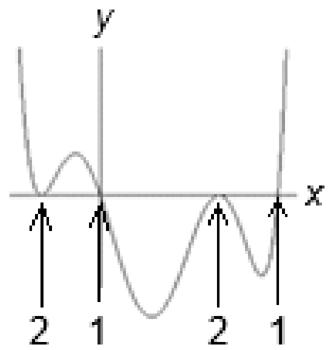
$t = \frac{12\log 3}{\log 2} \approx 19$ years.

We can check by substituting back into the original equation. $P(19) = P_0(2)^{\frac{19}{12}} \approx 3P_0$.



Choice **C**.

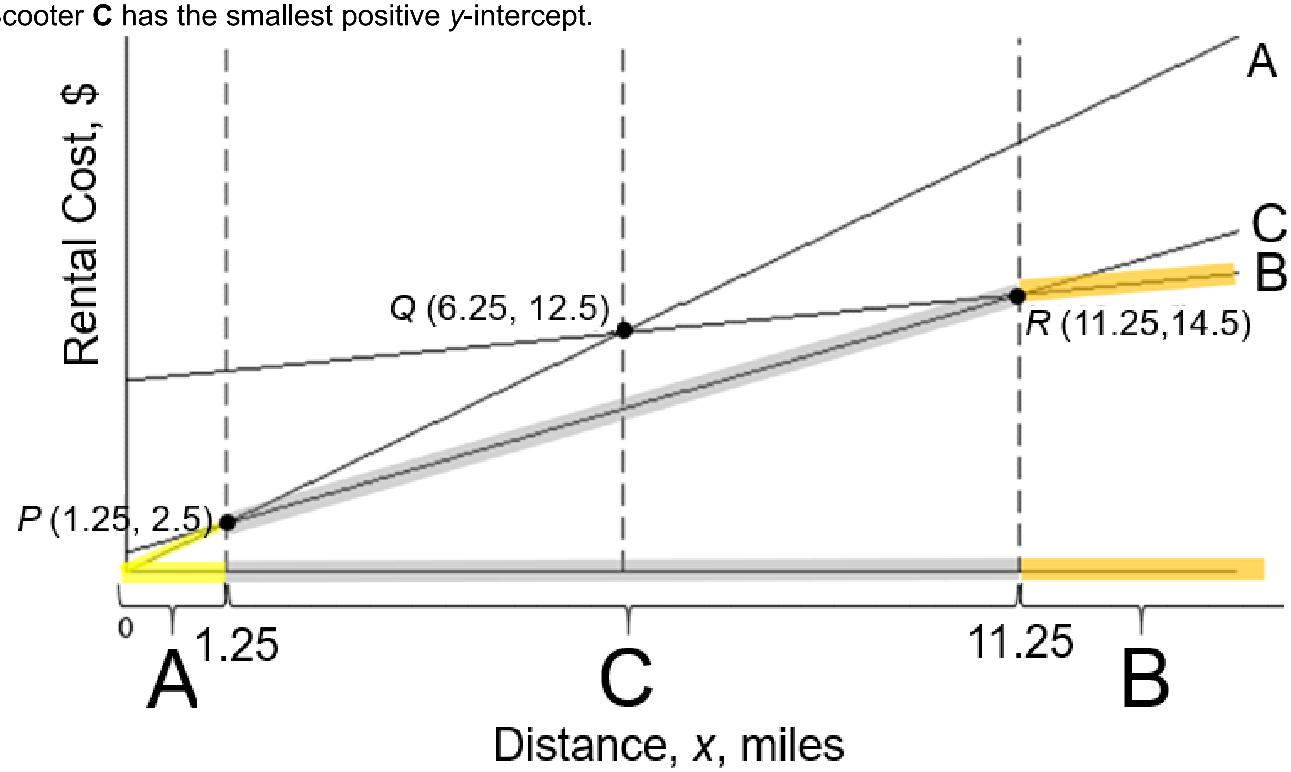
61) The end behavior (up-up) indicates the degree of the polynomial is even. Count the minimum multiplicities of each zero based on the shape of the graph (chair, bounce, line) near the zero.



Lowest multiplicities of each zero

We have 2 + 1 + 2 + 1 = 6. This matches the result that the end behavior is even. Choice **D**.

62) The graphs are labeled below.
Scooter **A** passes through the origin and has the steepest slope since it has the highest cost per mile.
Scooter **B** has the most gradual slope since it has the lowest cost per mile. It has the highest *y*-intercept.
Scooter **C** has the smallest positive *y*-intercept.



To find when Scooter **C** is cheapest, look for the values of x for which the graph of C is *below* the graph of A and C. Scooter **C** is cheapest on the interval $1.25 < x < 11.25$.

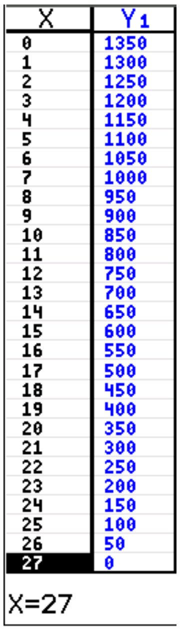
Although not needed to answer this question, we can see the following is true:
Scooter **A** is cheapest on the interval $0 < x < 1.25$.
Scooter **B** is cheapest on the interval $x > 11.25$.
We do not use the *y*-coordinates in reporting the interval. The correct choice is Choice **E.**

63) When the price p = \$11, the number of customers N who will come to the Seabreeze Park is 800. For each \$1.00 increase in the entrance price p, Seabreeze Park would lose an average of 50 daily customers: $N = f(p)$ is linear. When Δp = \$1, then ΔN = −50. The slope is $\frac{\Delta N}{\Delta p} = \frac{-50}{\$1} = -50$ and it passes through (\$11, 800). We have $N = b - 800p$. Substitute $p = 11$, $N = 800$:

$p = 11$, $N = 150 \Longrightarrow$	N	=	$b - 800p$	Substitute.
	800	=	$b - 50\cdot 11$	Subtract 275 from both sides.
	800	=	$b - 550$	Add 550 to both sides.
	b	=	1350	

Therefore $N = f(p) = 1350 - 50p$. Check each response in the question by scrolling the table feature of a grapher. The table confirms that if p = \$11, N = 800 and the rate of change is −50 customers per dollar increase, so the formula is correct.

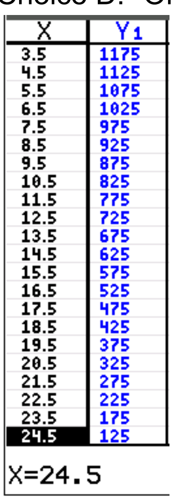
- Choice A: "A \$27 ticket price would result in no customers." True
- Choice B: "If the park had free admission, they would have as many as 1,350 daily customers." True



X	Y1
0	1350
1	1300
2	1250
3	1200
4	1150
5	1100
6	1050
7	1000
8	950
9	900
10	850
11	800
12	750
13	700
14	650
15	600
16	550
17	500
18	450
19	400
20	350
21	300
22	250
23	200
24	150
25	100
26	50
27	0

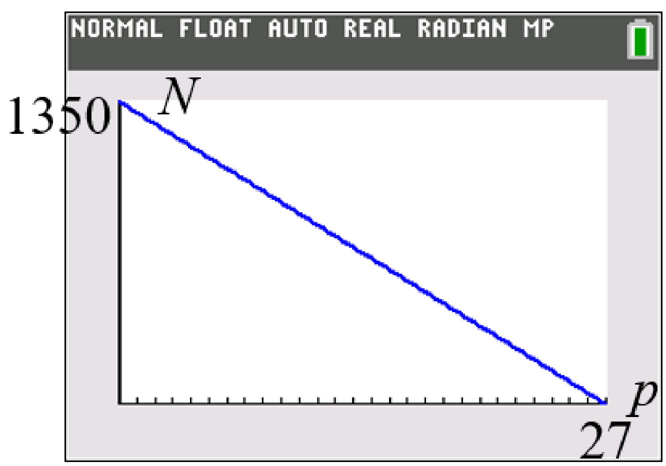
X=27

- Choice C: "If the ticket price were \$3.50, they would have 1175 daily customers." True.
- Choice D: "Only 125 customers would be willing to pay a \$24.50 admission price." True.



X	Y1
3.5	1175
4.5	1125
5.5	1075
6.5	1025
7.5	975
8.5	925
9.5	875
10.5	825
11.5	775
12.5	725
13.5	675
14.5	625
15.5	575
16.5	525
17.5	475
18.5	425
19.5	375
20.5	325
21.5	275
22.5	225
23.5	175
24.5	125

X=24.5



- Choice **E**. All of the above.

You can also use algebra to find the intercepts and to evaluate the formula at p = \$3.50 and p = \$24.50. To find the p-intercept of $N = f(p)$, set $N = 0$ and solve for p.

64) We add a third column to the table in the previous question which gives the daily revenue, *R*, for each entrance price *p.* The *revenue* is the total amount received by Seabreeze Park before any costs are deducted, which is *R* = *N*·*p*.
For example, if the price p = \$11, then N = 800 tickets are sold and the revenue *R* = 800·11 = \$8800.
One approach is to manually compute the product *N*·*p* for the rows you are interested in.

- The point *p* = 0, *R* = 0 means that if the tickets were free, there would be no revenue (even though 1350 customers would come.
- The point *p* = 27, *R* = 0 means that if the tickets were \$27, there would be no revenue (since no customers would buy them.)

The table shows that Choice A and B are both false.

- Choice A. “The higher they set the ticket price, the more revenue they will make.
- Choice B. “A ticket price of \$27 gives them the most revenue.”
- See the shaded row in the table below.

price, *p*	tickets sold, *N*	revenue, *R* = *N*·*p*
\$0	1350	\$0
\$11	800	\$8800
\$13	700	\$9100
\$14	650	\$9100
\$27	**0**	**\$0**
\$0	1350	\$0

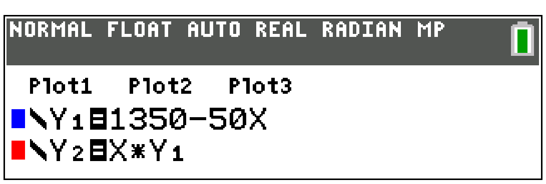
The table suggests also that C and D are false.

- Choice C: If the ticket price were \$13, they would have the highest revenue of \$9100.
- Choice D: If the ticket price were \$14, they would have the highest revenue of \$9100.
- See the shaded row in the table below.

price, *p*	tickets sold, *N*	revenue, *R* = *N*·*p*
\$0	1350	\$0
\$11	800	\$8800
\$13	700	\$9100
\$13.50	**675**	**\$9112.50**
\$14	650	\$9100
\$27	0	\$0

Examine the formula for the product *R* = *N*·*p* .
R(*x*) = *x*(1350 − 50*x*). If we distribute, we get $R = 1350x - 50x^2$, which is quadratic. How can you find the vertex? You *could* memorize the formula $x = \frac{b}{2a}$ to find the vertex of $y = ax^2 + bx + c$. But no need!
Use instead the natural symmetry of the parabola to your advantage.
The vertex always lies midway between the zeros (or midway between any two values of *x* which have the same *y*-values). Recall: this handy tip is also used in the Cannonball Problem # 59.

So the maximum revenue occurs if tickets were sold at *p* = \$13.50.
You can use a grapher instead of manual calculation by entering the formula for R in Y2. Below is one way:

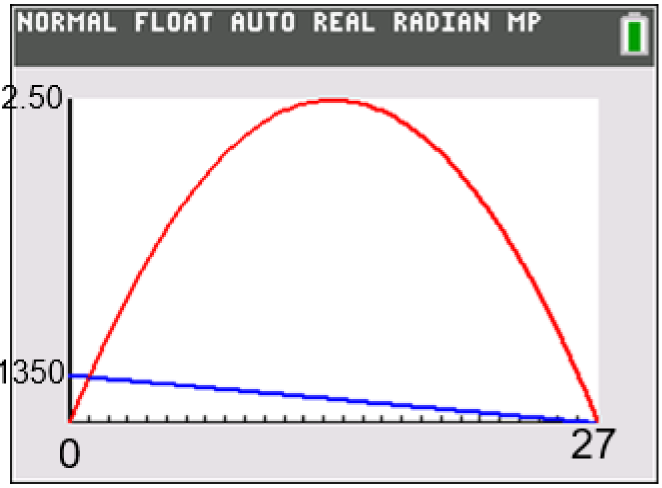


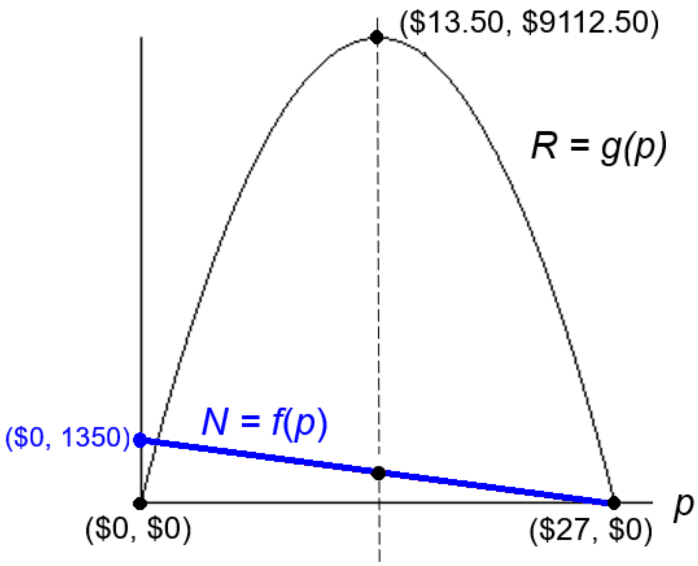
Choice **E** is correct. None of the response are true.
It can be illuminating to view the graph and table of both on a grapher. We show these on the next page.

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y_1	Y_2
12	750	9000
12.5	725	9062.5
13	700	9100
13.5	675	9112.5
14	650	9100
14.5	625	9062.5
15	600	9000
15.5	575	8912.5
16	550	8800
16.5	525	8662.5
17	500	8500

X=13.5





65) Because the zero of the function is 2, we have (*x* – 2) as a factor of the numerator since the function is 0 when the numerator is 0.

Since the vertical asymptote is *x* = 3, we have (*x* – 3) as a factor of the denominator. (The vertical asymptotes are found where the denominator is 0 and the numerator is not).

So we can write $y = \frac{k(x-2)}{(x-3)}$. Since the horizontal asymptote is *y* = 4 and it is found by the ratio of the leading terms, we must have *k* = 4.

Therefore the function must be $f(x) = \frac{4(x-2)}{(x-3)}$. Use a table feature to find Choice **C** is correct. Use the division key (/) in entering the formula. Or, use the formula: $f(403) = \frac{4(403-2)}{(403-3)} = \frac{4 \cdot 401}{400} = \frac{401}{100}$ = 4.01.

Y_1=4(X−2)/(X−3)

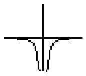
NORMAL FLOAT AUTO REAL DEGREE MP
TABLE SETUP
TblStart=403
△Tbl=1
Indpnt: Auto Ask
Depend: Auto Ask

X	Y_1
403	4.01
404	4.01
405	4.01
406	4.0099

66) Because the zeros of the function are 0 and –3 and are both single zeros, the factors of the numerator are $x(x+3)$, since the function is 0 when the numerator is 0.

There is one vertical asymptote at $x = –2$, so $(x + 2)$ is a factor of the denominator.

However, the short run behavior near this asymptote looks like $y = \frac{k}{x^2}$, i.e.



so the factor must have a power of 2.

We can write $y = \frac{kx(x+3)}{(x+2)^2}$. Since the horizontal asymptote is $y = 2$, we must have $k = 2$.

Note: $y = \frac{kx(x+3)}{(x+2)^2} \approx \frac{kx^2}{x^2} = k$ as $x \to \pm\infty$ so $k = 2$.

Therefore, the rational function has the formula $y = \frac{2x(x+3)}{(x+2)^2}$.

Use a table to confirm:

X	Y1
-6	2.25
-5	2.2222
-4	2
-3	0
-2	ERROR
-1	-4
0	0
1	.88889
2	1.25
3	1.44
4	1.5556

This should match the information provided.

Use a table to find determine if $f(-1) = -4$, $f(1) = 1$, and $f(-6) = 2.25$:

X	Y1
-6	2.25
-5	2.2222
-4	2
-3	0
-2	ERROR
-1	-4
0	0
1	.88889
2	1.25
3	1.44
4	1.5556

$f(-6) = 2.25$

$f(-1) = -4$

$f(1) \neq 1$

Since only Choices A and C are true, Choice **D** is correct.

67) Because there is a horizontal asymptote of $y = 0$, the degree of the numerator is less than the degree of the denominator. The numerator has a factor of $(x – 4)$ since it has a single zero. Because the curve approaches the vertical asymptote from the same side, the lowest degree possible for the denominator must be 2 and the denominator has a factor $(x – 4)^2$. It has the form $y = \frac{k(x-4)}{(x-2)^2}$,

and we can find k if we use the fact that when $x = 0$, $y = -8$.

$x = 0,\ y = -8 \Rightarrow$

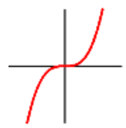
y	=	$\frac{k(x-4)}{(x-2)^2}$	Substitute.
-8	=	$\frac{k(0-4)}{(0-2)^2}$	Calculate.
-8	=	$\frac{-4}{4}k$	Divide both sides by −1
8	=	k	

So $f(x) = \frac{\mathbf{8(x-4)}}{\mathbf{(x-2)^2}}$. To find $f(3)$, we let $x = 3$ and find y.

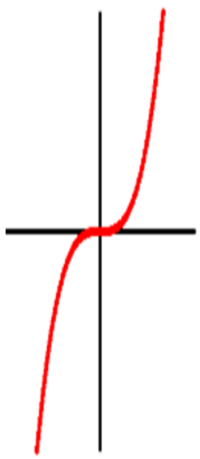
$f(3) = \frac{\mathbf{8(3-4)}}{\mathbf{(3-2)^2}} = \frac{\mathbf{8(-1)}}{\mathbf{1}} = -8$.

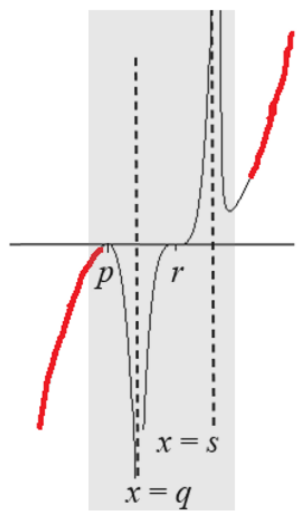
Alternatively, you can enter the formula in a grapher and use a table as in the previous question.
Choice **B.**

68) For very large x, the function looks very much like the graph of $y = \frac{kx^7}{x^4} = kx^3$, which is a chair shape:



In Graph B, cover everything but the end behavior to unveil how the graph looks like a chair at the ends.





Choice **B.**

69) We find formulas of Graphs A, C, and D in the previous question.

The formula for Graph A

- Locally at p, the curve crosses the x-axis like a slanted line, indicating that p is a single zero and factor $(x-p)$ has a power of 1.
- Locally at r, the curve crosses the x-axis like a chair, indicating that the factor $(x-r)$ has an odd power greater than 1. The lowest power possible is 3.
- Locally near $x = q$ and $x = s$, the curve approaches both vertical asymptotes from the same side, so the power for $(x-q)$ and $(x-s)$ must each be even.
- Since we have a horizontal asymptote of $y = k$, the degree of the numerator and denominator must be the same, which is 4. So the power of $(x-q)$ and $(x-s)$ must each be 2.

Putting all of this together gives us the formula $y = \frac{k(x-p)(x-r)^3}{(x-q)^2(x-s)^2}$.

The formula for Graph C

- Locally at p, the curve bounces on the x-axis, indicating that the factor $(x-p)$ has an even power. The lowest power possible is 2.
- Locally at r, the curve crosses the x-axis like a chair, indicating that the factor $(x-r)$ has an odd power greater than 1. The lowest power possible is 3.
- Locally near $x = q$, the curve approaches its vertical asymptotes from opposite sides, so $(x-q)$ must have an odd power (1, 3, 5, …).
- Locally near $x = s$, the curve approaches both vertical asymptotes from the same side, so $(x-s)$ must have an even power.
- Since we have a horizontal asymptote of $y = k$, the degree of the numerator and denominator must be the same. The combined powers of $(x-p)$ and $(x-r)$ must be 5, so the combined powers of the factors in the numerator must add up to 5. In other words, we have
  $y = \frac{k(x-p)^2(x-r)^3}{(x-q)^{ODD}(x-s)^{EVEN}}$.
- This means we could have $(x-q)^3(x-s)^2$ or $(x-q)(x-s)^4$.

Putting all of this together gives us either $y = \frac{k(x-p)^2(x-r)^3}{(x-q)^3(x-s)^2}$ or $y = \frac{k(x-p)^2(x-r)^3}{(x-q)(x-s)^4}$ but the first one uses the lowest powers possible.

The formula for Graph D

- Locally at p, the curve bounces on the x-axis, indicating that the factor $(x-p)$ has an even power. The lowest power possible is 2.
- Locally at r, the curve crosses the x-axis like a chair, indicating that the factor $(x-r)$ has an odd power greater than 1. The lowest power possible is 3.
- Locally near $x = q$ and $x = s$, the curve approaches both vertical asymptotes from the same side, so the power for $(x-q)$ and $(x-s)$ must each be even.
- Since we have a horizontal asymptote of $y = 0$, the degree of the numerator must be less than the degree of the denominator. The combined powers of $(x-p)$ and $(x-r)$ must be 5, so the combined powers of the factors in the numerator must add up to 6 or higher. In other words, we have
  $y = \frac{k(x-p)^2(x-r)^3}{(x-q)^{EVEN}(x-s)^{EVEN}}$.
  This means we could have $(x-q)^4(x-s)^2$ or $(x-q)^2(x-s)^4$.

Putting all of this together gives us either $y = \frac{k(x-p)^2(x-r)^3}{(x-q)^4(x-s)^2}$ or $y = \frac{k(x-p)^2(x-r)^3}{(x-q)^2(x-s)^4}$.

70) Given the graph, we have the following:

- The degree of the factor $(x - a)$ must be even since there is a bounce at the zero.
- The degree of the factors $(x - b)$ and $(x - d)$ must both be even since the curve approaches the vertical asymptotes $x = b$ and $x = d$ from the same side.
- The degree of the factor $(x - c)$ must be 3, 5, … since there is a chair at the zero.
- The long run behavior is the same as the power function $y = kx$, so the degree of the numerator must be one more than the degree of the denominator. Therefore, of the possible choices, it must be
  $f(x) = \frac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^2}$ Choice **B.**

71) *The function has a positive rate of change and concave up.*
Graphs B and D are concave up, but B has a negative rate of change since it decreases and D has a positive rate of change since it increases. Choice **D**.

72) *The function is concave down.*
Choices **A** and **C** are concave down.
Some students remember this by thinking of a concave down graph as a frown.

73) *The function is decreasing.* Choices **A**, **B**, and **F**.
Some students remember this by thinking of going down a descending path.

74) *The function is constant.* Choice **G**.
This statement describes the outputs, or *y*-values, of the function, which remain the same. The slope is 0.
Think of a flat parking lot, a billiard table, or the EKG monitor on a sad episode of a medical drama.

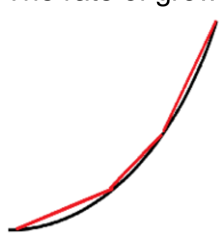
75) *The average rate of change of the function over every interval is constant.* Choices **E, F**, and **G**.
This statement describes the slopes of the lines drawn on the function similar to what was done above.
A linear function has a constant rate of change. Slopes of lines can be positive, negative, or zero.

76) *Even though the child's temperature is still rising, the penicillin seems to be taking effect.* Choice **C**.
Sketch a graph of temperature vs. time that increases and will eventually flatten out.
The rate of growth is modeled by the slopes of the line segments shown:

Since the rate of change is decreasing, the segments must have smaller and smaller slopes, i.e., they are less and less steep. The graph is concave down and increasing.

77) *Your distance from the Atlantic Ocean in kilometers, increases at a constant rate.* Choice **E**.
Sketch a linear graph of distance vs. time that increases since it climbs steadily.

78) *At first your balance grows slowly, but its rate of growth continues to increase.* Choice **D**.
Sketch a graph of balance vs. time that increases faster and faster, i.e., concave up.
The rate of growth is modeled by the slopes of the line segments shown:

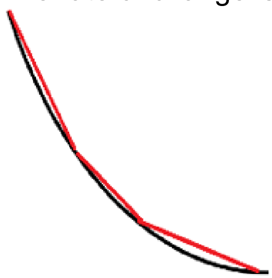


Since the rate of change is increasing, the segments must have larger and larger slopes, i.e., they are steeper and steeper. The graph is concave up and increasing.

79) *The annual profit is decreasing. Each year it falls more steeply than the previous year.* Choice **C**.
The rate of change is modeled by the slopes of the line segments shown:

Since the profit is decreasing, the segments must have negative slopes.
Since they fall faster and faster, the graph is concave down.

80) *The population of rhinos isn't decreasing as quickly as it used to be*. Choice **B**.
The rate of change is modeled by the slopes of the line segments shown:



Since the population is decreasing, the segments must have negative slopes.
Since they are less and less steep, the graph is concave down.

81) The graph of $p(x)$ is a vertical shift of $f(x)$ down 5 and horizontal shift left 2.
The transformation is $p(x) = f(x + 2) - 5$. Choice **A**.

82) The graph of $q(x)$ is a vertical compression of $f(x)$ by a factor of one half, followed by a vertical shift down 6 and horizontal shift 5 right. The transformation is $q(x) = 0.5f(x - 5) - 6$. Choice **E**.

83) The graph of r(x) is a horizontal reflection of f(x) or a reflection of f(x) about the y-axis, followed by a vertical shift down 4. The transformation is $r(x) = f(-x) - 4$. Choice **D**.

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- This Answer Key with worked out solutions.
- NotebookLM: AI-generated review tool. (Always double-check its math!)
- Practice Sets & Flash Cards: 100% human-created practice
- Live Review Session: Exam day from 1:00 PM – 3:00 PM. Hosted by a real human.
  Check Brightspace for the room location!