Practice Questions to Review for the MA 15300 Final

<u>Bring the following items to the final</u>:

- ✓ **your graphing calculator**
- ✓ **Number 2 pencils**
- ✓ **ID Number**
  (You can get this by logging into **http://go.pfw.edu**. You can also get it from your instructor.)

<u>The final exam will evaluate how well you meet the course goals of MA 15300:</u>

- Highlight the link of mathematics to the real world.
- Develop a wide base of mathematical knowledge, including
  - basic skills and concepts,
  - a functional view of mathematics, including graphical, analytical, numerical, and contextual viewpoints (Note: using these four representations is the *Rule of Four*),
  - properties and applications of some of the basic families of functions,
  - geometric visualization,
  - problem solving, predicting, critical thinking, and generalizing.
- Incorporate the use of general academic skills such as
  - communicating mathematics concepts,
  - understanding and using technology.

Just like the chapter exams throughout the semester, this exam will assess your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.

<u>Format of the exam:</u> The final exam will consist of entirely multiple-choice questions. You may certainly use your calculator (but not its manual). In fact, some questions will *require* a graphing calculator.

NO formula sheets, notes, books, or other external sources may be used.

<u>How to prepare for the exam</u>: Some of the practice questions that follow are from previous final exams over this material. Note that the exam will NOT look exactly like these questions, so you should also review previous homework assignments, *eHW*, quizzes, and tests, as well as material worked on during class meetings. **Topics from the last chapter on *Polynomial and Rational Functions* will receive more of an emphasis than earlier chapters.** Keep the *Rule of Four* (representing functions in words, graphs, tables, and formulas) in mind when solving problems, just as you have done throughout the semester.

The actual final exam will have less questions than what is provided in this review.
The worked out key is in your Brightspace course in the folder *Supplemental Resources for the Final Exam.* There you can also see the date, time, and location of the exam, which typically is **not** at the usual class meeting time or location.

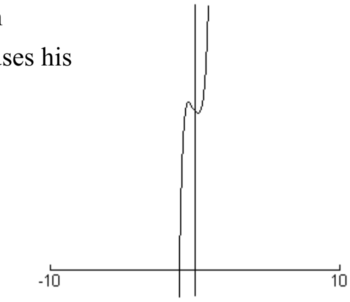
Date of Final: ___________________________________

Time: _________________________

Room Location: __________________ (typically this is not our usual classroom location)

Sample Questions for the Final Exam

1. Suppose you and Charlie are working together in a group to determine the long run behavior of $f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$. Charlie uses his graphing calculator in the window $-10 \le x \le 10$ and $-10 \le y \le 100$ and sees the graph shown. Charlie concludes that the long run behavior is as follows:
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to \infty$. How should you respond?



$-10 \le x \le 10 \quad -10 \le y \le 100$

A. "Good job, Charlie!"
B. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$."
C. "Sorry, Charlie!
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to -\infty$."
D. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to -\infty$."
E. "Sorry, Charlie!
As $x \to 0^+$, then $y \to -\infty$; as $x \to 0^+$, then $y \to \infty$."

Questions 2-3

The volume of pollutants (in millions of cubic feet) in Smirch Reservoir is given by

$$P(t) = 360 + 9t$$

where *t* is in years. The total volume of Smirch Reservoir (which includes both pollutants and water and also in millions of cubic feet) is gradually increasing and is given by

$$R(t) = 12{,}000 + 12t$$

Let $C(t)$ be the fraction of the reservoir's total volume that consists of pollutants.

Write an expression for $C(t) = \dfrac{P(t)}{R(t)}$ in terms of *t* and use your expression to answer the questions below.

2. In year $t = 0$, what percent of the reservoir's total volume consists of pollutants?
A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$% E. None of these

3. According to the mathematical model, if these trends were to continue for many, many years, about what percentage of Smirch Reservoir's total volume would eventually consist of pollutants?
A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$% E. None of these

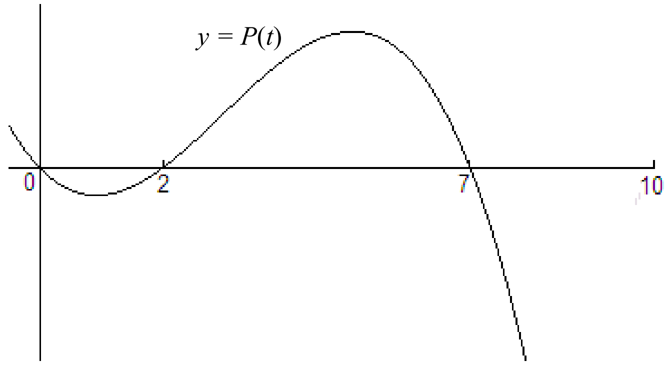
Questions 4-5

The amount Q of drug present in a person's body is $Q = 20(0.4)^t$, where Q is in milligrams at time t, and t is in hours.

4. What percent of the drug is lost per hour?
A. 4% B 20% C. 40% D. 6% E. 60% F. 80%

5. What is the growth factor?
A. 0.4 B. 4.0 C. 6.0 D. 20 E. 60 F. 80

For **Questions 6-7**, $P(t)$ is a polynomial of degree 3 whose graph is shown.



6. For $0 \le t \le 10$, $P(t)$ describes the temperature of a certain chemical reaction in degrees Celsius, t seconds after the reaction began. Suppose the temperature reached –1 degree Celsius exactly 1 second after the reaction began.

Determine the formula for $P(t)$.
Then find the minimum temperature that the chemical reaction reaches in the first 10 seconds after it began.

A. $-1^\circ C$ B. $-10^\circ C$ C. $-20^\circ C$ D. $-40^\circ C$ E. None of these

7. A certain power function $Q(t)$ has the same long run behavior as $P(t)$, so much that $Q(t)$ and $P(t)$ look nearly indistinguishable if you graph both of these functions with technology and zoom out for very large values of t. This tells us that it would not be sensible to use $P(t)$ to model the temperature of the reaction for *all* $t \ge 0$. What is the formula for $Q(t)$?

A. $Q(t) = -t^3$ B. $Q(t) = t^3$ C. $Q(t) = -\frac{1}{6}t^3$ D. $Q(t) = \frac{1}{6}t^3$ E. None of these.

Questions 8-10

The EDI pharmaceutical company has recently acquired the abandoned but historic Rotting Hill building and has decided to move its employees into this renovated building one month at a time. The table gives the number, $E(t)$, of EDI employees who have moved to the Rotting Hill building t months after the building was acquired.

t	$E(t)$
0	0
1	30
2.02	48
2.83	60

8. The data for $E(t)$ is modeled by a power function.
Find the formula for $E(t)$. Which of the following would be closest to the value of $E(7)$?

A. 100 B. 110 C. 120 D. 130 E. 210

9. Unfortunately, many of the employees of EDI who have their offices located in the Rotting Hill building have contracted a mysterious disease which incapacitates them for weeks at a time. The table gives the number, $S(t)$, of EDI employees who have their offices located in the Rotting Hill building *and* are sick t months after the initial acquisition of the building.

t	$S(t)$
0	0
1.05	6
2.05	15
2.98	25

The data for $S(t)$ is modeled by a power function. Find the formula for $S(t)$.
Which of the following would be closest to the value of $S(11)$?

A. 30 B. 70 C. 110 D. 120 E. 150

10. When the ratio of number of sick employees in a building to total number of employees in a building is greater than 0.75 the building is declared to have sick building syndrome and is closed down for health inspection. How many months after the Rotting Hill building is first acquired by EDI will it be closed for health inspection? Select the one closest to your answer.

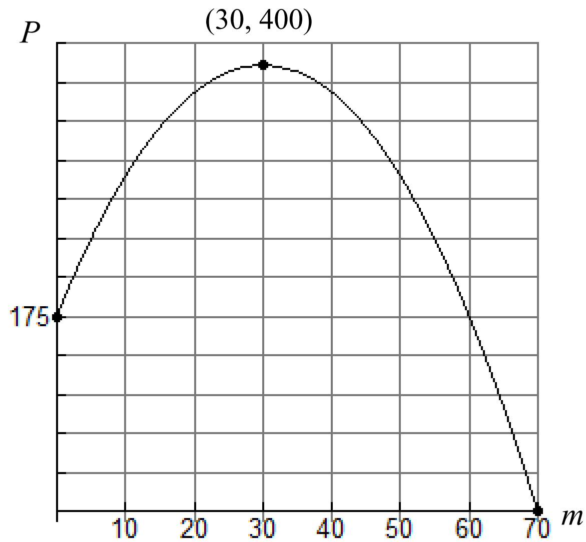
A. 10 days B. 2 months C. 5 months D. 7 months E. 16 months

Questions 11-17

Peter grows peppers. The yield, P, of peppers (in pecks) that Peter picks is a function of the amount, m, of fertilizer (in pounds) used, which is given by $P = f(m)$. See the graph below.

11. The statement $f(30) = 400$ means
A. The yield ranges from 30 to 400 pecks of peppers.
B. When 30 lb of fertilizer is applied, the yield is a maximum of 400 pecks of peppers.
C. For every 30 lb of fertilizer added to the orchard, you increase the yield by 400 pecks.
D. When 400 lb of fertilizer is applied, the yield is 30 pecks of peppers.
E. You apply 30 to 400 pounds of fertilizer to the orchard.

12. The vertical intercept for the graph represents:
A. The maximum yield of the orchard.
B. The amount of fertilizer that must be applied to produce a maximum yield.
C. The yield without applying any fertilizer at all.
D. The initial amount of fertilizer applied to the orchard.
E. The amount of fertilizer that will kill all the trees and produce no yield at all.



13. Estimate the range.
A. $0 \leq f(m) \leq 70$
B. $175 \leq f(m) \leq 400$
C. $70 \leq f(m) \leq 175$
D. $70 \leq f(m) \leq 400$
E. $0 \leq f(m) \leq 400$

14. For what values of m is the function increasing?
A. $175 < m < 400$
B. $0 < m < 400$
C. $0 < m < 30$
D. $30 < m < 70$
E. None of these

15. For what values of m is the function concave up?
A. $175 < m < 400$
B. $0 < m < 400$
C. $0 < m < 30$
D. $0 < m < 70$
E. None of these

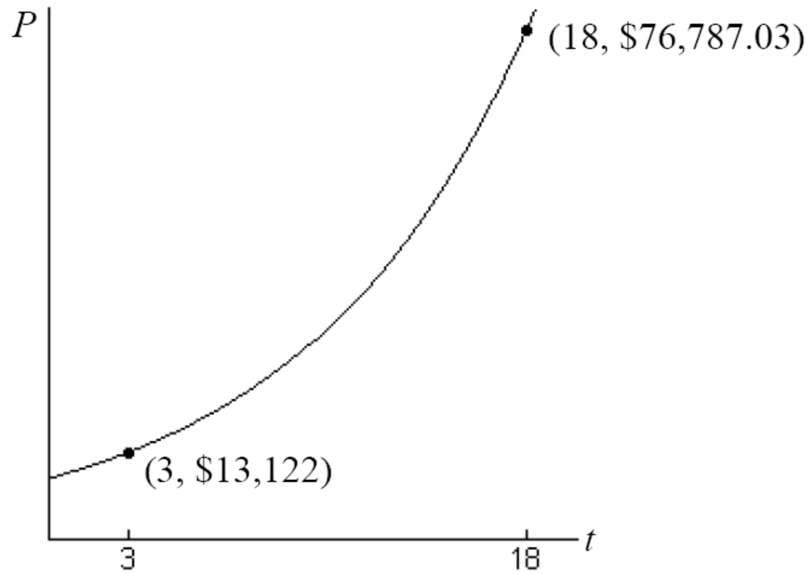
16. For what values of m is $P > 175$?
A. $175 < m < 400$
B. $60 < m < 400$
C. $60 < m < 70$
D. $0 < m < 60$
E. None of these

17. The function $f(m)$ is quadratic. Find a formula for $f(m)$.
Approximate the solutions to $f(m) = 261$.

a. What is one of the solutions less than 30? Select the best answer.
A. $m = 6.38$ B. $m = 6.40$ C. $m = 6.42$ D. $m = 6.44$ E. There are no solutions

b. What is another one of the solutions greater than 30? Select the best answer.
A. $m = 53.56$ B. $m = 53.58$ C. $m = 53.60$ D. $m = 53.62$ E. There are no solutions

Questions 18-19
The graph gives the balance, P, of an investment in year t. Find a possible formula for $P = f(t)$ assuming the balance grows exponentially.



18. Which amount is closest to the initial balance?
A. $5,424
B. $5,454
C. $9,216
D. $10,122
E. $11,664
F. $41,812

19. What annual interest rate does the account pay?
A. 1.125%
B. 11.25%
C. 12.5%
D. 34%
E. 112.5%
F. 125%

20. Use inverse properties and properties of logs to simplify the expression $e^{x\ln a}$. (Circle one.)

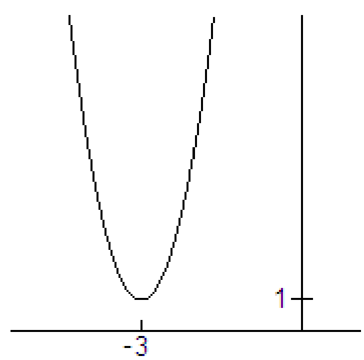
A. x^a B. e^{ax} C. $\frac{a}{x}$ D. ax E. a^x F. None of these

21. Water is pumped into a swimming pool at a constant rate. The table shows the volume of water every 30 minutes after the pumping began. What is the average rate of change?
A. 75 gallons
B. 0.4 minutes per gallon
C. 0.4 gallons per minute
D. 2.5 minutes per gallon
E. 2.5 gallons per minute

Time, t (min)	Volume, V (gal)
30	1075
60	1150
90	1225
120	1300

22. The graph of the function is a translation of $y = 5x^2$, shifted left 3 and up 1. What is the **range** of the function?

A. all real numbers
B. $y \geq 1$
C. $y \geq -3$
D. $y \geq -1$
E. $y \leq 1$

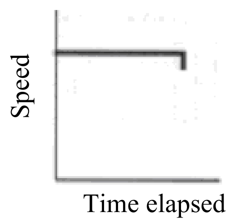


Questions 23-26

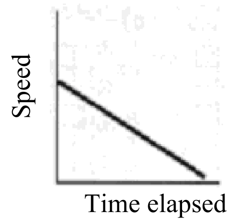
Indicate which graph matches the statements. Note the choice of axes.

23. A train pulls into a station and lets off passengers.

I.



II.



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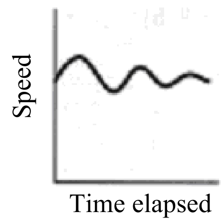
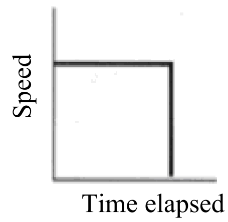
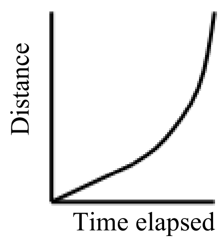


IV.

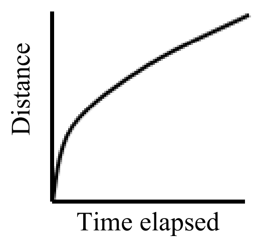


24. I start to walk to class at a slow steady rate. I hear the clock chimes and walk faster and faster.

I.



II.



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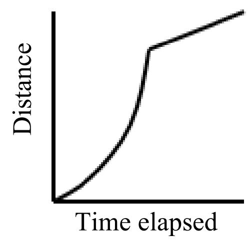

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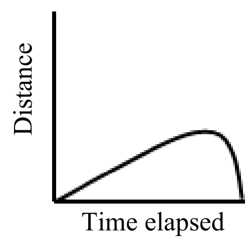
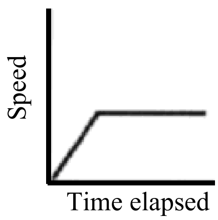
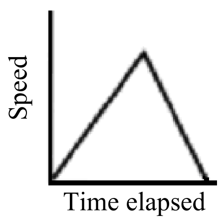


25. A rhino climbs a hill at a steady pace and then starts to run down one side.

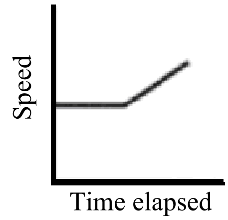
I.



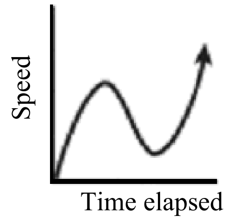
II.



III.

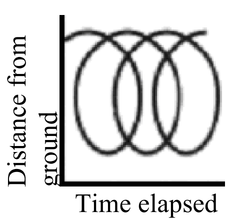


IV.

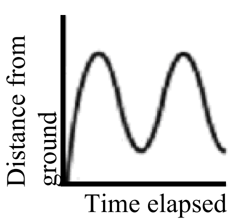


26. I ride on a Ferris Wheel.

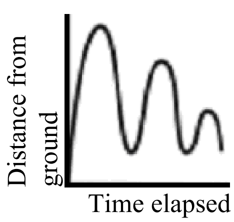
I.



II.



III.



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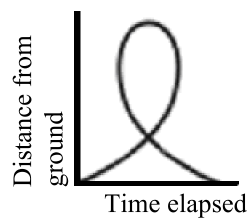
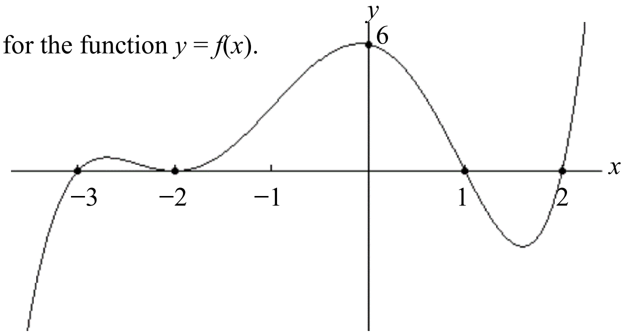


Questions 27-29

For the graphs in this problem assume all global (or long run) behavior is shown.

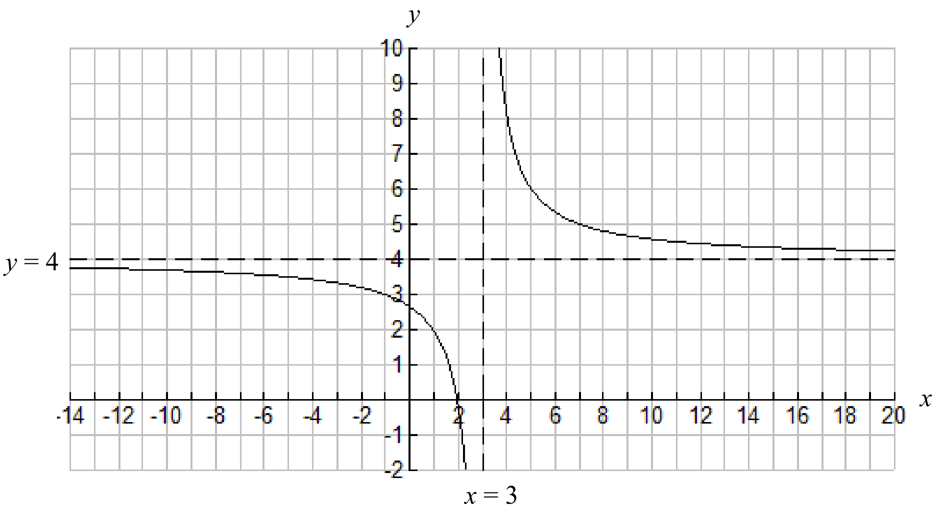
27. Find a possible formula of least possible degree for the function $y = f(x)$. Then use your formula to find $f(3)$.

A. 30
B. 75
C. 306
D. 501
E. None of these



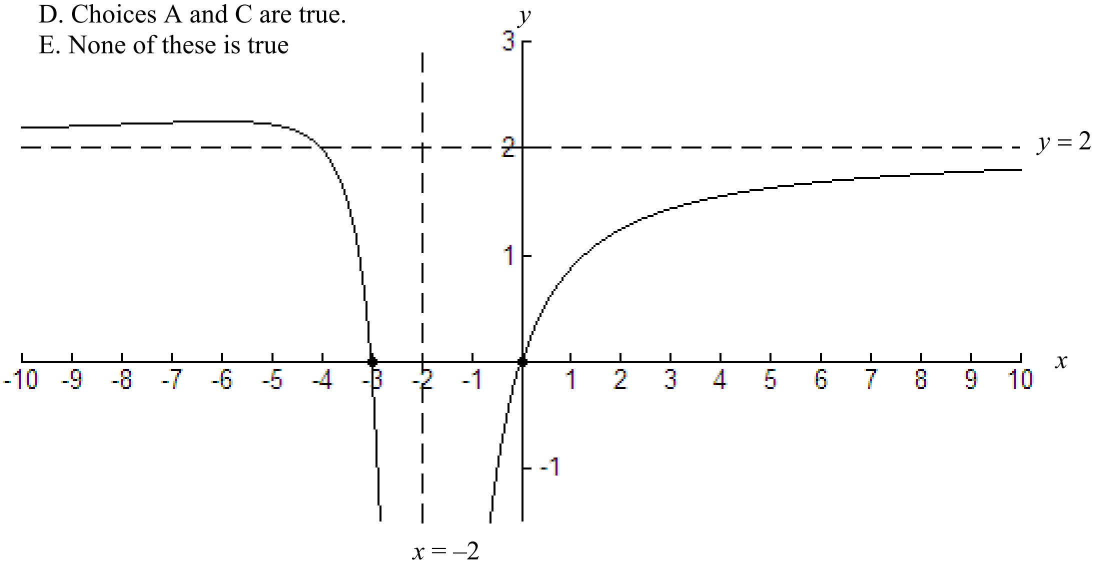
28. Find the formula for $f(x)$. It has a single zero at 2 and asymptotes shown. Use your formula to find $f(403)$. Tip: Use a table.

A. 4.1
B. 4.3
C. 4.01
D. 4.03
E. None of these

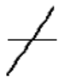
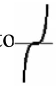
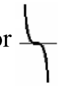
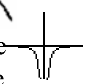
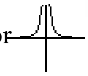


29. Find the formula for $f(x)$. It has zeros and asymptotes shown.
Using your formula, determine which of the following must be true? Tip. Use a table.

A. $f(-1) = -4$
B. $f(1) = 1$
C. $f(-6) = 2.25$
D. Choices A and C are true.
E. None of these is true



30. A rational function $y = f(x)$ has the following properties:

- there is only one zero at 4,
- the short run behavior near that zero looks like  or  (as opposed to  or )
- there is one vertical asymptote at $x = 2$,
- the short run behavior near the vertical asymptote looks like  or 
- the degree of the denominator is the lowest degree possible,
- there is a horizontal asymptote of $y = 0$, and
- $f(0) = -8$

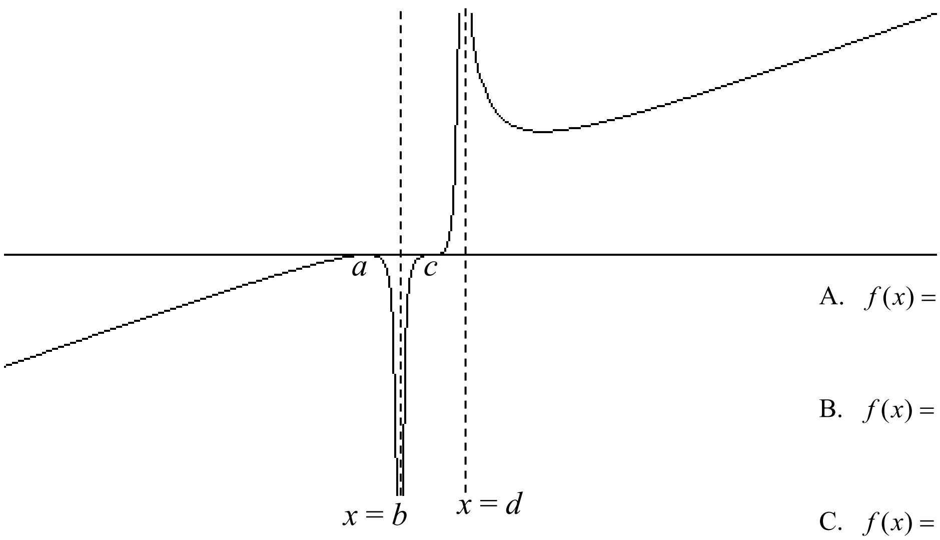
Find the formula for $f(x)$. Then use your formula to find $f(3)$.

A. -16
B. -8
C. -4
D. 8
E. 16

31. Assume a, b, c, and d are positive real numbers.
The rational function $f(x)$ graphed below has the following properties:

- short run behavior:
  - zeros are at a, c
  - vertical asymptotes are at $x = b$ and $x = d$
- long run behavior:
  as $x \to -\infty, y \to -\infty$
  as $x \to \infty, y \to \infty$
  Consequently, there is no horizontal asymptote.

Assume k is some positive real number. Which could be its equation?



A. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^4}$

B. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^2}$

C. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)}$

D. $f(x) = \dfrac{k(x-a)^4(x-c)^3}{(x-b)^3(x-d)^4}$

E. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)(x-d)^3}$

32. Assuming x, y, and w are positive real numbers, which of the following is $\log\frac{x^3y^2}{\sqrt{w}}$?

A. $x^3 + y^2 - \sqrt{w}$ B. $\frac{1}{3}\log x + \frac{1}{2}\log y - 2\log w$ C. $3\log x + 2\log y - \frac{1}{2}\log w$

D. $\frac{3\log x + 2\log y}{\frac{1}{2}\log w}$ E. None of these

33. Solve for x to the nearest hundredth: $25^x = 3^{600}$
(Most calculators are unable to solve this numerically or graphically due to overflow problems.)
A. 409.56 B. 530.44 C. 204.78 D. No solution E. None of these

34. Report all the zeros of the polynomial function: $f(x) = 400x(6x^2 - 42)$
A. 0, $\sqrt{7}$ B. 0, $\sqrt{7}$, 400 C. 0, $-\sqrt{7}$, $\sqrt{7}$, 400 D. 0, $-\sqrt{7}$, $\sqrt{7}$ E. None of these

35. Report all the zeros of the polynomial function: $f(x) = -3(x^4 - 7x^2 - 6x)$. Hint: Use a graph or table.
A. 1, −3 B. −1, 3 C. −2, −1, 0, 3 D. −3, −2, −1, 0, 3 E. None of these

36. Report all possible values of x for which $9x^2(x + 6)(x - 6)^2 \geq 0$. Support your reason graphically.
A. $-6 \leq x \leq 6$ B. $-6 \leq x \leq 0$ or $x \geq 6$ C. $x \geq -6$ D. $x \leq 6$ E. None of these

37. Report the domain of $f(x) = \sqrt{x - 100}$.
A. $x \leq 100$ B. $x \geq 100$ C. $x \geq -100$ D. $x \leq -100$ E. None of these

38. An initial deposit of \$4000 is made in a savings account for which the interest is compounded continuously. If the interest rate is 7.3%, how long will it take, to the nearest 0.01 year, for the investment to triple? Use $A = Pe^{rt}$.
A. 0.15 years B. 2.79 years C. 6.54 years D. 15.05 years E. None of these

39. Simplify $\ln\left(\frac{1}{\sqrt{e^x}}\right)$

A. $\frac{1}{x}$ B. $-x$ C. $-\frac{x}{2}$ D. $\frac{1}{\sqrt{x}}$ E. None of these

40. Which of the following is true about the graph of $y = f(x) = b^x$? List **all** correct answers.

I. It increases if $b > 1$
II. It decreases if $b < 0$
III. It has y-intercept (0, 1) if $b > 0$.

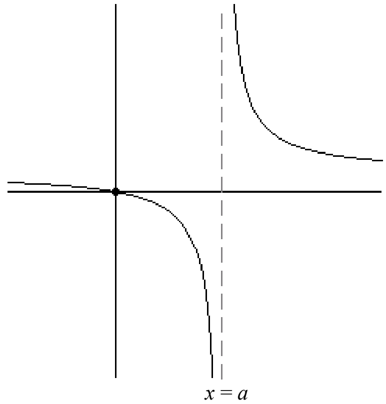
A. I, II and III B. I and II C. II and III D. I and III E. III only.

41. Use what you know about transformations and the graph of $y = \log x$ to answer the following about the graph of $f(x) = 2 + \log(x - 1)$. Which are true? List **all** correct answers.
The graph of $f(x) = 2 + \log(x - 1)$

I. increases for all values of x in its domain.
II. crosses the x-axis at 1
III. never touches the y-axis
IV. passes through the point (2, 2).

Note: Don't be misled by technology when answering this question.
A. I, II and III B. I and II C. II and IV D. I and IV E. I, III and IV only.

42. A function passes through the origin and has a vertical asymptote at $x = a$, where $a > 0$. It has the graph shown. Which could be its equation?



A. $f(x) = \dfrac{1}{x-a}$ B. $f(x) = \dfrac{1}{x+a}$ C. $f(x) = \dfrac{x}{x-a}$

D. $f(x) = \dfrac{x}{x+a}$ E. $f(x) = \dfrac{x}{(x-a)^2}$

43. Let a be some constant.

Which is true about $f(x) = \dfrac{2ax}{(x-a)^2}$?

A. Its horizontal asymptote is $y = 2a$.
B. Its horizontal asymptote is $y = 2$.
C. Its horizontal asymptote is $y = \dfrac{2a}{x}$.
D. Its horizontal asymptote is $y = 0$.
E. It has no horizontal asymptote.

44. The relationship of pH to the hydrogen ion concentration, C, is pH $= -\log C$.
If the pH is 2.1, what is the hydrogen ion concentration?
A. 0.74 B. 0.008 C. 125.89 D. −0.322 E. −125.89

Questions 45-46:

45. Which of the following is an acceptable first step to solve the equation $\ln 2x^3 = 5$?

A. $3\ln 2x = 5$ B. $2x^3 = e \cdot 5$ C. $2x^3 = \dfrac{5}{\ln}$ D. $2x^3 = e^5$ E. $\ln 2x^3 = \ln 5$

46. What is the correct **exact** solution to the equation $\ln 2x^3 = 5$?

A. $\sqrt[3]{\dfrac{5}{2\ln}}$ B. $\sqrt[3]{\dfrac{5e}{2}}$ C. $\sqrt[3]{\dfrac{e^5}{2}}$ D. $\sqrt[3]{\dfrac{5}{2}}$ E. $\frac{1}{2}e^{5/3}$

Questions 47-48:

47. To solve the equation $20 = 3e^x + 5$, which of the following is an acceptable first step?
A. $4 = 3e^x$ B. $\ln 20 = x\ln(3 \cdot e) + \ln 5$ C. $\ln 20 = x\ln(3) + \ln 5$ D. $15 = 3e^x$ E. $25 = 3e^x$

48. Report the correct **exact** solution to the equation $20 = 3e^x + 5$.

A. $\dfrac{\ln 4}{\ln 3}$ B. $\ln(4/3)$ C. $\dfrac{\ln 25}{\ln 3}$ D. $\ln(25/3)$ E. $\ln(5)$

49. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded quarterly. Find the amount in year t.
A. $2200(1.382)^{4t}$ B. $2200(1+\frac{3.82}{4})^{4t}$ C. $2200(1+\frac{0.0382}{4})^{4t}$ D. $2200(1+\frac{3.82}{4})^{t}$ E. $2200(1+\frac{0.382}{4})^{4t}$

50. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded continuously. Find the amount in year t.
A. $2200e^{1.382t}$ B. $2200e^{1.0382t}$ C. $2200(e \cdot 1.382)^t$ D. $2200e^{0.382t}$ E. None of these

51. Euphemia is solving the equation $(x+1)(x+2)=6$
She completes the problem as shown in the steps below.

Original Equation:	$(x+1)(x+2)=6$	
Step 1	$x+1=2$ and $x+2=3$	Since $2\cdot 3=6$, set the factors equal to 2 and 3, respectively.
Step 2	$x=1$	Solve each linear factor.

First determine if $x=1$ is the complete solution to the original equation $(x+1)(x+2)=6$
Then determine if there was an error made in the solution process.

A. Unfortunately, $x=1$ **is not** the complete solution to $(x+1)(x+2)=6$.
Her mistake is in **Step 1**.

B. Unfortunately, $x=1$ **is not** the complete solution to $(x+1)(x+2)=6$.
Her mistake is in **Step 2**.

C. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
She did not make any mistakes in the solution process.

D. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
However, her mistake is in **Step 1**.

E. $x=1$ **is** the complete solution to $(x+1)(x+2)=6$.
However, her mistake is in **Step 2**.

52. Plutarch is solving the equation $(x+2)(x-2)=4$
He completes the problem as shown in the steps below.

Original Equation:	$(x+2)(x-2)=4$	
Step 1	$x+2=2$ and $x-2=2$	Since $2\cdot 2=4$, set the factors equal to 2 and 2, respectively.
Step 2	$x=0$ and $x=4$	Solve each linear factor.

First determine if $x=0$ and $x=4$ is the complete solution to the original equation $(x+2)(x-2)=4$.
Then determine if there was an error made in the solution process.

A. Unfortunately, $x=0$ and $x=4$ **is not** the complete solution to $(x+2)(x-2)=4$.
His mistake is in **Step 1**.

B. Unfortunately, $x=0$ and $x=4$ **is not** the complete solution to $(x+2)(x-2)=4$.
His mistake is in **Step 2**.

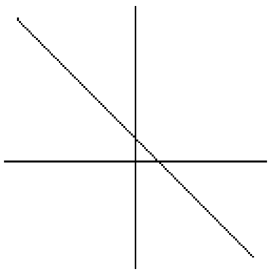
C. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
He did not make any mistakes in the solution process.

D. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
However, his mistake is in **Step 1**.

E. $x=0$ and $x=4$ **is** the complete solution to $(x+2)(x-2)=4$.
However, his mistake is in **Step 2**.

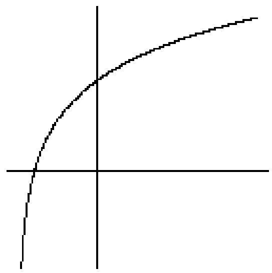
53. For each of the graphs below, select the formula beneath the graph which best fits the behavior of the graph. In each case, assume that A, B, and C are positive real numbers. (Circle your choice.)
Scales are not shown on the axes, so the graphs may not have a true geometric perspective.

(I)



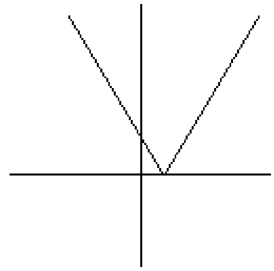
(a) $y = Ax + B$
(b) $y = -Ax - B$
(c) $y = B - Ax$
(d) $y = \dfrac{x+A}{x+A}$

(II)



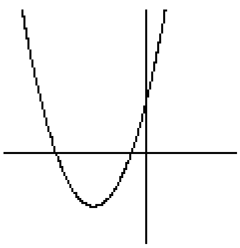
(a) $y = e^{-Ax}$
(b) $y = \log(x - A)$
(c) $y = \log(x + A)$
(d) $y = A^{(x+B)}$

(III)



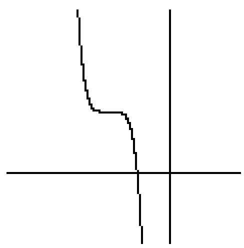
(a) $y = |x - A|$
(b) $y = |x + A|$
(c) $y = |x| - A$
(d) $y = |x| + A$

(IV)



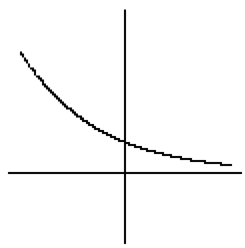
(a) $y = Ax^2 - B$
(b) $y = C - A(x+B)^2$
(c) $y = A(x+B)^2 - C$
(d) $y = A(x-B)^2 - C$

(V)



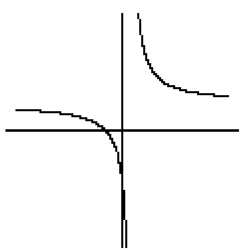
(a) $y = -Ax^5 + B$
(b) $y = Ax^3 + B$
(c) $y = -A(x+B)^5 + C$
(d) $y = A(x+B)^5 + C$

(VI)



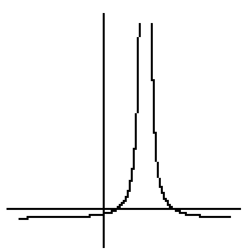
(a) $y = -\ln(x + A)$
(b) $y = -(1/A)^x$
(c) $y = -A^x$
(d) $y = (1/A)^x$

(VII)



(a) $y = \dfrac{A(x-B)}{x+C}$
(b) $y = -\dfrac{A(x-B)}{x+C}$
(c) $y = \dfrac{A(x+B)}{x-C}$
(d) $y = \dfrac{-A(x+B)}{x-C}$

(VIII)

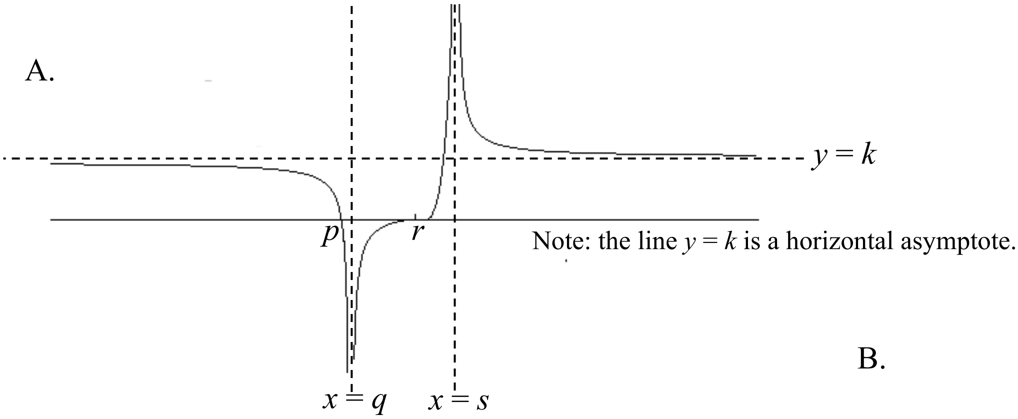


(a) $y = \dfrac{A}{(x-B)^2} - C$
(b) $y = \dfrac{A}{(x+B^2)} - C$
(c) $y = \dfrac{A}{(x-B)} - C$
(d) $y = \dfrac{-A}{(x-B)} - C$

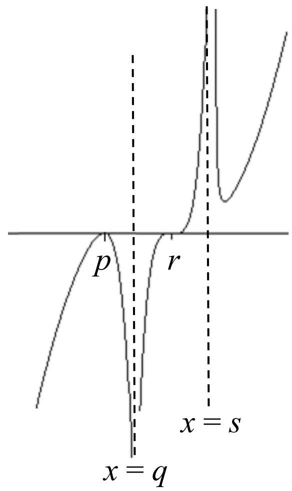
Questions 54-55:

54. Which looks most like the graph of $y = \dfrac{k(x-p)^2(x-r)^5}{(x-q)^2(x-s)^2}$, if k, p, q, r, and s are all positive numbers?

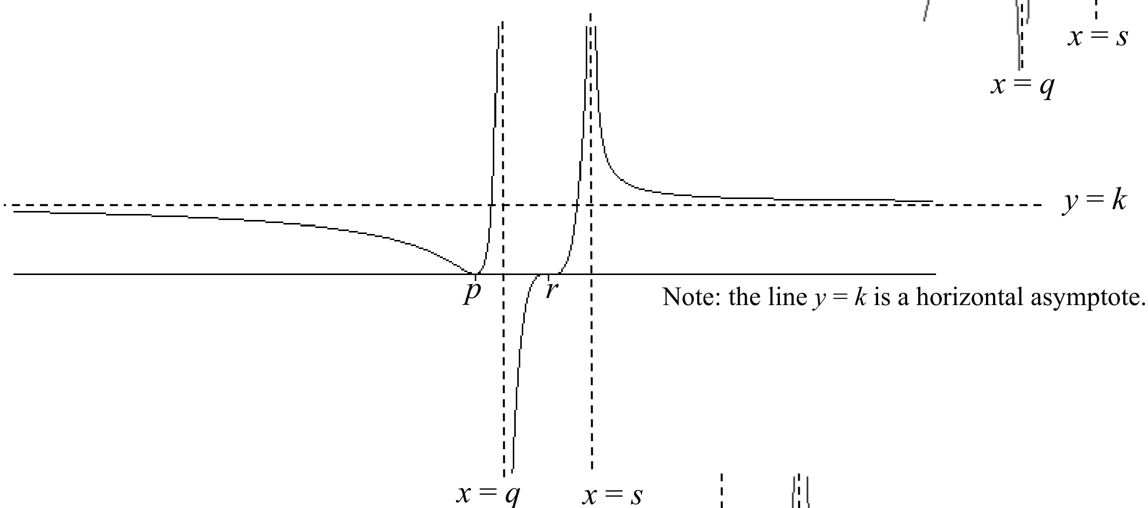
A.



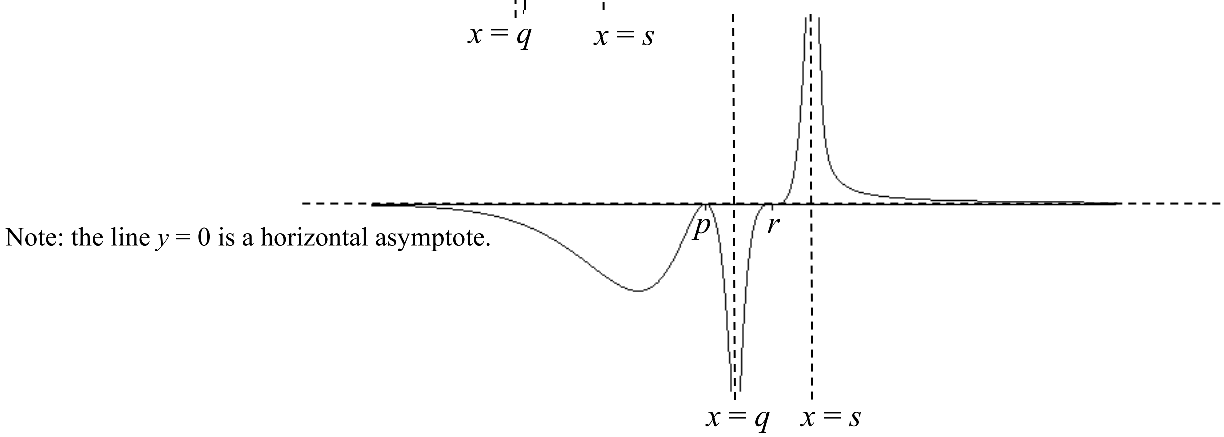
B.



C.



D.



55. For each of the graphs not selected in the previous problem, write all possible formulas such that the power on each factor is as small as possible. Your rational function formulas would involve k, p, q, r, and s.

56. Given $f(x)=\dfrac{4}{x^2}$ and $g(x)=\sqrt{x^2+4}$, find $f(g(x))$ and simplify.

A. $f(g(x))=\dfrac{4}{x^2+4}$ B. $f(g(x))=\dfrac{4}{\sqrt{x^2+4}}$ C. $f(g(x))=x^2+4$ D. $f(g(x))=\dfrac{4}{x^2\sqrt{x^2+4}}$ E. $f(g(x))=\dfrac{1}{x^2}$

57. Given $f(x)=\dfrac{\sqrt{x+1}}{2}$ and $g(x)=x^2+3$, find $f(g(x))$ and simplify.

A. $f(g(x))=\dfrac{x+2}{2}$ B. $f(g(x))=\dfrac{\sqrt{x^2+4}}{2}$ C. $f(g(x))=x$ D. $f(g(x))=x+1$ E. $f(g(x))=\dfrac{(x^2+3)\sqrt{x+1}}{2}$

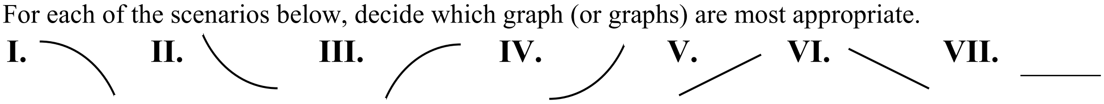
Questions 58-59:

A small amusement park charges $11.00 for admission. An average of only 800 people visit the park each day. Consultants predict that for each $1.00 increase in the entrance price, the park would lose an average of 50 daily customers.

58. Construct a table of values which shows the entrance price, p, and number of tickets sold, N.
Use the table to create a formula $N=f(p)$. Use the formula or your table to find the axes intercepts.
According to the model, which of the following are predicted?
A. A $27 ticket price would result in no customers.
B. If the park had free admission, they would have as many as 1,350 daily customers.
C. If the ticket price were $3.50, they would have 1175 daily customers.
D. Only 125 customers would be willing to pay a $24.50 admission price.
E. All of the above.

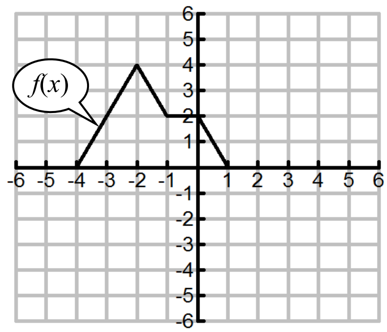
59. Add a third column to your table in Question **58** which gives the daily revenue, R, for each entrance price p.
The *revenue* is the total amount received by the park before any costs are deducted, which is $R=N\cdot p$.
For example, if the price $p=\$11$, then $N=800$ tickets are sold and the revenue $R=800\cdot 11=\$8800$.
Let $R=g(p)$. Find a formula for this function. According to the model, which is true?
A. The higher they set the ticket price, the more revenue they will make.
B. A ticket price of $27 gives them the most revenue.
C. If the ticket price were $13, they would have the highest revenue of $9100.
D. If the ticket price were $14, they would have the highest revenue of $9100.
E. None of these.

60. For each of the scenarios below, decide which graph (or graphs) are most appropriate.

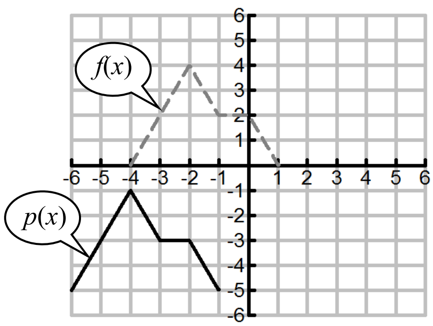


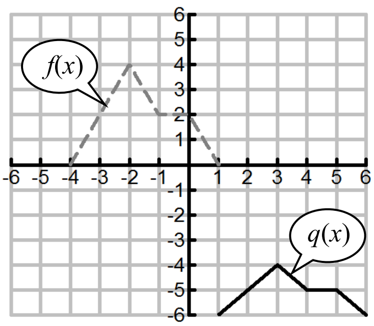
(a) "Even though the child's temperature is still rising, the penicillin seems to be taking effect."
(b) "Your distance from the Atlantic Ocean in kilometers, increases at a constant rate. "
(c) "At first your balance grows slowly, but its rate of growth continues to increase."
(d) "The annual profit is decreasing. Each year it falls more steeply than the previous year."
(e) "The function has a positive rate of change and the rate of change is decreasing."
(f) "The population of rhinos isn't decreasing as quickly as it used to be."
(g) The function is concave down.
(h) The function is decreasing.
(i) The function is constant.
(j) The average rate of change of the function is constant.

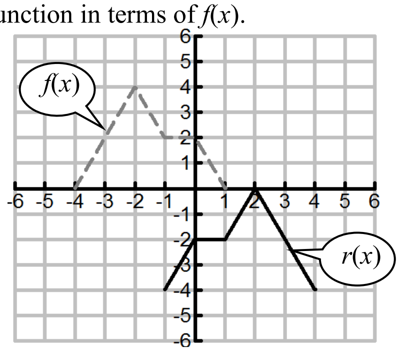
61-63. The graph of $y = f(x)$ is shown.



The functions shown below are transformations of $f(x)$. Write a rule for each function in terms of $f(x)$.







61. Select the choice for $p(x)$.
- A. $p(x) = f(x+2) - 5$
- B. $p(x) = f(x-2) - 5$
- C. $p(x) = f(x+2) + 5$
- D. $p(x) = f(x-2) + 5$
- E. None of these

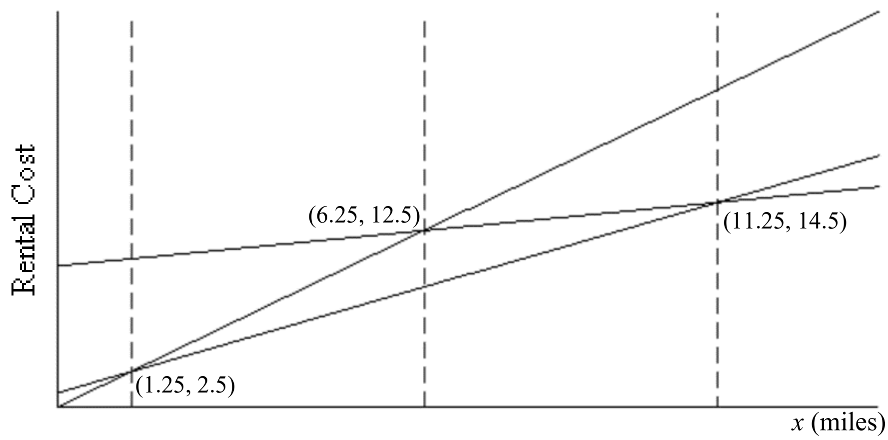
62. Select the choice for $q(x)$.
- A. $q(x) = f(x+5) - 6$
- B. $q(x) = f(x-5) - 6$
- C. $q(x) = f(x+5) + 6$
- D. $q(x) = 0.5f(x+5) - 6$
- E. $q(x) = 0.5f(x-5) - 6$

63. Select the choice for $r(x)$.
- A. $r(x) = -f(x+3) - 4$
- B. $r(x) = -f(x-3) - 4$
- C. $r(x) = f(-x) - 4$
- D. $r(x) = -f(x) - 4$
- E. $r(x) = f(-x-3) - 4$

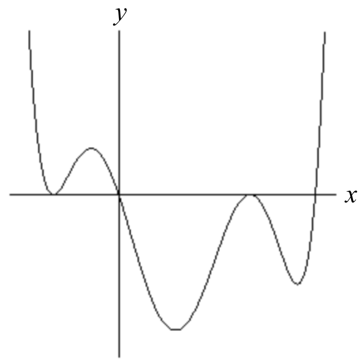
64. To rent an electronic scooter you have three available options.
- To rent **Scooter A** you will be charged \$2 per mile.
- To rent **Scooter B** you will be charged \$10 plus \$0.40 per mile.
- To rent **Scooter C** you will be charged only \$1 plus \$1.20 per mile.

The graphs for the three scooter's rental costs are shown below.
Carefully label each graph with the correct scooter. Then answer the following question:

If x represents the number of miles driven, for which interval is **Scooter C** the cheapest?
- A. $2.5 < x < 14.5$
- B. $2.5 < x < 11.25$
- C. $6.25 < x < 11.25$
- D. $0 < x < 6.25$
- E. None of these



65. The graph of a polynomial $f(x)$ is to the right.
Assume all global behavior is shown.
What is the least possible value for the **degree** of $f(x)$?



A. The least possible value for the degree of the polynomial is 3.
B. The least possible value for the degree of the polynomial is 4.
C. The least possible value for the degree of the polynomial is 5.
D. The least possible value for the degree of the polynomial is 6.
E. The least possible value for the degree of the polynomial is 7.

Questions 66-69 deal with the function $y = \dfrac{8x^2 - 8}{2x^2 - 4x}$.

66. Report all zeros
A. The zeros are 0, 2, and −2.
B. The zeros are 0, 1, and −1.
C. The zeros are 1 and −1.
D. The zeros are 2 and −2.
E. The zeros are 1, −1, and 8.

67. Report where the graph of $y = \dfrac{8x^2 - 8}{2x^2 - 4x}$ crosses the y-axis.
A. The graph crosses at $y = 0$.
B. The graph crosses at $y = 4$.
C. The graph crosses at $y = 1$ and $y = -1$.
D. The graph crosses at $y = 16$.
E. The graph never crosses the y-axis.

68. Report the equations of all vertical asymptotes.
A. The vertical asymptotes are $x = 0$, $x = -1$, $x = 1$, $x = 2$, and $x = -2$.
B. The vertical asymptotes are $x = 0$, $x = 1$, and $x = -1$.
C. The vertical asymptotes are $x = 0$, $x = 2$, and $x = -2$.
D. The vertical asymptotes are $x = 2$ and $x = -2$.
E. None of the above.

69. Report the equation of the horizontal asymptote.
A. The horizontal asymptote is $y = 0$.
B. The horizontal asymptote is $y = 1$.
C. The horizontal asymptote is $y = 2$.
D. The horizontal asymptote is $y = 4$.
E. The horizontal asymptote is $y = 8$.

70. In the year 1900 the population P of a town was 1160 people but it grew by 10 people every year.
In the year 1900 the population Q of a town was 1000. The town grew by 1.13% every year.
Find when the population of Q overtakes the population of P.
Select the response which is closest to the answer.
A. 1.3 years B. 39 years C. 70.9 years D. 116 years E. Q will never overtake P.

71. The path of an artillery shell, in feet, fired from a military base is given by $h(x) = 0.96x - 0.004x^2$.
Report the maximum height of the artillery shell.
A. 250 feet B. 240 feet C. 120 feet D. 57.6 feet E. None of these

72. The doubling time of a population is 12 years. What is its *tripling* time?
A. 4 years B. 15 years C. 19 years D. 36 years E. None of these