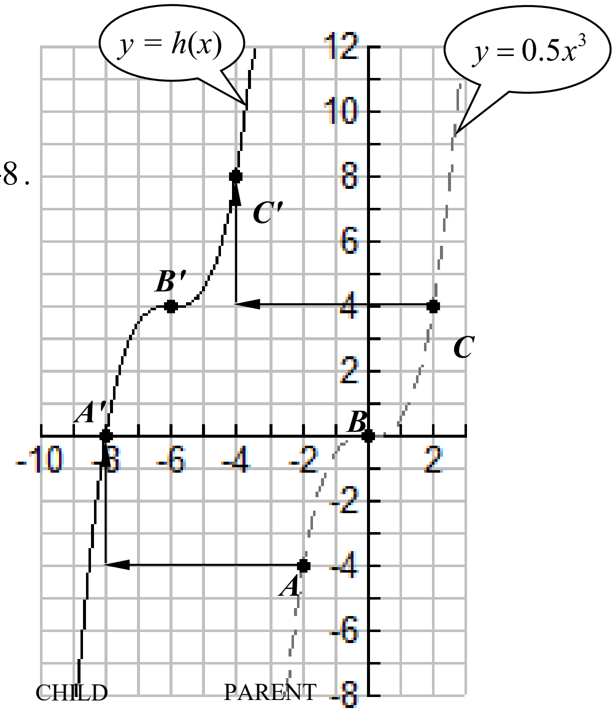
1. **a.** Horizontal shift 6 left and vertical shift 4 up.
Notice B' is $(-6, 4)$ and B is $(0, 0)$.

b. $h(x) = 0.5(x + 6)^3 + 4$ (Enter in a grapher to check.)

c. Use the graph. Notice A' to see $h(x)$ crosses the x-axis at -8.
Check with the formula.

If $x = -8$, $h(x) = 0.5(x + 6)^3 + 4$
$= 0.5(-8 + 6)^3 + 4$
$= 0.5(-2)^3 + 4$
$= 0.5(8) + 4 = 0$.

d. Use the formula. It crosses the y-axis when $x = 0$.
$h(0) = 0.5(0 + 6)^3 + 4 = 112$. You can also use the table.

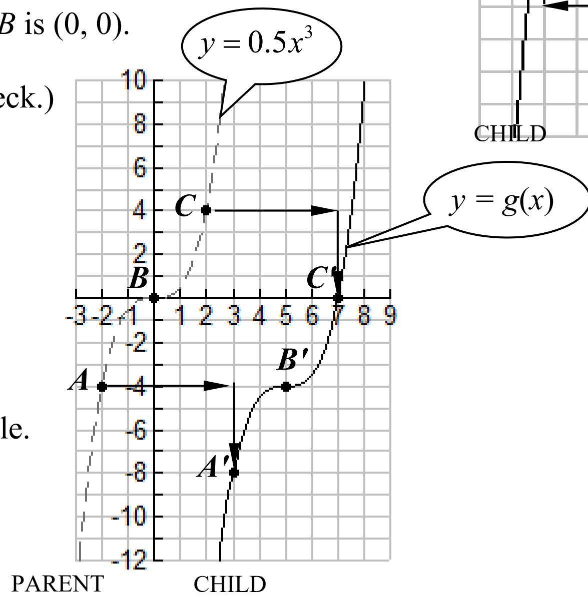


2. **a.** Horizontal shift 5 right and vertical shift 4 down.
Notice B' is $(5, -4)$ and B is $(0, 0)$.

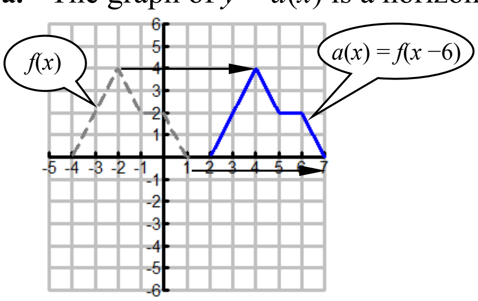
b. $g(x) = 0.5(x - 5)^3 - 4$
(Enter in a grapher to check.)

c. Notice C' to see $g(x)$ crosses the x-axis at 7.

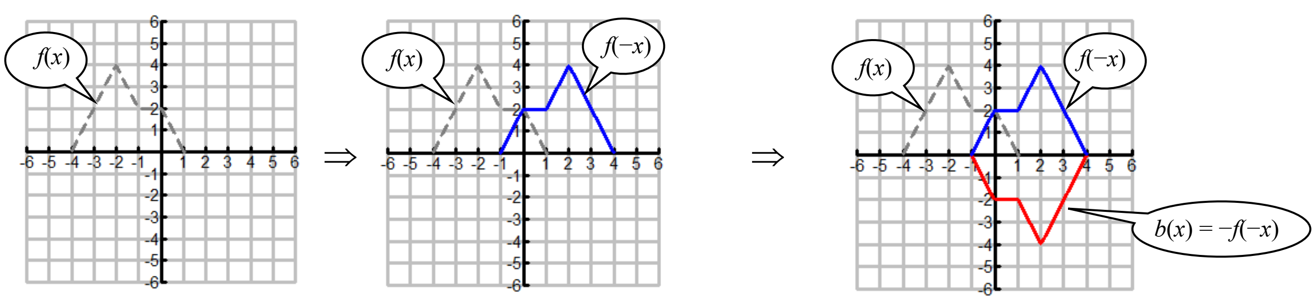
d. Use the formula.
It crosses the y-axis when $x = 0$.
$x = 0$.
$g(0) = 0.5(0 - 5)^3 - 4$
$= -66.5$
You can also use the table.



3. **a.** The graph of $y = a(x)$ is a horizontal shift of the graph of $y = f(x)$ to the right 6 so $a(x) = f(x - 6)$.



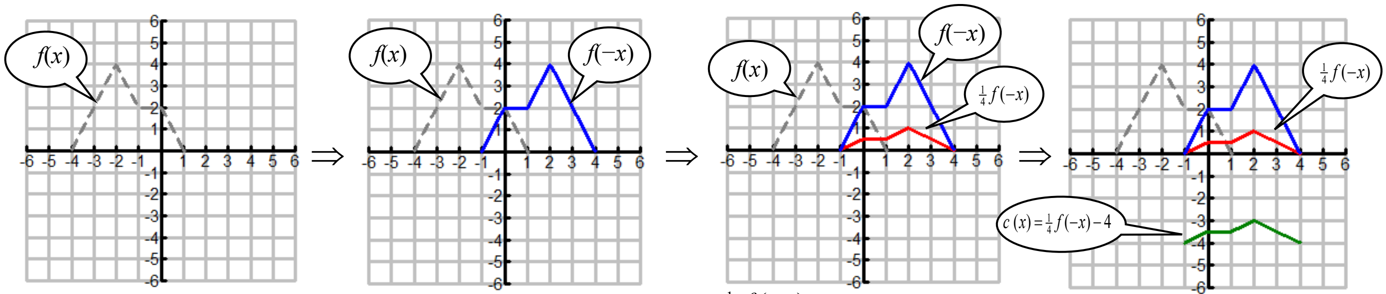
b. The graph of $y = b(x)$ is a horizontal and vertical reflection of the graph of $y = f(x)$ so $b(x) = -f(-x)$.



$f(-x)$ is a horizontal reflection of $f(x)$.

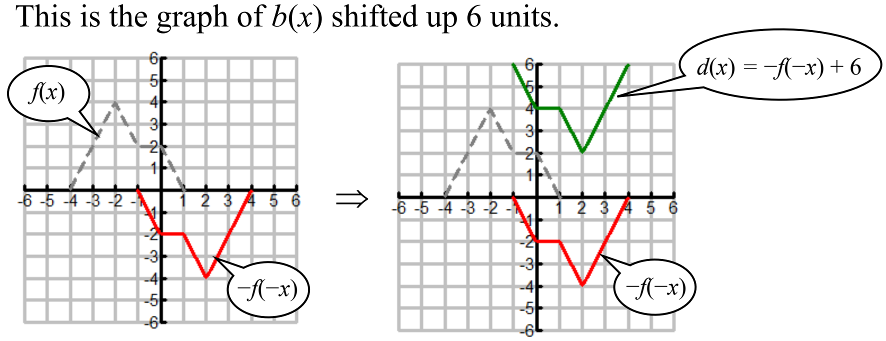
$-f(-x)$ is a vertical reflection of $f(-x)$.

c. The graph of $y = c(x)$ is a horizontal reflection, followed by a vertical compression by a factor of $\frac{1}{4}$, followed by a vertical shift down 4 units, so $c(x) = \frac{1}{4}f(-x) - 4$.



$f(-x)$ is a horizontal reflection of $f(x)$. $\frac{1}{4}f(-x)$ is a vertical compression of $f(-x)$. $\frac{1}{4}f(-x)-4$ is a vertical shift of $\frac{1}{4}f(-x)$ down 4.

d. The graph of $y = d(x)$ is a horizontal and vertical reflection, followed by a vertical shift up 6. This is the graph of $b(x)$ shifted up 6 units.

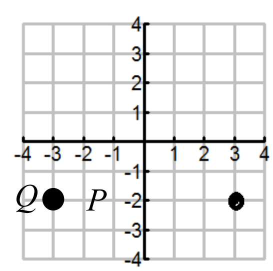


4. Suppose the point $P(3,–2)$ is a point on the graph of $y = f(x)$

a. Suppose $f(x)$ is **even:**

i. Report the coordinates of another point Q, which corresponds to P. (<u>−3</u> , <u>−2</u>)

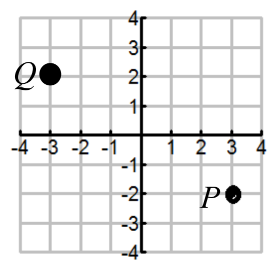
ii. Plot the point Q on the grid provided.



b. Suppose $f(x)$ is **odd**:

i. Report the coordinates of another point Q, which corresponds to P. (<u>−3</u> , <u>2</u>)

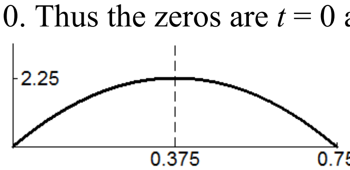
ii. Plot the point Q on the grid provided.



5. a. To factor $h(t) = -16t^2 + 12t$, remove a greatest common factor of $-4t$: $h(t) = -16t^2 + 12t = -4t(4t - 3)$. *Alternatively:* $h(t) = 4t(-4t + 3)$ is also correct. So also is $h(t) = -16t(t - 0.75)$ or $h(t) = 16t(-t + 0.75)$.

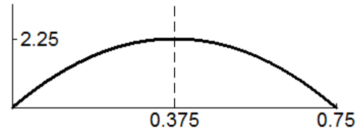
b. To find the zeros of the function, set each factor equal to 0. Thus the zeros are $t = 0$ and $t = 0.75$.

c. To find the vertex of the function, plot the zeros. The vertex is on the axis of symmetry which is midway between them. The x-coordinate of the vertex is $\frac{1}{2} \times 0.75 = 0.375$.



Find the y-coordinate of the vertex by substituting t = 0.375 in the formula or use a table with TblStart = 0 and ΔTbl = 0.375. We have $y = 2.25$. So the vertex is (0.375, 2.25).

d. The equation of the axis of symmetry is $x = 0.375$. The equation $t = 0.375$ is also correct.

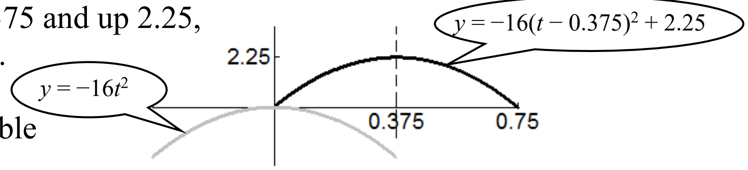


e. How much time in seconds is the dancer in the air? 0.75 sec

f. What is the maximum height of the jump? 2.25 ft

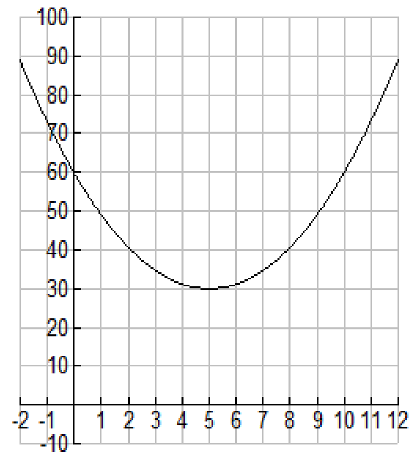
g. When does the maximum height of the jump occur? 0.375 sec

h. To write the formula in *vertex form*, use the fact that the parabola is a translation of $y = at^2$.
We are given the formula in standard form $y = at^2 + bt + c = -16t^2 + 12t$ so we know $a = -16$.
If we shift $y = -16t^2$ to the right 0.375 and up 2.25,
we have $y = -16(t - 0.375)^2 + 2.25$.
We can check with a grapher that
this produces the same graph and table
as $h(t) = -16t^2 + 12t$.



6. **a.** Use vertex form $y = a(x - h)^2 + k$ with $h = 5$, $k = 30$.
We have $y = a(x - 5)^2 + 30$
Plug in a point $x = 0$, $y = 60$ $\Rightarrow$ $y = a(x - 5)^2 + 30$

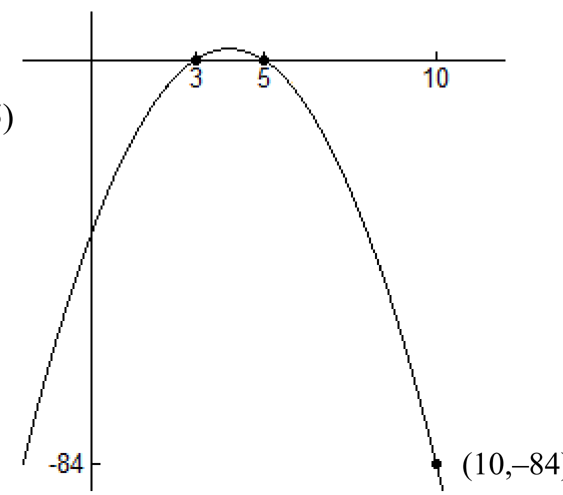
$$60 = a(0-5)^2 + 30$$
$$60 = 25a + 30$$
$$30 = 25a$$
$$a = \tfrac{30}{25} = 1.2$$



So the formula is $y = 1.2(x - 5)^2 + 30$

b. Use factored form: $y = a(x-3)(x-5)$
Plug in a point $x = 10$, $y = -84$ $\Rightarrow$ $y = a(x-3)(x-5)$

$$-84 = a(10-3)(10-5)$$
$$-84 = 7 \cdot 5a$$
$$-84 = 35a$$
$$a = \tfrac{-84}{35} = -2.4$$

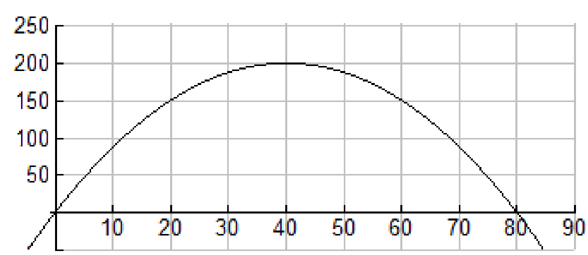


So the formula is $y = -2.4(x-3)(x-5)$

c. You can use factored form or vertex form.

Use factored form: $y = a(x)(x-80)$
Plug in a point $x = 40$, $y = 200$ $\Rightarrow$ $y = a(x)(x-80)$

$$200 = a(40)(40-80)$$
$$200 = -1600a$$
$$a = \tfrac{200}{-1600} = -0.125$$



So the formula is $y = -0.125(x)(x-80)$
Or use vertex form: $y = a(x - 40)^2 + 200$
Plug in a point $x = 80$, $y = 0$ $\Rightarrow$ $y = a(x - 5)^2 + 30$

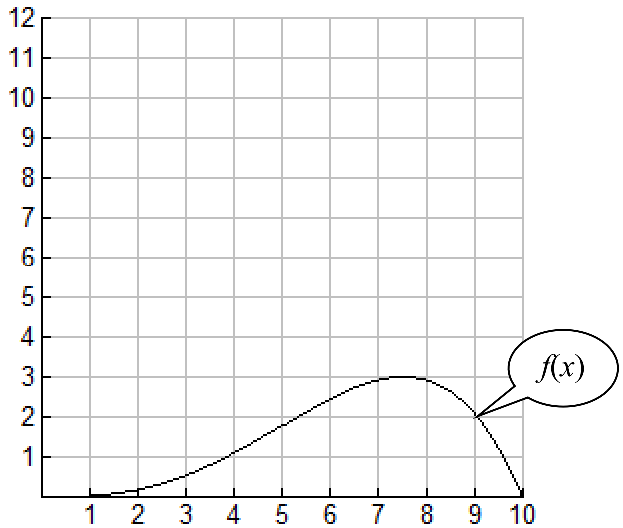
$$0 = a(80-40)^2 + 200$$
$$0 = 1600a + 200$$
$$-200 = 1600a$$
$$a = \tfrac{-200}{1600} = -0.125$$

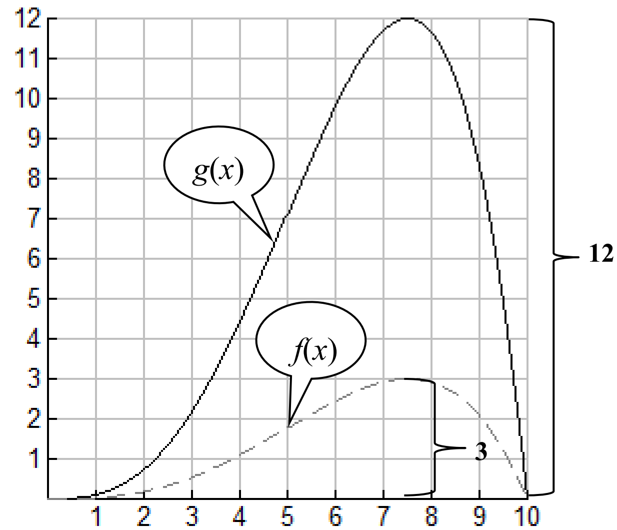
So the formula is $y = -0.125(x - 40)^2 + 200$

These two formulas are equivalent and either one is correct.

7. The graph of $y = f(x)$.is shown. Use the graph of $f(x)$ to write $g(x)$ as a transformation of $f(x)$. Find a formula for *g(x)* in **<u>terms of *f(x)*.</u>**

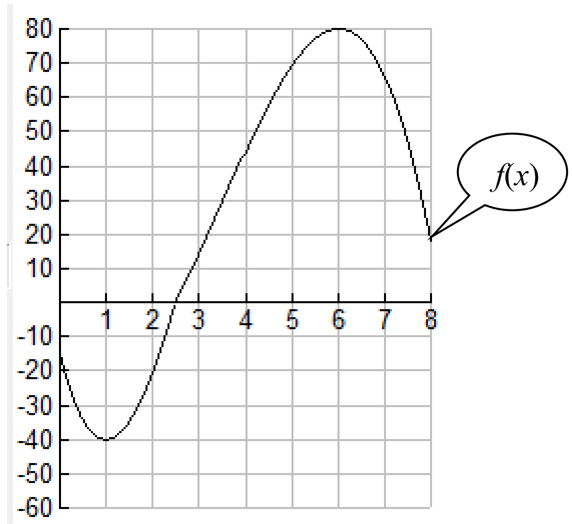
a.

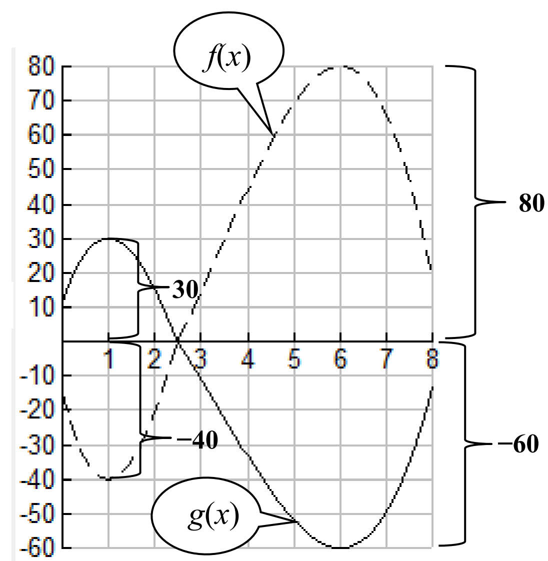




The outputs of $g(x)$ are *larger* than those for $f(x)$ so it is a *vertical stretch*. Compare maximum points. The graph of $g(x)$ is a vertical stretch of the graph of $f(x)$ by a factor of k, where $3k = 12$. Thus $k = 4$ and $g(x) = 4f(x)$.

b.



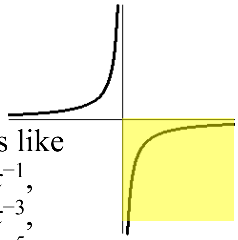


The outputs of $g(x)$ are *smaller* than those for $f(x)$ so it is a *vertical shrink*. Compare maximum points. The graph of $g(x)$ is a vertical compression of the graph of $f(x)$ by a factor of k, where $80k = -60$. You could also compare minimum points: $-40k = 30$. In either case, $k = -0.75$ and $g(x) = -0.75\, f(x)$.

8. The graphs below are power functions of the form $y = k\,x^p$.

Determine the following information. Circle the appropriate bold face words.

a.



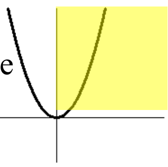
This is like $y=-x^{-1}$, $y=-x^{-3}$, $y=-x^{-5}\ldots$

The leading coefficient, k, is **negative**. Positive inputs yield **negative** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

b.



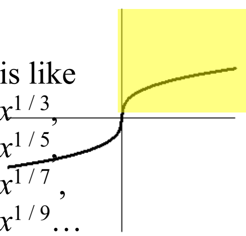
This is like $y=3x^2$, $y=3x^4$, $y=3x^6\ldots$

The leading coefficient, k, is **positive** Positive inputs yield **positive** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

c.



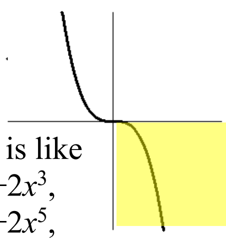
This is like $y=2x^{1/3}$, $y=2x^{1/5}$, $y=2x^{1/7}$, $y=2x^{1/9}\ldots$

The leading coefficient, k, is **positive** Positive inputs yield **positive** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

d.



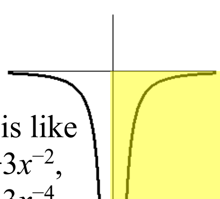
This is like $y=-2x^3$, $y=-2x^5$, $y=-2x^7\ldots$

The leading coefficient, k, is **negative.** Positive inputs yield **negative** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

e.



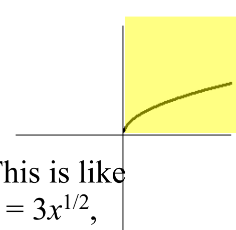
This is like $y=-3x^{-2}$, $y=-3x^{-4}$, $y=-3x^{-5}\ldots$

The leading coefficient, k, is **negative.** Positive inputs yield **negative** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

f.



This is like $y=3x^{1/2}$, $y=3x^{1/4}$, $y=3x^{1/6}\ldots$

The leading coefficient, k, is **positive.** Positive inputs yield **positive** outputs

The power, p, is **even** (like ±2, ±4,…) / **odd** (like ±1, ±3,…) / **fractional.** (like $\pm\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\ldots$)

The symmetry of the graph is **even** / **odd** / **neither even nor odd** .

9. **a**.

x	y
1	2
16	128

I'm k.

$$x = 1, y = 2 \Rightarrow y = kx^p$$
$$2 = k(1)^p$$
$$= k \cdot 1$$
$$= k$$

$$x = 16, y = 128 \Rightarrow y = 2x^p$$
$$128 = 2(16)^p$$ Divide both sides by 2.
$$64 = 16^p \longrightarrow 16^p = 64$$
$$p \log 16 = \log 64$$ Take logs of both sides .
$$p = \frac{\log 64}{\log 16} = 1.5$$

Therefore, $y = 2x^{1.5}$.
Check with a grapher that your table matches.

b.

x	y
81	900
625	1500

$$x = 81, y = 900 \quad \Rightarrow 900 = k81^p$$
$$x = 625, y = 1500 \Rightarrow 1500 = k625^p$$
$$\frac{900}{1500} = \frac{k(81)^p}{k(625)^p}$$
$$0.6 = \frac{(81)^p}{(625)^p}$$
$$0.6 = \left(\frac{81}{625}\right)^p$$

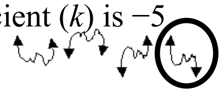
$$\left(\frac{81}{625}\right)^p = 0.6$$
$$\log\left(\frac{81}{625}\right)^p = \log 0.6$$
$$p \log\left(\frac{81}{625}\right) = \log 0.6$$
$$p = \frac{\log 0.6}{\log(81/625)}$$
$$= 0.25$$

$$x = 81, y = 900 \Rightarrow y = kx^{0.25}$$
$$900 = k(81)^{0.25}$$
$$k = \frac{900}{(81)^{0.25}}$$
$$= 300$$

Therefore, $y = 300x^{0.25}$.
Check with a grapher that your table matches.

10. For $f(x) = 80 + 70x - 30x^3 - 5x^7$

a. leading term (kx^p) is $-5x^7$ **b**. leading coefficient (k) is -5 **c**. degree (p) is 7

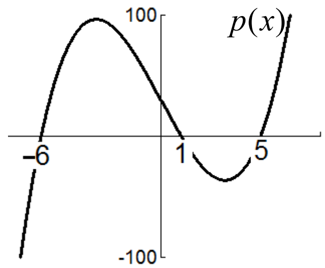
d. Report the long run behavior of $f(x)$. Specify as 

11. For $g(x) = -20(x-50)^4(x+200)^2 = -20x^6$ + terms of lower degree

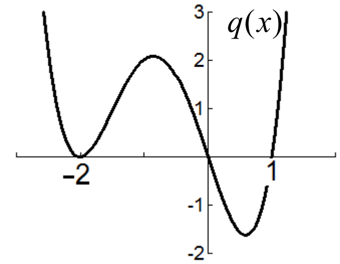
a. leading term (kx^p) is $-20x^6$ **b**. leading coefficient (k) is -20 **c**. degree (p) is 6

d. Report the long run behavior of $f(x)$.
Specify as 

12. $p(x) = x^3 - 31x + 30 = (x+6)(x-1)(x-5)$
All single zeros. Check both have same degree!

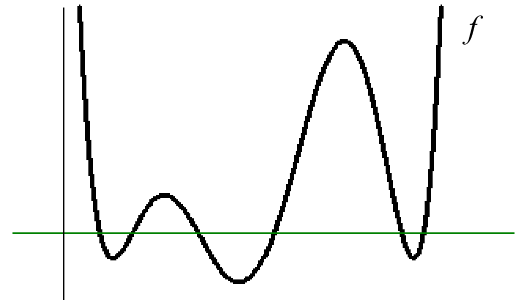


$q(x) = x^4 + 3x^3 - 4x = x(x+2)^2(x-1)$
-2 is a double zero.
0 and 1 are single zeros.
Check both have same degree!

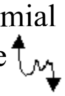


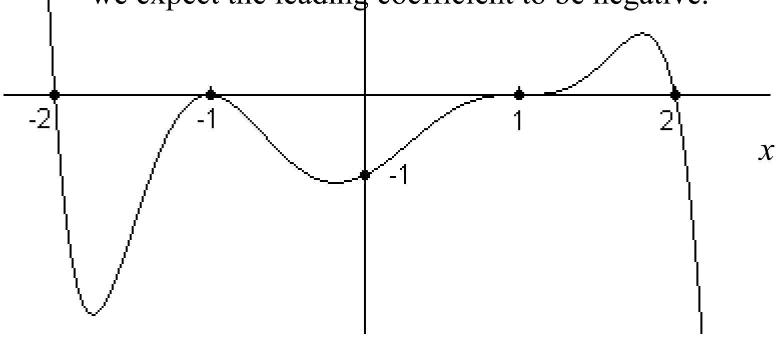
13. $f(x) = ax^n$ + remaining terms of lower degree

a. a is positive since it goes 
b. n is even since both arms go the same way (up).
c. $n \geq 6$ since there are 6 zeros (so 6 linear factors)



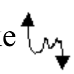
14. Write a possible formula for each polynomial function.

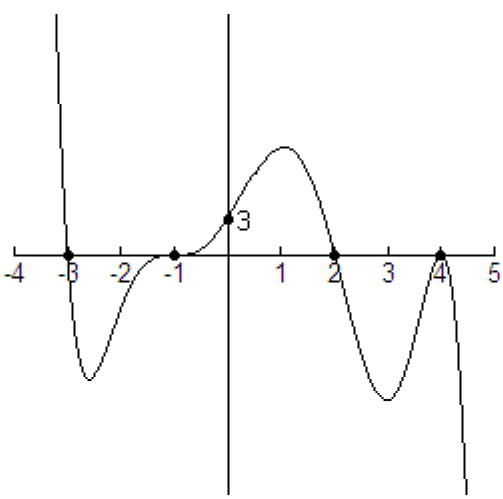
a. Since the long run behavior looks like 
we expect the degree to be odd and
we expect the leading coefficient to be negative.



Zeros: -2 (single)
-1 (double)
1 (triple)
2 (single)

$$y = a(x+2)(x+1)^2(x-1)^3(x-2)$$
$$x=0, y=-1 \Rightarrow y = a(x+2)(x+1)^2(x-1)^3(x-2)$$
$$-1 = a(0+2)(0+1)^2(0-1)^3(0-2)$$
$$-1 = a(4)$$
$$a = -\tfrac{1}{4} = -0.25$$
$$y = -0.25(x+2)(x+1)^2(x-1)^3(x-2)$$

b. Since the long run behavior looks like 
we expect the degree to be odd and
we expect the leading coefficient to be negative

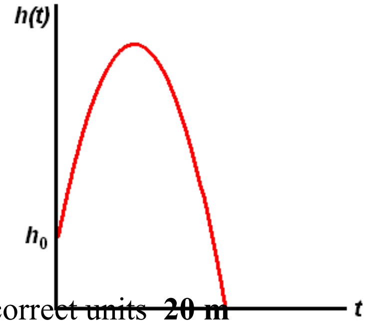


Zeros: -3 (single)
-1 (triple)
2 (single)
4 (double)

$$y = a(x+3)(x+1)^3(x-2)(x-4)^2$$
$$x=0, y=3 \Rightarrow y = a(x+3)(x+1)^3(x-2)(x-4)^2$$
$$3 = a(0+3)(0+1)^3(0-2)(0-4)^2$$
$$3 = a(-96)$$
$$a = -\tfrac{1}{32} = -0.03125$$

$$y = -0.03125(x+3)(x+1)^3(x-2)(x-4)^2$$

15. A model rocket is launched from the roof of a building with height h_0. Its height above ground (in meters) t seconds later is given by



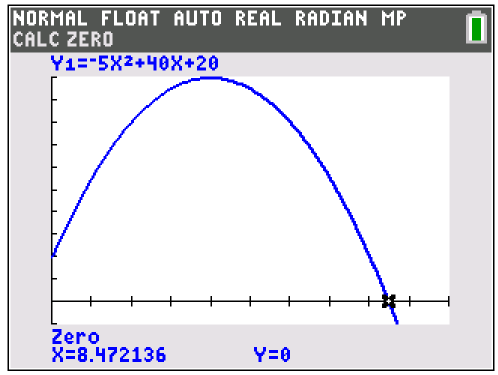
$$h = f(t) = -5t^2 + 40t + 20$$

Answer the following.
All work may be done on the calculator. No work need be shown!

a. What is the value of h_0, the initial height of the rocket? Please report with correct units. **20 m**

b. When will the rocket hit the ground?
Report accurate to two decimal places. **8.47 sec**

Use the table to find a viewing window.



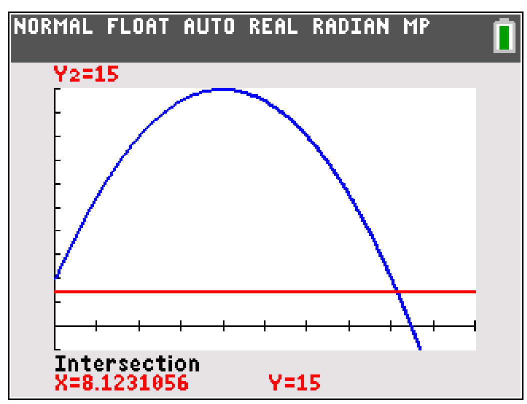
c. What is the **exact** maximum height of the rocket?
Please report with correct units. **100**

d. When will the rocket reach its maximum height?
Please report with correct units. **4 sec**

X	Y1	
0	20	
1	55	
2	80	
3	95	
4	100	← vertex
5	95	
6	80	
7	55	
8	20	
9	-25	
10	-80	

e. What length of time will the rocket be 15 feet or higher?
Report accurate to two decimal places. **8.12 s**

Y1 = $-5X^2+40X+20$
Y2 = 15



f. Give the domain of the height of the rocket function (restricted according to the ***context of the problem situation***.)

$0 \le t \le 8.47$

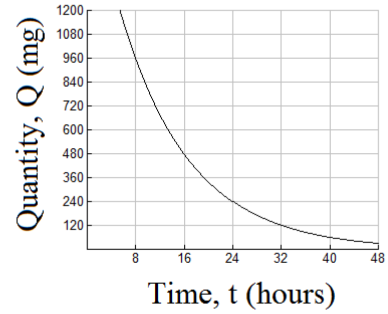
g. Give the range of the height of the rocket function (restricted according to the ***context of the problem situation***.)

$0 \le f(t) \le 100$

16. A function Q gives the amount, in mg of drug in a patient's body.
The function Q decays exponentially.
Assume the pattern holds.

a. Complete the first entry in the table.
Complete the next row in the table.

t, hours	$f(t)$
0	**1920** = 960 ·2
8	960
16	480
24	240
32	120
40=32+8	**60** = 120 ·0.5



b. Report the half-life, in hours
Ans: 8 hours (This is the Δt.)

c. Find a formula for this function

$Q = 1920(0.5)^{t/8}$ or $Q = 1920(0.917)^t$ since $b = (0.5)^{1/8} \approx 0.917$

d. What was the original amount of medication taken? Ans. 1920 mg

e. Every hour the patient loses **8.3** % and keeps **91.7** % of the drug. Report each to the nearest 0.1 percent.

f. Find, to the nearest 0.01 hour, the time it takes for the amount of drug to first fall below 1000 mg.
Show work. $t \approx$ **7.53** hours

Method 1: Solve using logarithms. We can use common or natural logarithms. We will use natural.

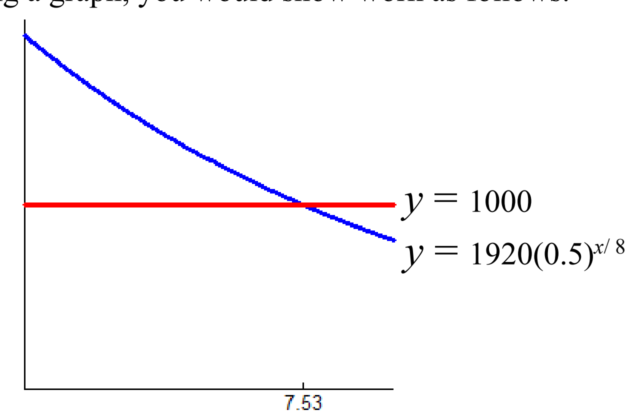
Solve $1920(0.5)^{t/8} = 1000$	Divide both sides by 1920
$(0.5)^{t/8} = 1000/1920$	Take natural logs of both sides.
$\ln(0.5)^{t/8} = \ln(1000/1920)$	Use the Bob Barker Property.
$\frac{t}{8}\ln(0.5) = \ln(1000/1920)$	Multiply both sides by 8.
$t\ln(0.5) = 8\ln(1000/1920)$	Divide both sides by ln(0.5)

$$t = \frac{8\ln(1000/1920)}{\ln(0.5)} \approx 7.53$$

Method 2: Using a table, you would show work as follows.
Enter your formula **1920(0.5)^(x/8)** in Y= and scroll.
Set your step size to 0.01

X	Y1
7.51	1001.6
7.52	1000.8
7.53	999.9
7.54	999.03
7.55	998.17

Method 3: Using a graph, you would show work as follows.



17. The relationship of pH to the hydrogen ion concentration C is pH = −log C.
If the pH is 2.15 what is the hydrogen ion concentration? Report to three decimal places.

Since the pH is 2.15 and pH = −log C we have 2.15 = −log C.
Multiply both sides by −1: $\log C = -2.15$
Make both sides a power of 10: $10^{\log C} = 10^{-2.15}$

$$C = 10^{-2.15} = \frac{1}{10^{2.15}} \approx 0.007$$ Choice C.

18. Sales of an item increase by 50% every 9 years.
Assume sales $f(t)$ continue to grow exponentially, where t is in years.

t	$f(t)$
0	100
9	**150 = 100 x 1.5**
18	**225 = 150 x 1.5**

a. If 100 items were sold at year $t = 0$, complete the table to determine the number sold in year $t = 9$ and year $t = 18$.
Report **<u>whole number</u>** of values. Ans: See the table to the right.

b. At what effective percent rate does it increase **<u>per year</u>**? Round to the nearest **0.1** percent. Ans: 4.6%
The formula for f(t) is $y = 100b^t$. Plug in a point, say $t = 9$, $y = 150$. Then solve for b.

$$100b^9 = 150$$

$$b^9 = \frac{150}{100} = 1.5$$

$$b = 1.5^{1/9} \approx 1.046$$

(Note that we didn't need the initial value 100 to find the value of b. We just needed the growth factor per 9-month period.)
The growth factor per year is 1.046, so the growth rate is 4.6%.

c. $f(t) = 100(1.5)^{t/9}$ or $f(t) = 100(1.046)^t$ Either of these is correct.

19. Sales of an item decrease by 98% every 6 years.
Assume sales $f(t)$ continue to decay exponentially, where t is in years.

t	$f(t)$
0	100,000
6	**2000 = 100,000 x 0.02**
12	**40 =2000 x 0.02**

a. If 100,000 items were sold at year $t = 0$, complete the table to determine the number sold in year $t = 6$ and year $t = 12$.
Report **<u>whole number</u>** of values.
Ans: If we lose 98%, we keep only 2% = 0.02 every 6 years. See the table to the right.

b. At what effective percent rate does it decrease **<u>per year</u>**? Round to the nearest **0.1** percent. Ans: 47.9%
The formula for f(t) is $y = 100000b^t$. Plug in a point, say $t = 6$, $y = 2000$. Then solve for b.

$$100{,}000b^6 = 2000$$

$$b^6 = \frac{2000}{100{,}000} = 0.02$$

$$b = 0.02^{1/6} \approx 0.521$$

(Note that we didn't need the initial value 100,000 to find the value of b.
We just needed the decay factor per 6-month period.)
The decay factor per year is 0.521, so we keep 52.1%, and thus lose 100% − 52.1% = **47.9%** per year.

c. $f(t) = 100000(0.02)^{t/6}$ or $f(t) = 100000(0.521)^t$ Either of these is correct.

20. a. In 53 seconds, $P = 400(0.5)^{53/53} = 400 \times 0.5 = 200$

so the time it takes to decay to half its original amount is 53 minute.

b. The function $Q = 400(0.5)^{t/53}$ can be written $Q = 400(0.5^{1/53})^t$

by laws of exponents and the fact that $\frac{t}{53} = \frac{1}{53}t$. Use a calculator to find $(0.5)^{1/53} \approx 0.987007$ so

$Q \approx 400\left(0.987007\right)^t$. Each second we keep about 98.7%,

so we lose 100% − 98.7% = 1.3% per minute.

21. a. $Q = Q_0(1.1776)^t$

b. Set $Q_0(1.1776)^t = 2Q_0$

$Q_0(1.1776)^t = 2Q_0$	Divide both sides by Q_0
$(1.1776)^t = 2$	Take natural logs of both sides.
$\ln(1.1776)^t = \ln(2)$	Use the Bob Barker Property.
$t\ln(1.1776) = \ln(2)$	Divide both sides by ln(1.1776)
$t = \dfrac{\ln(2)}{\ln(1.1776)} \approx 4.24$	

c. Set $Q_0(1.1776)^t = 3Q_0$

$Q_0(1.1776)^t = 3Q_0$	Divide both sides by Q_0
$(1.1776)^t = 3$	Take natural logs of both sides.
$\ln(1.1776)^t = \ln(3)$	Use the "Bob Barker Property".
$t\ln(1.1776) = \ln(3)$	Divide both sides by ln(1.1776)
$t = \dfrac{\ln(3)}{\ln(1.1776)} \approx 6.72$	