Logarithms KEY

Find the logarithms. Do not use a calculator. If they do not exist, state so.

1. $\log_3\left(\frac{1}{27}\right) = \boxed{?}$

   Rewriting in exponential form, we have $3^{\boxed{?}} = \frac{1}{27}$.
   Write each side to the same base. Since $27 = 3^3$, then $\frac{1}{27} = 3^{-3}$.
   We need to find what number is 3 raised to in order to get $3^{\boxed{?}} = 3^{-3}$.
   This is -3. Therefore, $\log_3\left(\frac{1}{27}\right) = \boxed{-3}$.

2. $\log_{\frac{1}{3}} 9 = \boxed{?}$

   Rewriting in exponential form, we have $\frac{1}{3}^{\boxed{?}} = 9$.
   Write each side to the same base. Since $9 = 3^2$, and $\frac{1}{3} = 3^{-1}$, then

   $$\left(\tfrac{1}{3}\right)^{\boxed{?}} = 9$$

   $$\left(3^{-1}\right)^{\boxed{?}} = 3^2$$

   $$(3)^{-1\times\boxed{?}} = 3^2$$

   To find the answer, we need to find what number multiplied by -1 is 2.
   This is -2. Therefore, $\log_{\frac{1}{3}} 9 = \boxed{-2}$.

3. $\log_3\sqrt{3} = \boxed{?}$

   Rewriting in exponential form, we have $3^{\boxed{?}} = \sqrt{3}$.
   Since $3^{1/2} = \sqrt{3}$, we have $\log_3\sqrt{3} = \boxed{1/2}$.

4. To find $\ln\frac{1}{e^{0.247}}$, write $\frac{1}{e^{0.247}}$ as e raised to some power and use the inverse property $\ln e^w = w$.

   $$\ln\frac{1}{e^{0.247}} = \ln e^{-0.247} = -0.247$$

5. To find $\ln\sqrt[3]{\frac{1}{e^2}}$, write $\sqrt[3]{\frac{1}{e^2}}$ as e raised to some power and use the inverse property $\ln e^w = w$.

   $$\ln\sqrt[3]{\frac{1}{e^2}} = \ln\sqrt[3]{e^{-2}} = \ln e^{-2/3} = -\tfrac{2}{3}$$

6. To find $\log\sqrt{1000}$, write $\sqrt{1000}$ as 10 raised to some power and use the inverse property $\log 10^{w} = w$.

Therefore, $\log\sqrt{1000} = \log\sqrt{10^{3}} = \log 10^{3/2} = 3/2$

7. $\log_{\sqrt{5}} 25 = \boxed{?}$.

Rewriting in exponential form, we have $\left(\sqrt{5}\right)^{\boxed{?}} = 25$.

Write each side to the same base. Since $\sqrt{5} = 5^{1/2}$, and $25 = 5^{2}$, then

$\left(\sqrt{5}\right)^{\boxed{?}} = 25$

$\left(5^{\frac{1}{2}}\right)^{\boxed{?}} = 5^{2}$

$(5)^{\frac{1}{2}\boxed{?}} = 5^{2}$

To find the answer, we need to find what number multiplied by $\frac{1}{2}$ is 2.

$(5)^{\frac{1}{2}\boxed{x}} = 5^{2}$ means $\frac{1}{2}x = 2$.

Multiply both sides by 2 to find that $x =$ 4. Therefore, $\log_{\sqrt{5}} 25 = \boxed{4}$.

8. $\log_{2}\left(\frac{1}{2}\right) = \boxed{?}$

Rewriting in exponential form, we have $(2)^{\boxed{?}} = \frac{1}{2}$.

Since $2^{-1} = \frac{1}{2}$, we have $\log_{2}\left(\frac{1}{2}\right) = \boxed{-1}$.

9. $\log_{5}\left(\frac{1}{125}\right) = \boxed{?}$

Rewriting in exponential form, we have $(5)^{\boxed{?}} = \frac{1}{125}$.

Since $5^{-3} = \frac{1}{125}$, we have $\log_{5}\left(\frac{1}{125}\right) = \boxed{-3}$.

10. $\log_{\pi} 0 = \boxed{?}$

Rewriting in exponential form, we have $(\pi)^{\boxed{?}} = 0$.

This equation has no solution, so the answer is undefined.

11. $\log_{\pi} 1 = \boxed{?}$

Rewriting in exponential form, we have $(\pi)^{\boxed{?}} = 1$.

Since $\pi^{0} = 1$, we have $\log_{\pi} 1 = \boxed{0}$.

12. $\log\left(\frac{1}{\sqrt[3]{10}}\right) = \boxed{?}$

To find $\log\left(\frac{1}{\sqrt[3]{10}}\right)$, write $\frac{1}{\sqrt[3]{10}}$ as 10 raised to some power and use the inverse property $\log 10^w = w$.

Therefore, $\log\left(\frac{1}{\sqrt[3]{10}}\right) = \log\left(\frac{1}{10^{1/3}}\right) = \log\left(10^{-1/3}\right) = -1/3$

13. $\log_{27} 9 = \boxed{?}$

Rewriting in exponential form, we have $(27)^{\boxed{?}} = 9$.

Write each side to the same base. Since $27 = 3^3$, and $9 = 3^2$, then

$(27)^{\boxed{?}} = 9$

$(3^3)^{\boxed{?}} = 3^2$

$(3)^{3\boxed{?}} = 3^2$

To find the answer, we need to find what number multiplied by 3 is 2.

$(3)^{3\boxed{x}} = 3^2$ means $3x = 2$.

Divide both sides by 3 to find that $x = \frac{2}{3}$. Therefore, $\log_{27} 9 = \boxed{2/3}$.

14. $\log_7 49$

To find $\log_7 49$, write 49 as 7^2 and use the inverse property: $\log_7 49 = \log_7 7^2 = 2$.

15. $\log_{\frac{1}{2}} 8 = \boxed{?}$

Rewriting in exponential form, we have $\left(\frac{1}{2}\right)^{\boxed{?}} = 8$.

Write each side to the same base. Since $\frac{1}{2} = 2^{-1}$, and $8 = 2^3$, then

$\left(\frac{1}{2}\right)^{\boxed{?}} = 8$

$(2^{-1})^{\boxed{?}} = 2^3$

$(2)^{-1\boxed{?}} = 2^3$

To find the answer, we need to find what number multiplied by -1 is 3.

$(2)^{-1\boxed{x}} = 2^3$ means $-x = 3$. Divide both sides by -1.

The answer is $x = -3$. Therefore, $\log_{\frac{1}{2}} 8 = \boxed{-3}$.

16. $\log_{0.07}\left(\dfrac{1}{0.07^4}\right)$

To find $\log_{0.07}\left(\dfrac{1}{0.07^4}\right)$, write $\dfrac{1}{0.07^4}$ as 0.07^{-4} and use the inverse property:

$$\log_{0.07}\left(\frac{1}{0.07^4}\right) = \log_{0.07}\left(0.07^{-4}\right) = -4$$

17. $\ln\dfrac{1}{\sqrt{e}}$

To find $\ln\dfrac{1}{\sqrt{e}}$, write $\dfrac{1}{\sqrt{e}}$ as e raised to some power and use the inverse property $\ln e^w = w$.

$$\ln\frac{1}{\sqrt{e}} = \ln\frac{1}{e^{1/2}} = \ln e^{-1/2} = -\tfrac{1}{2}$$

18. $\ln e^3$

To find $\ln e^3$, use the inverse property: $\ln e^3 = 3$

19. $\log 10{,}000{,}000{,}000^2$

To find $\log 10{,}000{,}000{,}000^2$, write $10{,}000{,}000{,}000^2$ as 10 raised to some power and use the inverse property: $\log 10^w = w$

$$\log 10{,}000{,}000{,}000^2 = \log(10^{10})^2 = \log 10^{20} = 20$$

20. $\log_{\sqrt{8}}\sqrt{8} = \boxed{?}$.

Rewriting in exponential form, we have $\left(\sqrt{8}\right)^{\boxed{?}} = \sqrt{8}$.

So $\log_{\sqrt{8}}\sqrt{8} = \boxed{1}$

21. $\log_{\frac{1}{4}} 4 = \boxed{?}$

Rewriting in exponential form, we have $\left(\tfrac{1}{4}\right)^{\boxed{?}} = 4$.

So $\log_{\frac{1}{4}} 4 = \boxed{-1}$

22. $\log_4 16$

To find $\log_4 16$, write 16 as 4^2 and use the inverse property: $\log_4 16 \;=\; \log_4 4^2 \;=\; 2$.

23. $\log_3 9$

To find $\log_3 9$, write 9 as 3^2 and use the inverse property: $\log_3 9 \;=\; \log_3 3^2 \;=\; 2$.

24. $\log_2 32$

To find $\log_2 32$, write 32 as 2^5 and use the inverse property: $\log_2 32 \;=\; \log_2 2^5 \;=\; 5$.

25. $\log_2\left(\frac{1}{32}\right)$

To find $\log_2\left(\frac{1}{32}\right)$, write $\frac{1}{32}$ as 2^{-5} and use the inverse property:

$\log_2\left(\frac{1}{32}\right) \;=\; \log_2 2^{-5} \;=\; -5$.

26. $\log_7 \sqrt{7}$

To find $\log_7 \sqrt{7}$, write $\sqrt{7}$ as $7^{1/2}$ and use the inverse property: $\log_7 \sqrt{7} = \log_7 7^{1/2} = \frac{1}{2}$

27. $\ln e^{1234567}$

To find $\ln e^{1234567}$, use the inverse property: $\ln e^{1234567} = 1234567$

28. $\log_{19}\left(\frac{1}{19}\right)$

To find $\log_{19}\left(\frac{1}{19}\right)$, write $\frac{1}{19}$ as 19^{-1} and use the inverse property:

$\log_{19}\left(\frac{1}{19}\right) \;=\; \log_{19} 19^{-1} \;=\; -1$.

29. $\log\sqrt{10^x}$

To find $\log\sqrt{10^x}$, write $\sqrt{10^x}$ as 10 raised to some power and use the inverse property:

$\log\sqrt{10^x} \;=\; \log 10^{x/2} \;=\; \frac{x}{2}$

30. $\ln\sqrt{e^{3x}}$

To find $\ln\sqrt{e^{3x}}$, write $\sqrt{e^{3x}}$ as e raised to some power and use the inverse property

$\ln\sqrt{e^{3x}} \;=\; \ln e^{3x/2} \;=\; \frac{3x}{2}$.

31. $\ln\sqrt[3]{e^{2}}$

To find $\ln\sqrt[3]{e^{2}}$, write $\sqrt[3]{e^{2}}$ as e raised to some power and use the inverse property

$\ln\sqrt[3]{e^{2}} \;=\; \ln e^{2/3} = \frac{2}{3}$.

32. $\log_{81} 3 = \boxed{?}$

Rewriting in exponential form, we have $81^{\boxed{?}} = 3$.

We need to find what number 81 is raised to in order to get $81^{\boxed{?}} = 3$.

Since $81 = 3^4$ we have

$81^{\boxed{?}} = 3$

$\left(3^4\right)^{\boxed{?}} = 3$

$(3)^{4\boxed{?}} = 3^1$

To find the answer, we need to find what number multiplied by 4 is 1.

$(3)^{4\boxed{x}} = 3^1$ means $4x = 1$. Divide both sides by 4.

The answer is $x = \frac{1}{4}$. Therefore, $\log_{81} 3 = \boxed{1/4}$.

33. $\log\left(\dfrac{1}{10^x}\right)$

To find $\log\left(\dfrac{1}{10^x}\right)$, write $\dfrac{1}{10^x}$ as 10^{-x} and use the inverse property:

$\log\left(\dfrac{1}{10^x}\right) \;=\; \log\left(10^{-x}\right) \;=\; -x$.

34. $\log_{\pi}\left(\dfrac{1}{\pi^4}\right)$. Use the inverse property: $\log_{\pi}\left(\dfrac{1}{\pi^4}\right) \;=\; \log_{\pi}\pi^{-4} \;=\; -4$

35. $\ln e^{3x}$

To find $\ln e^{3x}$, use the inverse property: $\ln e^{3x} = 3x$

36. $\log_{0.01}(1000) = \boxed{?}$

Rewriting in exponential form, we have $(0.01)^{\boxed{?}} = 1000$.

$\log_{0.01}(1000) = \boxed{?}$

Write each side to the same base. Since $0.01 = 10^{-2}$, and $1000 = 10^3$, then

$(0.01)^{\boxed{?}} = 1000$

$(10^{-2})^{\boxed{?}} = 10^3$

$(10)^{-2\boxed{?}} = 10^3$

To find the answer, we need to find what number multiplied by -2 is 3.

$(10)^{-2\boxed{x}} = 10^3$ means $-2x = 3$.

Divide both sides by -2 to find that $x = -\frac{3}{2}$. Therefore, $\log_{0.01}(1000) = \boxed{-3/2}$.

37. $\ln e^{3\sqrt{2\pi}}$

To find $\ln e^{3\sqrt{2\pi}}$, use the inverse property: $\ln e^{3\sqrt{2\pi}} = 3\sqrt{2\pi}$

38. $\log 10^{\sqrt{17}}$

To find $\log 10^{\sqrt{17}}$, use the inverse property: $\log 10^{\sqrt{17}} = \sqrt{17}$

39. $\ln(1/e^{123456789})$

To find $\ln(1/e^{123456789})$, write $1/e^{123456789}$ as $e^{-123456789}$ and use the inverse property:

$\ln(1/e^{123456789}) = \ln e^{-123456789} = -123456789$.

40. $\log\left(\frac{1}{10^{4x}}\right)$

To find $\log\left(\frac{1}{10^{4x}}\right)$, write $\frac{1}{10^{4x}}$ as 10^{-4x} and use the inverse property:

$\log\left(\frac{1}{10^{4x}}\right) = \log 10^{-4x} = -4x$.