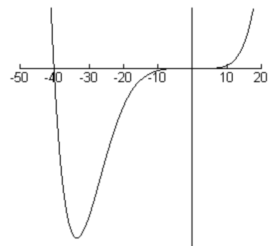
Solutions to Review for the MA 15300 Final



$-50 \le x \le 20$

$-300{,}000{,}000 \le y \le 100{,}000{,}000$

1. For positive or negative large values of x,

$$f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$$

looks like its leading term, the power function $y = x^6$.
We can describe its long run behavior as follows:
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$.
Enlarge the viewing window to see that eventually the graph turns around.
Choice **B**.

2. We have $C(t) = \dfrac{P(t)}{R(t)} = \dfrac{360+9t}{12{,}000+12t}$. Therefore $C(0) = \dfrac{360+9(0)}{12{,}000+12(0)} = \dfrac{360}{12{,}000} = 0.03$ or 3%. Choice **B**.

3. As t gets larger and larger, the function $C(t) = \dfrac{360+9t}{12{,}000+12t}$ approaches the ratio of the leading terms, namely $\dfrac{9t}{12t} = 0.75$. Eventually 75% of the reservoir's total volume would consist of pollutants. This can be confirmed with a graph of the function or a view of its table for large values of t. Choice **E**.

4. $Q = 20(0.4)^t = 20(1 - 0.6)^t$, so 60% of the drug is lost per hour. Choice **E**.

5. The growth factor of $y = ab^t$ is b. Choice **A**.

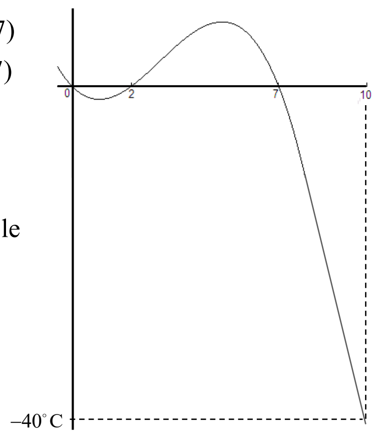
6. There are zeros at 0, 2, and 7. Therefore: $y = kt(t-2)(t-7)$

$$x = 1,\ y = -1) \Rightarrow\ -1 = k(1)(1-2)(1-7)$$
$$-1 = k(-1)(-6)$$
$$k = -\tfrac{1}{6}$$

The minimum value of $P(t)$ in the first ten seconds must be $P(10) = -40^\circ\text{C}$. This can be found using a graph or table or by evaluating $P(t) = -\frac{1}{6}t(t-2)(t-7)$ for $t = 10$.

$P(10) = -\frac{1}{6}(10)(10-2)(10-7) = -\frac{1}{6}(10)(8)(3) = -40$.

Choice **D**.

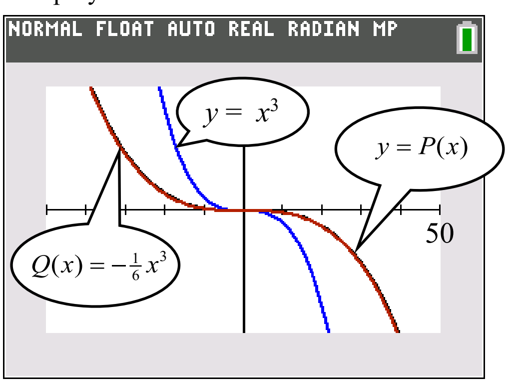


7. $Q(t) = -\frac{1}{6}t^3$ since+ *remaining terms of lower degree*

Therefore, $P(t)$ and $Q(t)$ look nearly indistinguishable for large values of t.

Note that the $-\frac{1}{6}$ is not optional. You can check this with a grapher in a large window.

This [lecture video] demonstrates why the leading term and the polynomial look so much alike (although a different polynomial is used. For those who still doubt, below is a graph of $Q(x) = -\frac{1}{6}x^3$, $P(x)$, and $y = x^3$.



$-500 \le x \le 500$ by $-10{,}000{,}000 \le y \le 10{,}000{,}000$

Such a graph is nice to confirm, but not necessary to solve this problem if you already know the concept that the polynomial has the same end behavior as its leading term (which is a power function).

Choice **C**.

8. $E(t)=30t^{0.668}$. To find $y = kt^{p}$, notice $E(1)=30$ so if $t=1$, then $y=30$.

Therefore we have $k=30$, since $30=k(1)^{p}=k(1)=k$.

Now use another point to find p for $y=30t^{p}$. We used (2.02, 48).

$$48=30(2.02)^{p}$$

$$\frac{48}{30}=(2.02)^{p}$$

$$1.6=(2.02)^{p} \qquad \text{So } p=\frac{\ln 1.6}{\ln 2.02}\approx 0.668.$$

This means $E(t)=30x^{0.67}$ and $E(7)=30(7)^{0.67}\approx 110$. Choice **B**.

9. $S(t)=5.61x^{1.37}$. To find $y = kt^{p}$, use two points. We used (2.05, 15) and (2.98, 25).

$$\frac{25}{15}=\frac{k(2.98)^{p}}{k(2.05)^{p}}$$

$$\frac{5}{3}=\left(\tfrac{2.98}{2.05}\right)^{p}$$

$$p=\frac{\ln(5/3)}{\ln(2.98/2.05)}\approx 1.3655$$

$y=kt^{1.366}$ Now use any other point to find k. We used (1.05, 6)

$$6=k(1.05)^{1.366}$$

$$k\approx 5.61$$

$S(t)=5.61x^{1.37}$ Choice **E**.

10. $E(t)=30x^{0.67}$

$$S(t)=5.61x^{1.37}$$

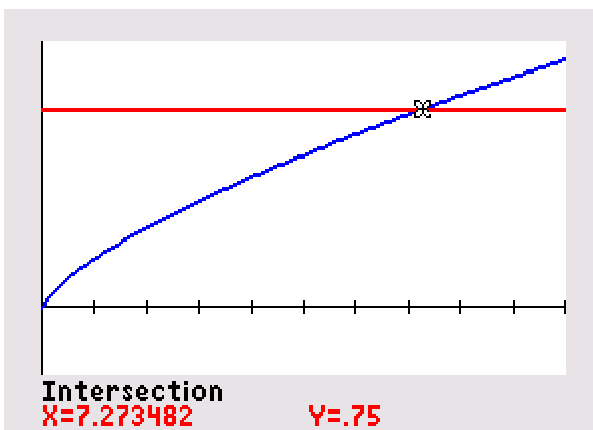
$$\frac{S}{E}=\frac{5.61x^{1.37}}{30x^{0.67}}=0.187x^{0.7}$$

Solve $0.187x^{0.7}>0.75$ by graphing $y = 0.187x^{0.7}$ and the target line $y=0.75$

Perform an INTERSECTION routine or solve $0.187x^{0.7}=0.75$ to find the first time after which the ratio $\frac{S}{E}$ is above 0.75. This is about 7.3 months. Choice **D**.

```
Y1=.187X^.7
Y2=.75
```

Window: $0\le x\le 10$, $-0.25\le y\le 1$



Note: You could also just enter

```
Y1=5.61X^1.37/(30X^.67)
Y2=.75
```

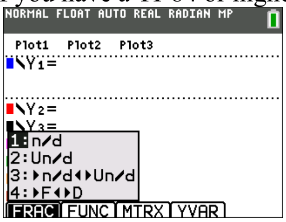
in a grapher, but the parentheses are crucial on a TI-83 or TI-83 Plus.

For example, you would NOT get the same function if you just typed

```
Y1=5.61X^1.37/30X^.67
Y2=.75
```

That would give you $y = \dfrac{5.61x^{1.37}}{30} \cdot x^{0.67}$ which is not what you want at all.

If you have a TI-84 or higher, use the fraction template **n/d** by pressing ALPHA Y=.



$$Y_1 = \frac{5.61X^{1.37}}{30X^{.67}}$$
$$Y_2 = .75$$

11. 30 lb of fertilizer produces a maximum yield of 400 pecks of peppers. Choice **B**.

12. Without applying any fertilizer at all, we see from the graph that the orchard will produce 175 pecks of peppers. Choice **C**.

13. The range is $0 \le f(m) \le 400$. Note: You can also write [0, 400]. Choice **E**.

14. The function $f(m)$ is **increasing** for $0 < m < 30$. Choice **C**.
Note: The function $f(m)$ is **decreasing** for $30 < m < 70$.

15. The function $f(m)$ is never **concave up** and is **concave down** for $0 < m < 70$. Choice **E**.

16. $f(m) > 175$ for $0 < m < 60$.
Determine where the graph of $y = f(m)$ is above the line $y = 175$.
The yield is more than 175 pecks of peppers when the amount of fertilizer applied is more than 0 lb and less than 60 lb. Choice **D**.

17. We first find a formula for $f(m)$. We can use vertex form or factored form.
Both are shown below but only one form is needed.

Vertex Form:
To find the vertex form, use a shift transformation of the graph of $y = ax^2$ (left 30 and up 400).
We have $y = a(x - 30)^2 + 400$. Plug in the point (0, 175).

$$\begin{aligned} y &= a(x-30)^2 + 400 \\ 175 &= a(0-30)^2 + 400 \\ -225 &= 900a \\ a &= -0.25 \end{aligned}$$

In vertex form, $f(x) = -0.25(x - 30)^2 + 400$.

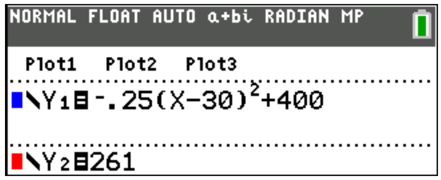
Factored Form:
The function has a positive zero of 70, which is $70 - 30 = 40$ units from the axis of symmetry.
The other zero is also 40 units from the axis of symmetry or at $30 - 40 = -10$.
The factored form is $y = a(x - 70)(x + 10)$, but from Question 20b, $a = -0.25$.
In factored form, $f(x) = -0.25(x - 70)(x + 10)$.

If we had not solved for a already,
you can also find a by plugging in the point (0, 175).

$$y = a(x-70)(x-10)$$
$$175 = a(0-70)(0-10)$$
$$175 = -700a$$
$$a = \frac{175}{-700} = -0.25$$

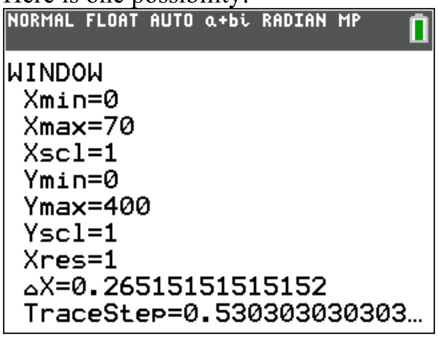
However, the leading coefficient a is the same for expanded form, vertex form, and factored form, so if you already have one of these formulas, you have a.

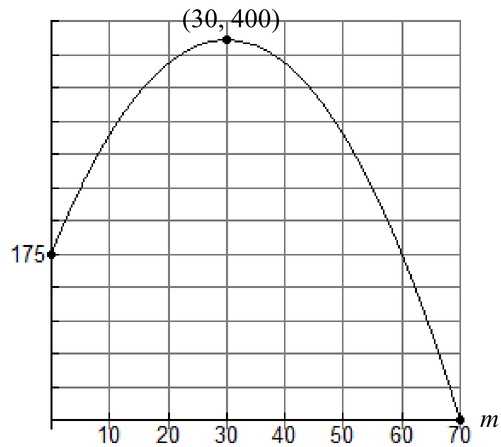
We now use the formula to approximate the solutions to $f(m) = 261$. Enter both the function in your grapher and the line $y = 261$.



The question provided a graph, so we can use that to set a viewing window.

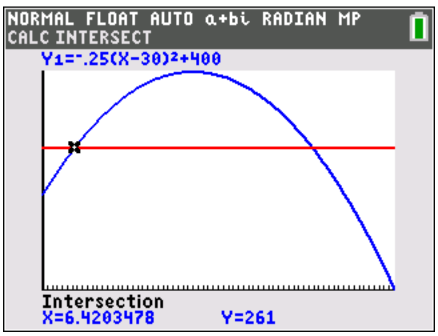
Here is one possibility:

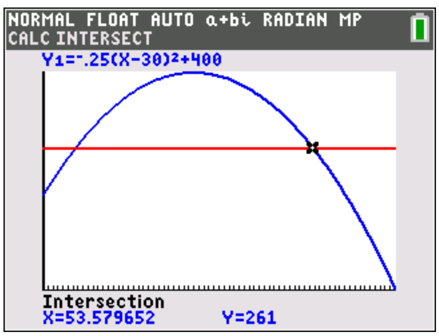


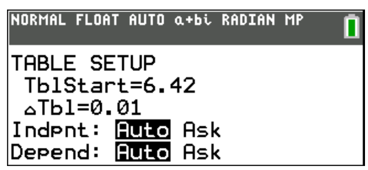


Check your graph passes through the given points (0, 175), (30, 400), and (70,0) before continuing.

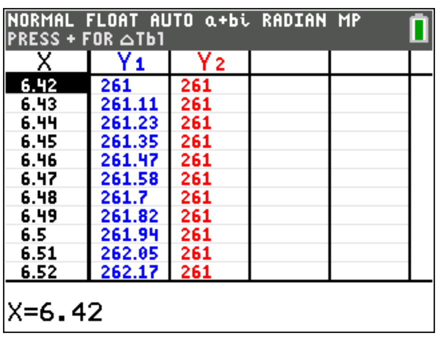
You can use the Intersection Feature of the grapher to find that $f(m) = 261$ when $m \approx 6.42, 53.58$.





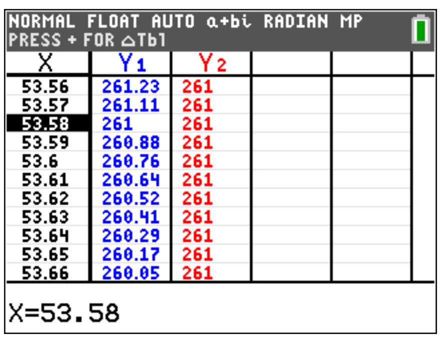


You can confirm with the Table Feature. Set TblStart = 6.42 and ΔTbl = 0.01


NORMAL FLOAT AUTO a+bi RADIAN MP
PRESS + FOR ΔTbl

X	Y1	Y2
6.42	261	261
6.43	261.11	261
6.44	261.23	261
6.45	261.35	261
6.46	261.47	261
6.47	261.58	261
6.48	261.7	261
6.49	261.82	261
6.5	261.94	261
6.51	262.05	261
6.52	262.17	261

X=6.42


NORMAL FLOAT AUTO a+bi RADIAN MP
PRESS + FOR ΔTbl

X	Y1	Y2
53.56	261.23	261
53.57	261.11	261
53.58	261	261
53.59	260.88	261
53.6	260.76	261
53.61	260.64	261
53.62	260.52	261
53.63	260.41	261
53.64	260.29	261
53.65	260.17	261
53.66	260.05	261

X=53.58

Thus we have **a.** Choice **C** and **b.** Choice **B**.

18. The equation is $P = 9216(1.125)^t$.
The initial amount when $t = 0$ is \$9,216. Choice **C**.

$$\frac{ab^{18}}{ab^3} = \frac{76787.03}{13122}$$

$$\frac{\cancel{a}b^{18}}{\cancel{a}b^3} = \frac{76787.03}{13122}$$

$$b^{15} = \frac{76787.03}{13122}$$

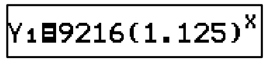
$$b = \sqrt[15]{\frac{76787.03}{13122}} = \left(\frac{76787.03}{13122}\right)^{\frac{1}{15}} = 1.125$$

$$y = a(1.125)^t$$

$$13122 = a(1.125)^3$$

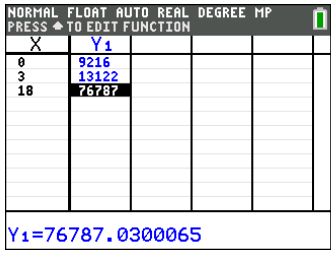
$$a = \frac{13122}{(1.125)^3} = 9216$$

$$P = 9216(1.125)^t$$



Press 2nd TBLSET and use the **Indpnt: Auto** feature to be able to enter nonconsecutive inputs.

The table confirms the equation is true. (Slight roundoff error is of no concern.)



19. Since the equation is $P = 9216(1.125)^t = 9216(1 + \underline{\mathbf{0.125}})^t$, the growth rate is 12.5%. Choice **C**.

20. $e^{x\ln a} = e^{\ln a^x} = a^x$. Choice **E**.

21. The average rate of change is $\frac{\Delta V}{\Delta t}$.

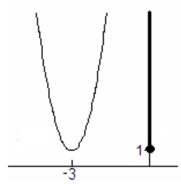
Δt	Time, t (min)	Volume, V (gal)	ΔV
	30	1075	
30 min	60	1150	75 gal
30 min	90	1225	75 gal
30 min	120	1300	75 gal

Find the change in time, Δt, and the change in volume, ΔV, over the intervals.

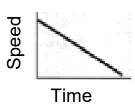
Then create ratios. The average rate of change will be the rate at which the water fills the pool.

$\frac{\Delta V}{\Delta t} = \frac{75\,\text{gal}}{30\,\text{min}} = 2.5$ gallons per minute. The answer is Choice **E**.

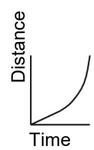
22. The range of the function $y = 5x^2$ is all real numbers greater than equal to 0.
The function shown is a translation of $y = 5x^2$ up 1, so the range is $[1, \infty)$.
Notice on the graph to the right, values of y begin at $y = 1$ and increase forever. Choice **B**.



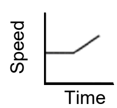
23. Choice II. The train's speed slows to a stop (speed is 0).



24. Choice I. My rate is constant at first, so the graph appears linear.
Once the chimes ring, my rate increases so the graph is concave up.



25. Choice III. First the rhino's speed is constant, or flat. The graph appears horizontal.
When the rhino runs, her speed increases. The slope represents speed.



26. Choice II. The ferris wheel car climbs to its highest point, then descends, then climbs again.
The radius of the wheel is fixed, so once you have boarded,
the high points are all the same and the low points are all the same.

27. The polynomial has formula $y = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$

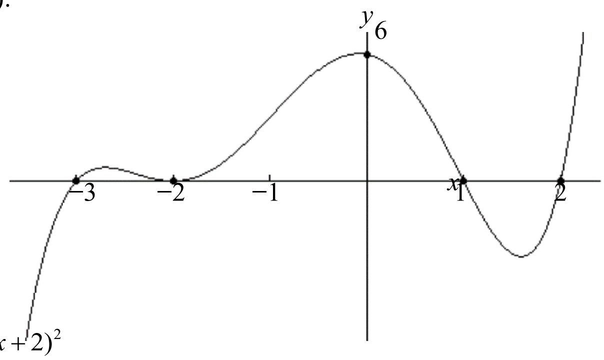
Because the function has single zeros at –3, 1, and 2 and a double zero at –2 we can write $y = a(x-2)(x-1)(x+3)(x+2)^2$ Now substitute the point (0,6):

$$\left.\begin{array}{l} x=0 \\ y=6 \end{array}\right\} y = a(x-2)(x-1)(x+3)(x+2)^2$$

$$6 = a(-2)(-1)(3)(2)^2$$

$$6 = 24a$$

$$a = \frac{6}{24} = \frac{1}{4}$$



Therefore, the polynomial is $f(x) = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$

To find $f(3)$, let $x = 3$: $f(3) = \frac{1}{4}(3-2)(3-1)(3+3)(3+2)^2 = \frac{1}{4}(1)(2)(6)(5)^2 = 75$

You could also use the table feature of a graphing calculator. Choice **B**.

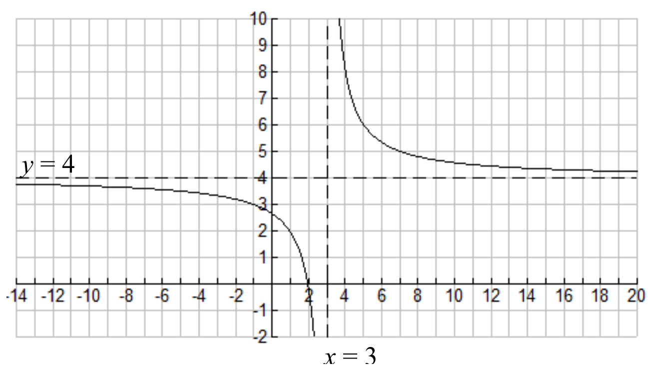
Important:
You should check with a graphing calculator to be sure that the function is correct.

X	Y1
-4	-30
-3	0
-2	0
-1	3
0	6
1	0
2	0
3	75
4	378
5	1176
6	2880

This should match the given information provided.

28. The rational function has the formula $y = \dfrac{4(x-2)}{(x-3)}$

Because the zeros of the function is 2, we have $(x-2)$ as a factor of the numerator since the function is 0 when the numerator is 0.



Since the vertical asymptote is $x = 3$, we have $(x-3)$ as a factor of the denominator. (The vertical asymptotes are found where the denominator is 0 and the numerator is not).

So we can write $y = \dfrac{a(x-2)}{(x-3)}$.

Since the horizontal asymptote is $y = 4$ and it is found by the ratio of the leading terms, we must have $a = 4$.

Therefore the function must be $f(x) = \dfrac{4(x-2)}{(x-3)}$. Use a table feature to find Choice **C** is correct.

Alternatively, use the formula: $f(403) = \dfrac{4(403-2)}{(403-3)} = \dfrac{4 \cdot 401}{400} = \dfrac{401}{100} = 4.01$

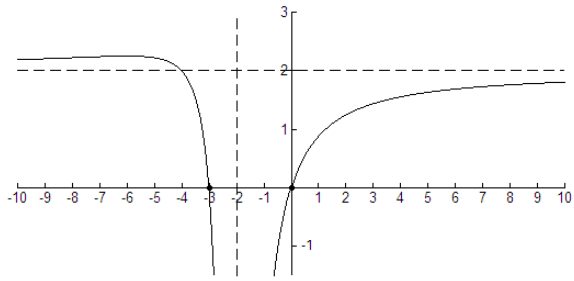
Y1=4(X-2)/(X-3)

NORMAL FLOAT AUTO REAL DEGREE MP
TABLE SETUP
TblStart=403
ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask

X	Y1
403	4.01
404	4.01
405	4.01
406	4.0099

$f(403) = 4.01$

29. The rational function has the formula $y = \dfrac{2x(x+3)}{(x+2)^2}$.



Because the zeros of the function are 0 and –3, the factors of the numerator are $x(x+3)$, since the function is 0 when the numerator is 0.

There is one vertical asymptote at $x = -2$, so $(x + 2)$ is a factor of the denominator. However, the short run behavior near this asymptote looks like $y = k/x^2$ () so the factor must have a power of 2.

We can write $y = \dfrac{ax(x+3)}{(x+2)^2}$. Since the horizontal asymptote is $y = 2$, we must have $a = 2$.

Note: $y = \dfrac{ax(x+3)}{(x+2)^2} \approx \dfrac{ax^2}{x^2} = a$ as $x \to \pm\infty$ so $a = 2$.

Therefore, the rational function has the formula $y = \dfrac{2x(x+3)}{(x+2)^2}$.

Use a table to confirm:

X	Y1
-6	2.25
-5	2.2222
-4	2
-3	0
-2	ERROR
-1	-4
0	0
1	.88889
2	1.25
3	1.44
4	1.5556

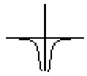
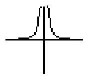
This should match the given information provided.

Use a table to find determine if $f(-1) = -4$, $f(1) = 1$, and $f(-6) = 2.25$:

X	Y1	
-6	2.25	$f(-6) = 2.25$
-5	2.2222	
-4	2	
-3	0	
-2	ERROR	
-1	-4	$f(-1) = -4$
0	0	
1	.88889	$f(1) \neq 1$
2	1.25	
3	1.44	
4	1.5556	

Since only Choices A and C are true, Choice **D** is correct.

30. The equation is $y = \dfrac{8(x-4)}{(x-2)^2}$

Because there is a horizontal asymptote of $y = 0$, the degree of the numerator is less than the degree of the denominator. The numerator has a factor of $(x-4)^1$ since it has a single zero. Because the short run behavior near the vertical asymptote looks like  or 
the lowest degree possible for the denominator must be 2.

So it has a factor of $(x-2)^2$. It has the form $y = \dfrac{a(x-4)}{(x-2)^2}$,
and we can find a if we use the fact that when $x = 0$, $y = -8$:

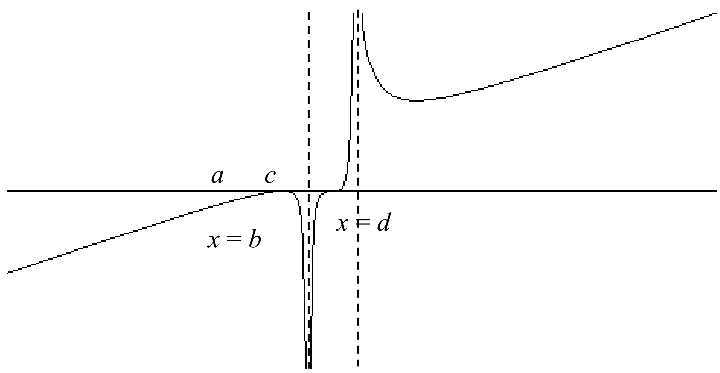
$$-8 = \frac{a(0-4)}{(0-2)^2}$$
$$-8 = \tfrac{-4}{4}a$$
$$a = 8$$

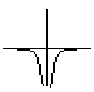
So $f(x) = \dfrac{8(x-4)}{(x-2)^2}$. To find $f(3)$, we let $x = 3$ and find y.

$$f(3) = \frac{8(3-4)}{(3-2)^2} = \frac{8(-1)}{1} = -8$$

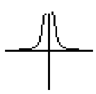
Alternatively, you can enter the formula in a grapher and use a table. Choice **B.**

31. The degree of the factor $(x-a)$ must be even since there is a bounce at the zero.



The degree of the factor $(x-b)$ must be even since the vertical asymptote appears as
 near b.

The degree of the factor $(x-c)$ must be 3, 5, … since there is a chair at the zero.

The degree of the factor $(x-d)$ must be even since the vertical asymptote appears as  near d.

The long run behavior is the same as the power function $y = kx$, so the degree of the numerator must be one more than the degree of the denominator. Therefore, it must be Choice **B**.

32. $$\log_b\left(\frac{x^3y^2}{\sqrt{w}}\right) = \log_b x^3 + \log_b y^2 - \log_b \sqrt{w}$$
$$= \log_b x^3 + \log_b y^2 - \log_b w^{1/2}$$
$$= 3\log_b x + 2\log_b y - \tfrac{1}{2}\log_b w$$

The correct answer is Choice **C**.

33. $25^x = 3^{600}$

$\ln 25^x = \ln 3^{600}$

$x \ln 25 = 600 \ln 3$

$x = \dfrac{600 \ln 3}{\ln 25} \approx 204.78$

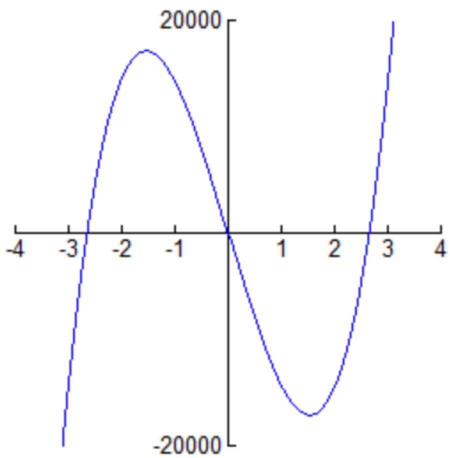
The correct answer is Choice **C**.

34. The zeros of $f(x) = 400x(6x^2 - 42)$ are 0, $\sqrt{7}$, and $-\sqrt{7}$
Check graphically.
This third degree polynomial crosses the x-axis three times.
Find the zeros by solving $f(x) = 0$. Factor.
Set each factor equal to 0 and solve.

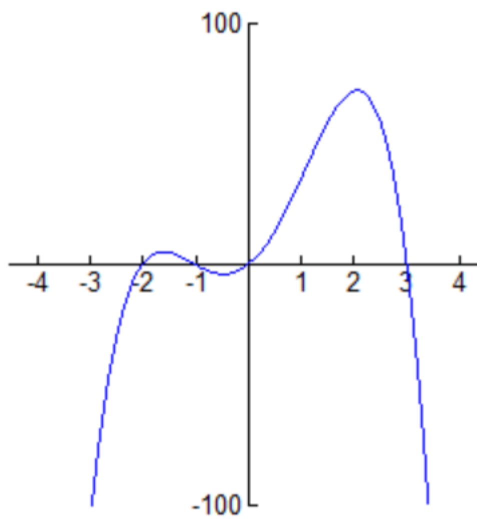
$$400x(6x^2 - 42) = 0$$

$$\begin{array}{r|l} 400x = 0 & 6x^2 - 42 = 0 \\ x = 0 & 6x^2 = 42 \\ & x^2 = 7 \\ & x = \pm\sqrt{7} \end{array}$$

Choice **D**.



35. The zeros of $f(x) = -3(x^4 - 7x^2 - 6x)$.
This fourth degree polynomial crosses the x-axis four times.
We can factor out x from each term of $x^4 - 7x^2 - 6x$:
$f(x) = -3x(x^3 - 7x - 6)$
However, try as you might $x^3 - 7x - 6$ cannot be factored.
The only way to find the zeros is with a graph or a table.
The zeros are −2, −1, 0, and 3. Choice **C**.



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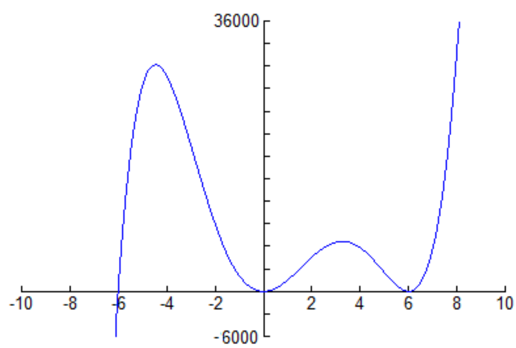
X	Y1
-2	0
-1	0
0	0
1	36
2	72
3	0
4	-360
5	-1260
6	-3024
7	-6048
8	-10800

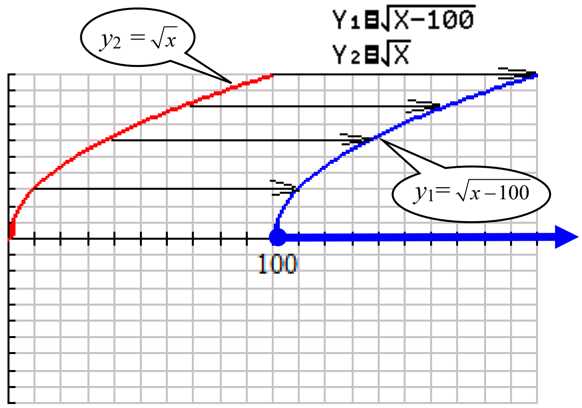
X= -2

36. Sketch a graph of the polynomial $f(x) = 9x^2(x + 6)(x - 6)^2$
by hand (or use a grapher, but it's difficult to find a window).
Determine the values of x for which f is above or on the x-axis,
which is $x \geq -6$.

The correct answer is Choice **C**.



37. We can find the domain of $f(x) = \sqrt{x-100}$ using the graph, the table or reason from the formula.
The graph of $f(x) = \sqrt{x-100}$ is a horizontal shift of the graph of the power function $y = \sqrt{x}$ right 100 units.
The domain is $x \geq 100$.
You can also write the domain $[100, \infty)$.



Use a table to confirm that 100 is included in the domain, as well as all reals larger than 100.
Values less than 100 cause the calculator to bail.

X	Y1
95	ERROR
96	ERROR
97	ERROR
98	ERROR
99	ERROR
100	0
101	1
102	1.4142
103	1.7321
104	2
105	2.2361

The formula $f(x) = \sqrt{x-100}$ tells you that $f(x)$ is defined if the radicand $x - 100 \geq 0$.
When you solve this inequality, you have $x \geq 100$.

The viewing window is $0 \leq x \leq 200$ by $-10 \leq y \leq 10$, with Xscl = 10 and Yscl = 1.

Choice **B**.

38. Solve $4{,}000e^{0.073t} = 12{,}000$

$4000e^{0.073t} = 12{,}000$ Divide both sides by 4000 to get $e^{0.073t}$ all by itself.

$e^{0.073t} = 3$ Take natural logarithms of both sides.

$\ln e^{0.073t} = \ln 3$ Use the inverse property: $\ln e^{0.073t} = 0.073t$.

$0.073t = \ln 3$ Divide both sides by 0.073 to solve for t.

$$t = \frac{\ln 3}{0.073} \approx 15.05$$

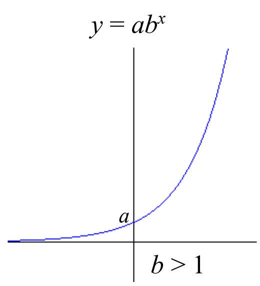
Choice **D**. TIP: Check by resubstituting:

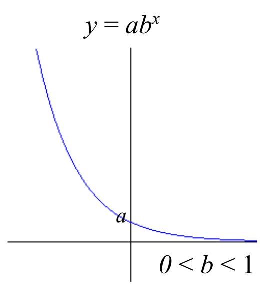
```
NORMAL FLOAT AUTO REAL RADIAN MP
ln(3)/.073
                    15.04948341
4000e^(.073Ans)
                          12000
```

39. $\ln\left(\frac{1}{\sqrt{e^x}}\right) = \ln\left(\frac{1}{e^{x/2}}\right) = \ln\left(e^{-x/2}\right) = -\frac{x}{2}$ Choice **C**.

40. In general, the graph of $y = f(x) = ab^x$ increases for $b > 1$ and decreases for $0 < b < 1$ and has y-intercept $(0, a)$.

✓	I.	It increases if $b > 1$
	II.	It decreases if $b < 0$
✓	III.	It has y-intercept (0, 1) if $b > 0$.



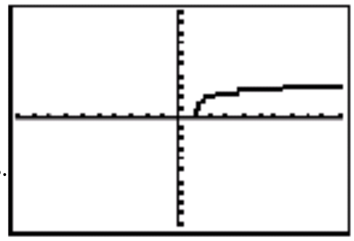


The function $y = b^x$ is a special case, with $a = 1$. Therefore, Items I and III are correct. Choice **D**.

41. The graph of $y = 2 + \log(x - 1)$ is a horizontal shift 1 unit to the right and a vertical shift 2 units up of the graph of $y = \log(x)$.

- Since the graph of $y = \log(x)$ has a vertical asymptote of $x = 0$, the graph of $y = 2 + \log(x - 1)$ has a vertical asymptote of $x = 1$.
- Since the domain of $y = \log(x)$ is the set of all real numbers $x > 0$, the domain of $y = 2 + \log(x - 1)$ is the set of all real numbers $x > 1$. Therefore it does **not** cross the x-axis at 1 and it never touches the y-axis.
- The graph of $y = 2 + \log(x - 1)$ passes through the point (2, 2): check: $x = 2, y = 2 \Rightarrow$

$$\begin{aligned} y &= 2 + \log(x-1) \\ 2 &= 2 + \log(2-1)? \\ 2 &= 2 + \log(1)? \\ 2 &= 2 + 0? \quad \textbf{YES} \end{aligned}$$



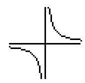
A graph of $y = 2 + \log(x-1)$ produced by technology in a standard window $-10 \le x \le 10$ $-10 \le y \le 10$ can look misleading!

- The range of the function $y = 2 + \log(x - 1)$ is all real numbers.

It is difficult for most technology to produce an accurate graph of a logarithm function. Don't be deceived by a misleading graph. Therefore Items I, III, and IV are correct. Choice **E**.

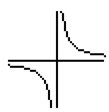
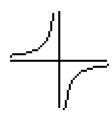
✓	I.	increases for all values of x in its domain.
	~~II.~~	~~crosses the x-axis at 1~~
✓	III.	never touches the y-axis
✓	IV.	passes through the point (2, 2).

42. Since the vertical asymptote is $x = a$, the **denominator** must have $(x - a)$ as a factor. Since the function has a single zero through the origin (0, 0), the **numerator** must be 0 when $x = 0$.

The short run behavior of the function near its vertical asymptote looks like  requiring the factor in the **denominator to be raised to an odd power**.

The equation $y = \dfrac{x}{x-a}$ is the only choice which meets these three criteria. Choice **C**.

43. As $x \to \infty$ or $x \to -\infty$, $f(x) = \dfrac{2ax}{(x-a)^2} \approx \dfrac{2ax}{x^2} = \dfrac{2a}{x}$.

In other words, the graph of $y = \dfrac{2ax}{(x-a)^2}$ and the graph of $y = \dfrac{2a}{x}$ have the same long run behavior. The graph of $y = \dfrac{2a}{x}$ has end behavior which looks like  or 

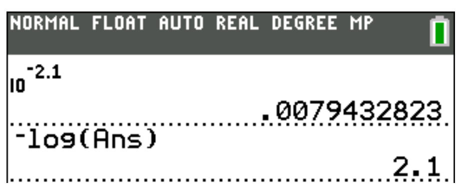
(depending on whether a is positive or negative).
In either case, as $x \to -\infty$ or as $x \to \infty$, the function approaches 0
The horizontal asymptote is $y = 0$. Choice **D**.

44. Since pH = –log C and pH = 2.1, we must solve the logarithmic equation $2.1 = -\log C$.

$-\log C = 2.1$	Multiply both sides by -1
$\log C = -2.1$	Make both sides a power of 10
$10^{\log C} = 10^{-2.1}$	Use an inverse property

$$C = 10^{-2.1} = \frac{1}{10^{2.1}} \approx 0.0008$$

Choice **B**.



45. To solve $\ln 2x^3 = 5$, exponentiate both sides to base e:

$$e^{\ln 2x^3} = e^5 \quad \text{Make both sides a power of } e.$$
$$2x^3 = e^5 \quad \text{Use the inverse property.}$$

The answer is Choice **D**.

46. To solve $\ln 2x^3 = 5$

$2x^3 = e^5$ From Question 57.

$x^3 = \frac{1}{2}e^5$ Divide both sides by 2.

$$x^3 = \frac{e^5}{2}$$

$x = \sqrt[3]{\frac{e^5}{2}}$ Take the cubed root of both sides

You can check by substitution: $\ln 2\left(\sqrt[3]{\frac{e^5}{2}}\right)^3 = \ln 2\left(\frac{e^5}{2}\right) = \ln e^5 = 5$. The answer is Choice **C**.

47. To solve $20 = 3e^x + 5$ first subtract 5 from both sides:

This gives us $15 = 3e^x$. The answer is Choice **D**.

48. To solve $20 = 3e^x + 5$

$3e^x - 15$ From Question 59.

$e^x = 5$ Divide both sides by 5.

$\ln e^x = \ln 5$ Take natural logs of both sides.

$x = \ln 5$ Use the inverse property.

You can check by substitution: $3e^{\ln 5} + 5 = 3 \cdot 5 + 5 = 20$. The answer is Choice **E**.

49. Use $P(1+\frac{r}{n})^{n \cdot t}$ with $P = 2200$, $r = 0.0382$, and $n = 4$. The balance in year t is $2200(1+\frac{0.0382}{4})^{4t}$.

Remember that 3.82 per cent is $\frac{3.82}{100} = 0.0382 = 3.82\%$.

The answer is Choice **C**.

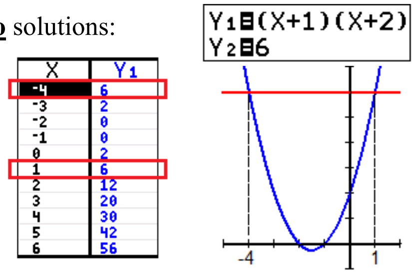
TIP: To divide 3.82 by 100, move the decimal point of 3.82 two places to the left.
For example: 03.82% becomes 0.0382

50. Since you are compounding continuously, use $Pe^{r \cdot t}$ with $P = 2200$, $r = 0.0382$. (See previous question.)
The balance in year t is $2200e^{0.0382t}$. Note: $2200e^{0.382t}$ grows at a continuous rate of $0.382 = 38.2$ %.
Since the balance is none of the choices listed, the answer is Choice **E.**
TIP: To multiply 0.382 by 100, move the decimal point of 0.382 two places to the right.
For example: 0.382 becomes 38.2%.

51. The answer is Choice **A**. Euphemia's answer is incorrect. Her error was in **Step 1**.
It is ***false*** to conclude that $A \cdot B = 6 \Leftrightarrow A = 2$ or $B = 3$.
There are infinitely many ways for a product of two numbers $A \cdot B = 6$.
(For example, $A = \frac{1}{2}, B = 12$ is one possibility. $A = \frac{6}{\sqrt{\pi}}, B = \sqrt{\pi}$ is another.)
Euphemia would have been correct if she had concluded $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$,
which is called the "Zero Factor Property". If Euphemia checks her answer in the formula she **does** get a correct result:

$$x = 1 \quad \Rightarrow (1+1)(1+2) = (2)(3) = 6$$

However, if she uses a grapher to check, she would have seen there are **two** solutions:
The correct solution can be solved analytically by expanding:



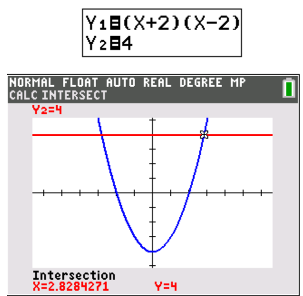
$$(x+1)(x+2) = 6$$
$$x^2 + 3x + 2 = 6$$
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$
$$x + 4 = 0 \quad \Big\| \quad x - 1 = 0$$
$$x = -4 \quad \Big\| \quad x = 1$$

So the complete solution **is not** $x = 0$; the complete solution is $x = 0$ and $x = -4$.

52. The answer is Choice **A**. Plutarch's answer is incorrect. His error was in **Step 1**.
It is ***false*** to conclude that $A \cdot B = 4 \Leftrightarrow A = 2$ or $B = 2$.
There are infinitely many ways for a product of two numbers $A \cdot B = 4$.
(For example, $A = \frac{1}{10}, B = 40$ is one possibility. $A = \sqrt{2}, B = \sqrt{8}$ is another. These are arbitrary wild examples.)
Plutarch would have been correct if he had concluded $A \cdot B = 0 \Leftrightarrow A = 0$ or $B = 0$, which, as described in Question **67**, is called the "Zero Factor Property". If Plutarch checks his answer in the formula he **does not** get a correct result:

$$x = 0 \quad \Rightarrow (0+2)(0-2) = -4 \neq 4$$
$$x = 4 \quad \Rightarrow (4+2)(4-2) = 6 \cdot 2 = 12 \neq 4$$

However, if he uses a grapher to check, he would have seen there are two solutions:
The correct solution can be solved analytically by expanding:



$$(x+2)(x-2) = 4$$
$$x^2 - 4 = 4$$
$$x^2 = 8$$
$$x = \pm\sqrt{8}$$

You could also write the solutions as $x = \pm 2\sqrt{2}$.

Note that $x = \pm\sqrt{8} = \pm 2\sqrt{2}$ is the **exact** solution.
The approximate solution, to a mere seven decimal places, is $x \approx \pm 2.8284271$.
(For the approximate solution to one million decimal places, see **https://apod.nasa.gov/htmltest/gifcity/sqrt8.1mil**)
So the complete solution **is not** $x = 0$ and $x = 4$.

53. I. Choice **C.**
$y = B - Ax$ since it has a positive y-intercept (B) and slope is negative ($-A$).

II. Choice **C**
$y = \log(x + A)$ since it is a shift of $y = \log x$ to the left A units. (Its vertical asymptote is at x = $-A$.)

III. Choice **A**
$y = |x - A|$ since it is a shift of $y = |x|$ to the right A units. (Its minimum is when $x = A$.)

IV. Choice **C**

$y = A(x+B)^2 - C$ since the x-and y-coordinate coordinates of the vertex are negative and the parabola is concave up.

V. Choice **C**

$y = -A(x+B)^5 + C$ since it is a vertical reflection of $y = x^5$ combined with a horizontal shift to the left and a vertical shift up.

VI. Choice **D**

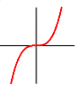
$y = (1/A)^x$ since it is exponential decay.

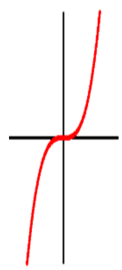
VII. Choice **C**

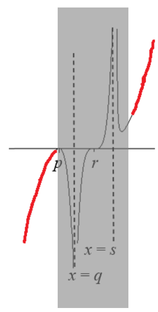
$y = \dfrac{A(x+B)}{x-C}$ since its vertical asymptote is $x = C$ with C positive,
it has a horizontal asymptote $y = A$ with A positive, and a negative zero (at $-B$).

VIII. Choice **A**

$y = \dfrac{A}{(x-B)^2} - C$ since it is a shift of $y = \dfrac{A}{x^2}$ to the right B units and down C units.

54. The graph of the function $y = \dfrac{k(x-p)^2(x-r)^5}{(x-q)^2(x-s)^2}$ looks very much like the graph of $y = \dfrac{kx^7}{x^4} = kx^3$
For very large x, which resembles a chair shape  (since k is positive).

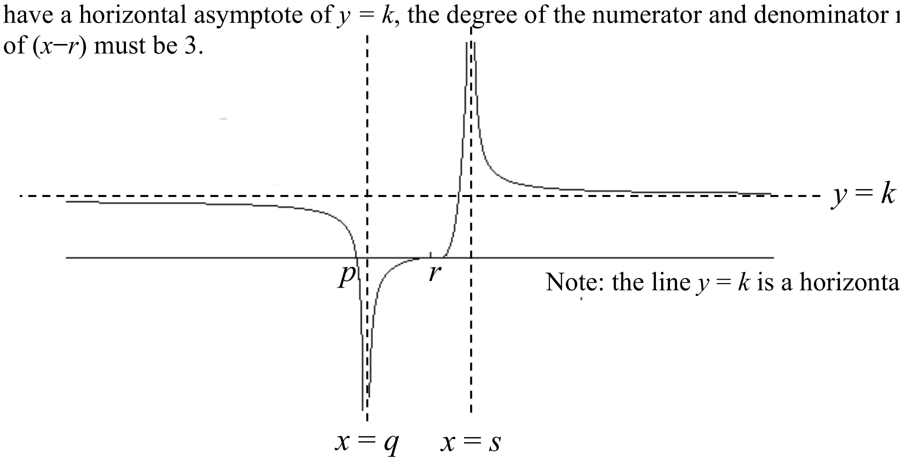




Choice **B.**

55. Choice **A** is $y = \dfrac{k(x-p)(x-r)^3}{(x-q)^2(x-s)^2}$

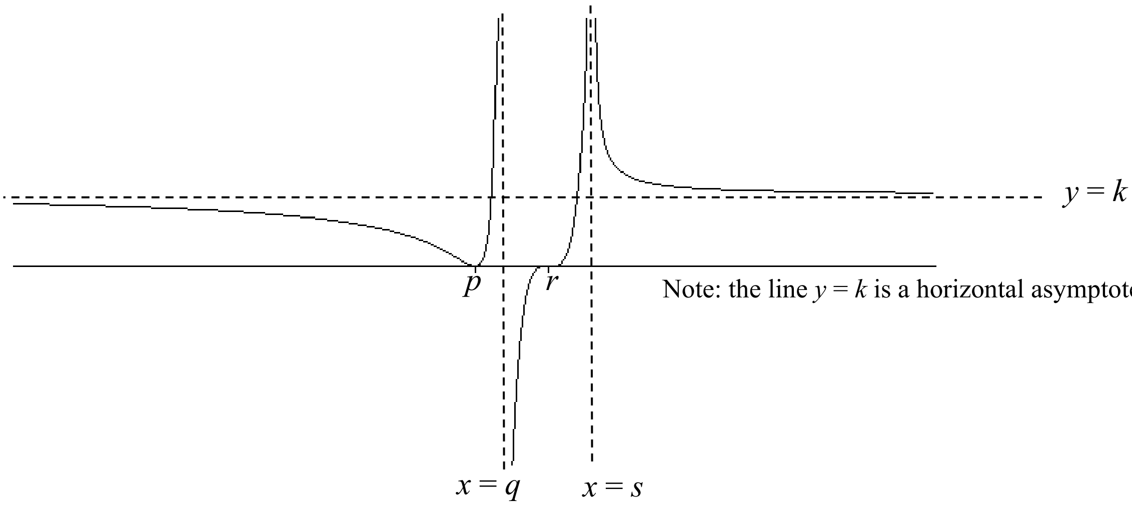
Since p is a single zero, the power of $(x-p)$ must be 1. The volcano shape near $x = q$ and $x = s$ mean the power for $(x-q)$ and $(x-s)$ must be even. To have the lowest power possible means these are both 2. So the degree of numerator must be 4. Since q is a multiple zero in the shape of a chair, the power of $(x-r)$ must be odd. Since we have a horizontal asymptote of $y = k$, the degree of the numerator and denominator must be the same, so the power of $(x-r)$ must be 3.



Note: the line $y = k$ is a horizontal asymptote.

Choice **C** is $y=\dfrac{k(x-p)^2(x-r)^3}{(x-q)^3(x-s)^2}$

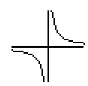
Note: $y=\dfrac{k(x-p)^4(x-r)^3}{(x-q)^5(x-s)^2}$ and $y=\dfrac{k(x-p)^4(x-r)^3}{(x-q)^3(x-s)^4}$ would also be options but the factors do not have the lowest degree. Since the zero at p is a bounce, the degree of $(x-p)$ must be even. The volcano shape near $x = s$ means the degree for $(x-s)$ must be even. The "twisted sister" shape near $x = q$ (with the graph approaching $\pm\infty$ on opposite sides means the degree for $(x-q)$ must be odd. Since the zero at r is a chair shape, the power of $(x-r)$ must be odd. To have the lowest power possible means the power of $(x-p)$ must be 2 and the power of $(x-r)$ must be 3. So the degree of numerator must be 5. Since we have a horizontal asymptote of $y = k$, the degree of the numerator and denominator must be the same, so the power of $(x-s)$ must be 2 and the power of $(x-q)$ must be 3.



Note: the line $y = k$ is a horizontal asymptote.

Choice **D** has two correct possibilities: it could either be $y=\dfrac{k(x-p)^2(x-r)^3}{(x-q)^2(x-s)^4}$ or $y=\dfrac{k(x-p)^2(x-r)^3}{(x-q)^4(x-s)^2}$

Since the zero at p is a bounce, the degree of $(x-p)$ must be even. The volcano shapes near $x = s$ and $x = q$ mean the degree for $(x-q)$ and $(x-s)$ must both be even. Since the zero at r is a chair shape, the power of $(x-r)$ must be odd. To have the lowest power possible means the power of $(x-p)$ must be 2 and the power of $(x-r)$ must be 3. So the degree of numerator must be 5.

Since we have a horizontal asymptote of $y = 0$ and the long run behavior looks like ,

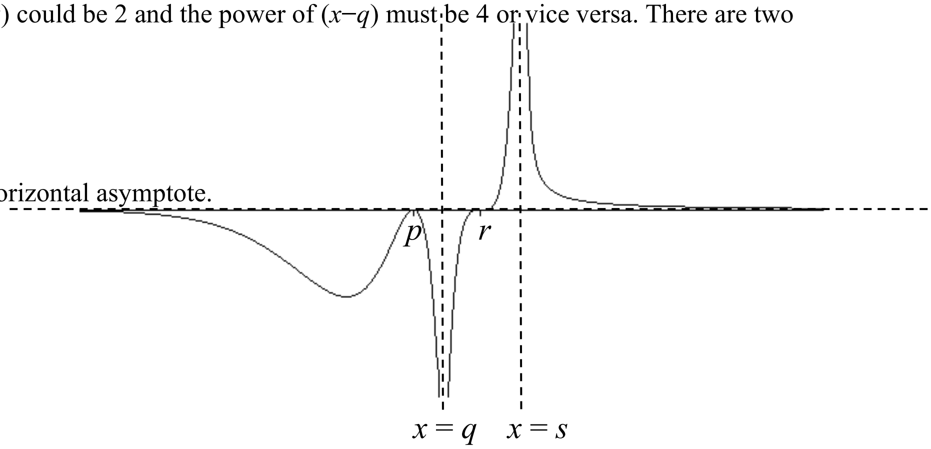
Then as $x \to \pm\infty$ the graph behaves like $y=\dfrac{k}{x^{ODD}}$. We want the powers to be as small as possible,

so we want the degree of the denominator to be 1 more than the degree of the numerator,

i.e., as $x \to \pm\infty$ the graph looks like $y=\dfrac{k}{x}$. So the degree of the denominator must be 6.

This means the power of $(x-q)$ could be 2 and the power of $(x-q)$ must be 4 or vice versa. There are two possibilities.

Note: the line $y = 0$ is a horizontal asymptote.



56. The function $f(x)=\dfrac{4}{x^2}$ takes any input and returns 4 divided by the square of the input.

We can replace x by a placeholder, such as an empty box, i.e. $f(\square)=\dfrac{4}{(\square)^2}$

If f takes the function $g(x)=\sqrt{x^2+4}$ as an input, then we have the following: $f\left(\boxed{g(x)}\right)=\dfrac{4}{\left(\boxed{\sqrt{x^2+4}}\right)^2}$

$$=\frac{4}{x^2+4}$$

This is as simplified as possible. The answer is Choice **A**.

While it is true that $\dfrac{4}{x^2\cdot 4}=\dfrac{\cancel{4}}{x^2\cdot\cancel{4}}=\dfrac{1}{x^2}$ since $\dfrac{4}{x^2\cdot 4}=\dfrac{1}{x^2}\cdot\dfrac{4}{4}=\dfrac{1}{x^2}\cdot 1=\dfrac{1}{x^2}$,

be careful not to incorrectly simplify $\dfrac{4}{x^2+4}$. In particular, $\dfrac{4}{x^2+4}\neq\dfrac{\cancel{4}}{x^2+\cancel{4}}\neq\dfrac{1}{x^2}$

You can check your answer: Y3=Y4 ≠ Y5

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PRESS + FOR △Tbl

X	Y3	Y4	Y5
0	1	1	ERROR
1	0.8	0.8	1
2	0.5	0.5	0.25
3	0.3077	0.3077	0.1111
4	0.2	0.2	0.0625
5	0.1379	0.1379	0.04
6	0.1	0.1	0.0278
7	0.0755	0.0755	0.0204
8	0.0588	0.0588	0.0156
9	0.0471	0.0471	0.0123
10	0.0385	0.0385	0.01

X=0

Y1=4/X²
Y2=√(X²+4)
Y3=Y1(Y2(X))
Y4=4/(X²+4)
Y5=1/X²

57. For the function $f(x)=\dfrac{\sqrt{x+1}}{2}$ we can replace x by a placeholder, such as an empty box, i.e.. $f(\square)=\dfrac{\sqrt{\square+1}}{2}$

If f takes the function $g(x)=x^2+3$ as an input, then we have the following:

$$f\left(\boxed{g(x)}\right)=\frac{\sqrt{\boxed{x^2+3}+1}}{2}$$
$$=\frac{\sqrt{x^2+3+1}}{2}$$
$$=\frac{\sqrt{x^2+4}}{2}$$

This is as simplified as possible. The answer is Choice **B**.

Careful! It can be tempting to incorrectly simplify $\dfrac{\sqrt{x^2+4}}{2}$

$$\frac{\sqrt{x^2+4}}{2}\neq\frac{\sqrt{x^2}+\sqrt{4}}{2}=\frac{x+2}{2}\quad(\text{assuming } x\geq 0)$$

For example, $\sqrt{25}=\sqrt{16+9}$

Compare: $\sqrt{16}+\sqrt{9}=4+3=7$

$\sqrt{25}=5$

So $\sqrt{16}+\sqrt{9}$ and $\sqrt{25}$ are not equal.

While it is true that $\dfrac{x\cdot 2}{2}=\dfrac{x\cdot\cancel{2}}{\cancel{2}}=x$ since $\dfrac{x\cdot 2}{2}=\dfrac{x}{1}\cdot\dfrac{2}{2}=x\cdot 1=x$, be careful not to incorrectly simplify $\dfrac{x+2}{2}$

$$\frac{x+2}{2}\neq\frac{x+\cancel{2}}{\cancel{2}}\neq x$$

You can check your answer with a grapher: Y3=Y4 ≠ Y5 ≠ Y6

NORMAL FLOAT AUTO REAL DEGREE MP
PRESS + FOR △Tbl

X	Y3	Y4	Y5	Y6
0	1	1	1	0
1	1.118	1.118	1.5	1
2	1.4142	1.4142	2	2
3	1.8028	1.8028	2.5	3
4	2.2361	2.2361	3	4
5	2.6926	2.6926	3.5	5
6	3.1623	3.1623	4	6
7	3.6401	3.6401	4.5	7
8	4.1231	4.1231	5	8
9	4.6098	4.6098	5.5	9
10	5.099	5.099	6	10

X=0

Y1=√(X+1)/2
Y2=X²+3
Y3=Y1(Y2(X))
Y4=√(X²+4)/2
Y5=(X+2)/2
Y6=X

58. We know that when the price p = \$11, the number of customers N who will come to the park is 800. For each \$1.00 increase in the entrance price p, the park would lose an average of 50 daily customers: $N = f(p)$ is linear. When Δp = \$1, then $\Delta N = -50$.

The slope is $\frac{\Delta N}{\Delta p} = \frac{-50}{\$1} = -50$ and it passes through (\$11, 800).

We have $N = b - 800p$. Substitute $p = 11$, $N = 800$:

$$800 = b - 50(11)$$
$$800 = b - 550$$
$$b = 1350$$

Therefore $N = f(p) = 1350 - 50p$.

Check each response by scrolling the table feature of a grapher. The table confirms that if p = \$11, N = 800 and the rate of change is −50 customers per dollar increase.

"If the park had free admission, they would have as many as 1,350 daily customers." True

"A \$27 ticket price would result in no customers." True

X	Y1
0	1350
1	1300
2	1250
3	1200
4	1150
5	1100
6	1050
7	1000
8	950
9	900
10	850
11	800
12	750
13	700
14	650
15	600
16	550
17	500
18	450
19	400
20	350
21	300
22	250
23	200
24	150
25	100
26	50
27	0

X=27

"If the ticket price were \$3.50, they would have 1175 daily customers." True.

"Only 125 customers would be willing to pay a \$24.50 admission price." True.

Choice **E**. All of the above.

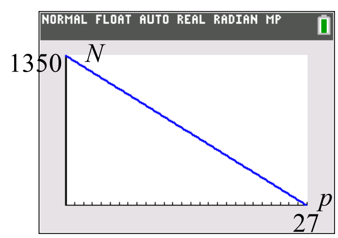
X	Y1
3.5	1175
4.5	1125
5.5	1075
6.5	1025
7.5	975
8.5	925
9.5	875
10.5	825
11.5	775
12.5	725
13.5	675
14.5	625
15.5	575
16.5	525
17.5	475
18.5	425
19.5	375
20.5	325
21.5	275
22.5	225
23.5	175
24.5	125

X=24.5

You can also use algebra to find the intercepts and to evaluate the formula at p = \$3.50 and p = \$24.50.

To find the p-intercept of $N = f(p)$, set $N = 0$ and solve for p:

$$0 = 1350 - 50p$$
$$50p = 1350$$
$$p = 27$$



The horizontal intercept or p-intercept is (27, 0).
This means that if the ticket price were \$27,
no customer would purchase one.
The vertical intercept or N-intercept can be found
by inspection from the formula $N = 1350 - 50p$: $N(0) = 1350$.
The values of $N(3.5)$ and $N(24.5)$ can be also be calculated
with arithmetic from the formula.

59. We add a third column to the table in Question **58** which gives the daily revenue, R, for each entrance price p.
The *revenue* is the total amount received by the park before any costs are deducted, which is $R = N \cdot p$.
For example, if the price $p = \$11$, then $N = 800$ tickets are sold and the revenue $R = 800 \cdot 11 = \$8800$.
One approach is to manually compute the product $N \cdot p$ for the rows you are interested in.

The point $p = 0$, $R = 0$ means that if the tickets were free, there would be no revenue (even though 1350 customers would come.

The point $p = 27$, $R = 0$ means that if the tickets were \$27, there would be no revenue (since no customers would buy them.)

p	N	$R = p \cdot N$
\$0	1350	\$0
\$11	800	\$8800
\$13	700	\$9100
\$14	650	\$9100
\$27	0	\$0

The table shows that Choice A and B are both false
A. "The higher they set the ticket price, the more revenue they will make."
B. "A ticket price of \$27 gives them the most revenue."

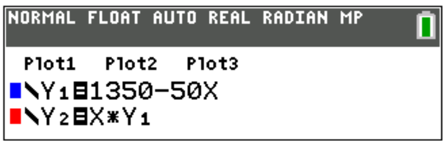
The table suggests also that C and D are false.

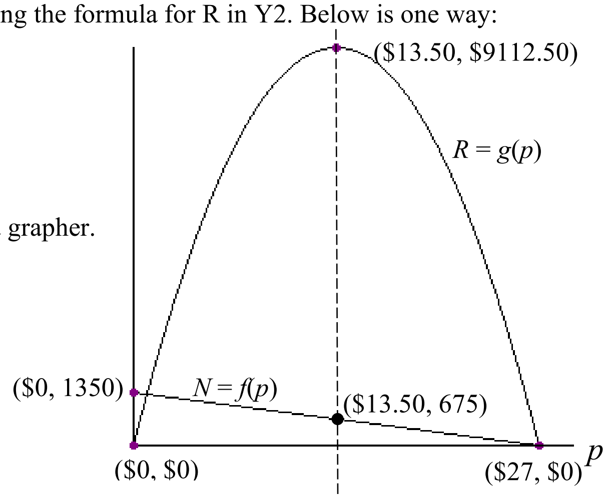
Examine the formula for the product $R = N \cdot p$.
$R(x) = x(1350 - 50x)$. If we multiply it out we get $R = 1350x - 50x^2$, which is quadratic.
This has a maximum on its axis of symmetry, which is midway between its intercepts
(or midway between any two values of x which have the same y-values.)

p	N	$R = p \cdot N$
\$0	1350	\$0
\$11	800	\$8800
\$13	700	\$9100
\$13.50	675	\$9112.50
\$14	650	\$9100
\$27	0	\$0

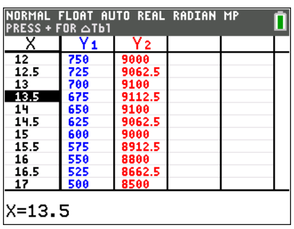
So the maximum revenue occurs if tickets were sold at $p = \$13.50$.

You can use a grapher instead of manual calculation by entering the formula for R in Y2. Below is one way:





Choice **E** is correct. None of the response are true.
It can be illuminating to view the graph and table of both on a grapher.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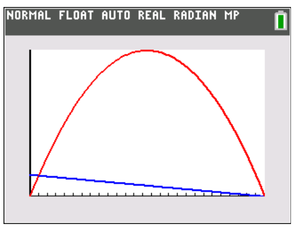
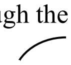
PRESS + FOR ΔTbl

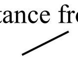
X	Y1	Y2
12	750	9000
12.5	725	9062.5
13	700	9100
13.5	675	9112.5
14	650	9100
14.5	625	9062.5
15	600	9000
15.5	575	8912.5
16	550	8800
16.5	525	8662.5
17	500	8500

X=13.5

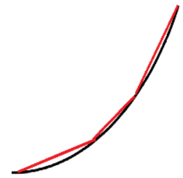


60. Some of these can be represented by more than one graph.

(a) "Even though the child's temperature is still rising, the penicillin seems to be taking effect."
Choice **III**. 
We sketch a graph of temperature vs. time that increases and will eventually flatten out.
The rate of growth is modeled by the slopes of the line segments shown to the right.
Since the rate of change is decreasing, the lines must have smaller and smaller slopes,
i.e., they are less and less steep. The graph is concave down and increasing.

(b) "Your distance from the Atlantic Ocean in kilometers, increases at a constant rate."
Choice **V**. 
We sketch a linear graph of distance vs. time that increases since it climbs steadily.

(c) "At first your balance grows slowly, but its rate of growth continues to increase."
Choice **IV**. 



We sketch a graph of balance vs. time that increases faster and faster, i.e., concave up.
The rate of growth is modeled by the slopes of the line segments shown to the right.
Since the rate of change is increasing, the lines must have larger and larger slopes,
i.e., they are steeper and steeper. The graph is concave up and increasing.

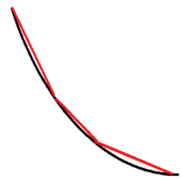
(d) "The annual profit is decreasing. Each year it falls more steeply than the previous year."
Choice **I**. 

The rate of change is modeled by the slopes of the line segments shown to the right.
Since the profit is decreasing, the lines must have negative slopes.
Since they fall faster and faster, the graph is concave down.

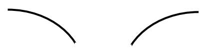
(e) "The function has a positive rate of change and the rate of change is decreasing."
Choice **III**. 

The rate of change is modeled by the positive slopes of the line segments shown to the right.
Since the rate of change is decreasing, the lines must have smaller and smaller slopes,
i.e., they are less and less steep. The graph is concave down and increasing.

(f) "The population of rhinos isn't decreasing as quickly it used to be."
Choice **II**. 



The rate of change is modeled by the negative slopes of the line segments
which are less and less steep.

(g) The function is concave down.
Choices **I** and **III**. 

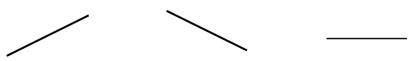
Some students remember this by thinking of a concave down graph as a frown. 

(h) The function is decreasing.
Choices **I**, **II**, and **VI**. 

Some students remember this by thinking of going down a descending path.

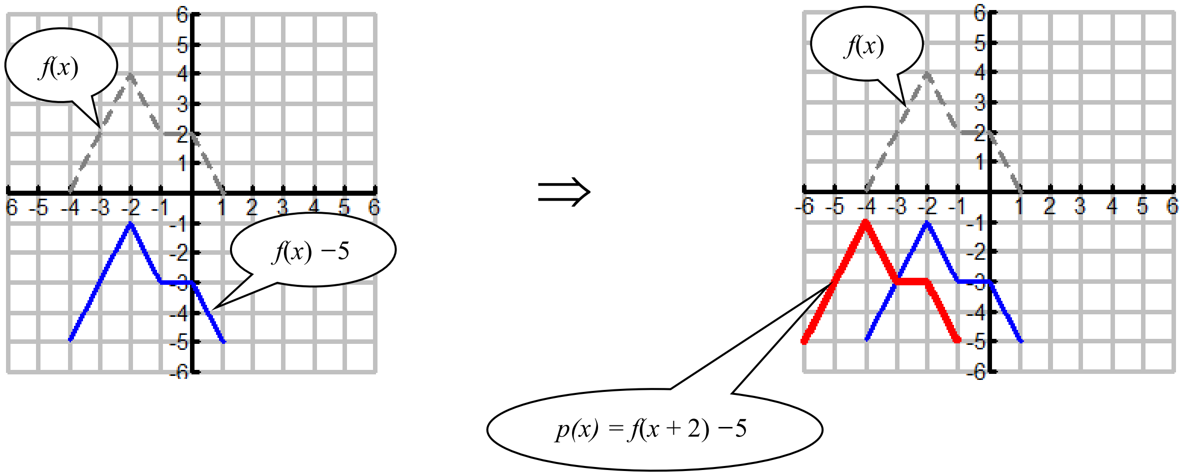
(i) The function is constant.
Choice **VII**.
This statement describes the outputs, or y-values, of the function. The slope is 0.
Think of a flat parking lot, a billiard table, or a the EKG monitor on a sad episode of a television medical drama.

(j) The average rate of change of the function is constant.
Choices **V**, **VI**, and **VII**.

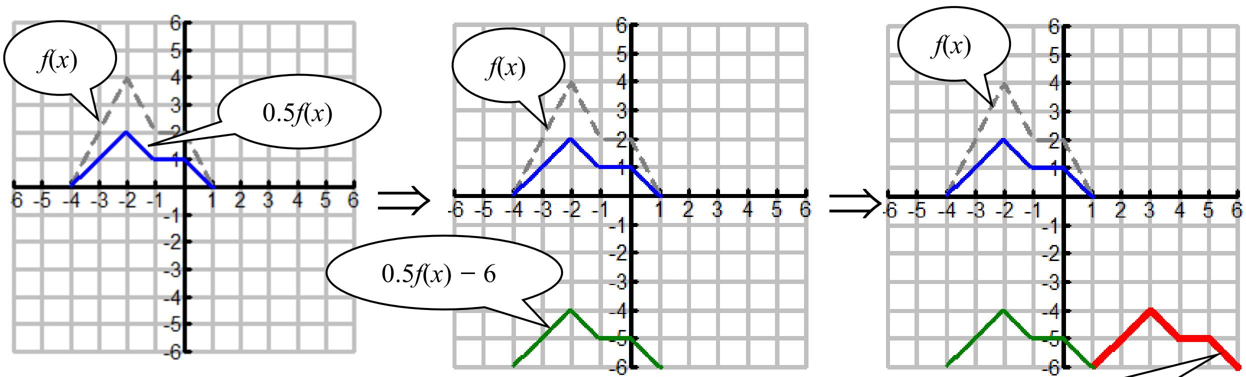


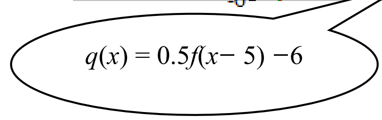
This statement describes the slopes of the lines drawn on the function similar to what was done above.
A linear function has a constant rate of change. Slopes of lines can be positive, negative, or zero.

61. The graph of $p(x)$ is a vertical shift of $f(x)$ down 5 and horizontal shift left 2.
The transformation is $p(x) = f(x + 2) - 5$. Choice **A**.

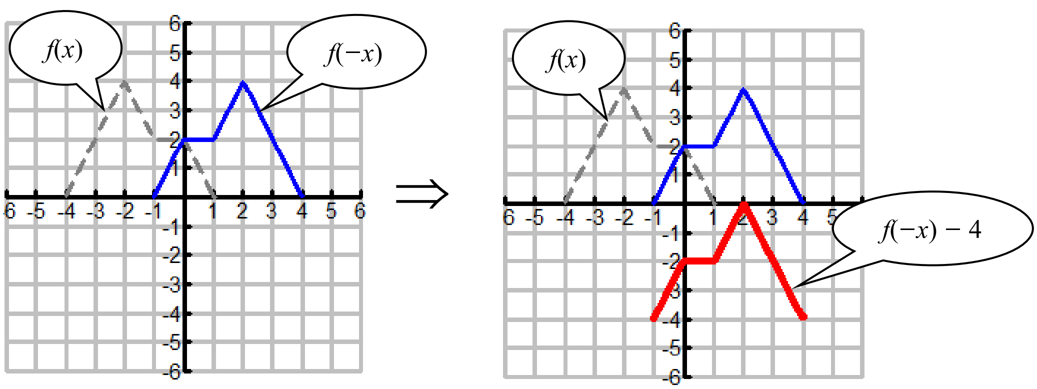


62. The graph of $q(x)$ is a vertical compression of $f(x)$ by a factor of $\frac{1}{2}$,
followed by a vertical shift down 6 and horizontal shift 5 right.
The transformation is $q(x) = 0.5f(x - 5) - 6$. Choice **E**.

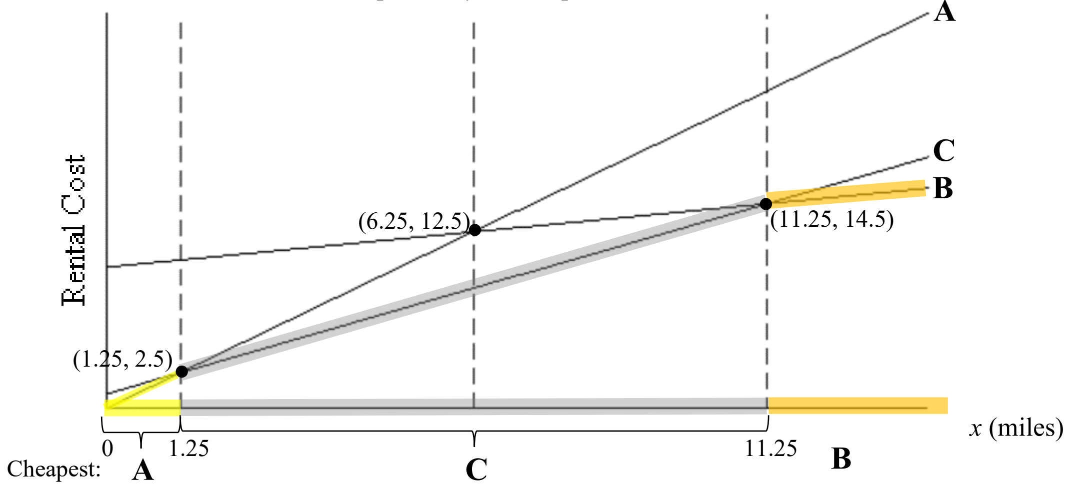




63. The graph of $q(x)$ is a horizontal reflection of $f(x)$
(or a reflection of $f(x)$ about the y-axis), followed by
a vertical shift down 4. The transformation is $r(x) = f(-x) - 4$. Choice **D**.



64. The graphs are labeled below.
Scooter **A** passes through the origin and has the steepest slope since it has the highest cost per mile.
Scooter **B** has the most gradual slope since it has the lowest cost per mile. It has the highest y-intercept.
Scooter **C** has the smallest positive y-intercept.

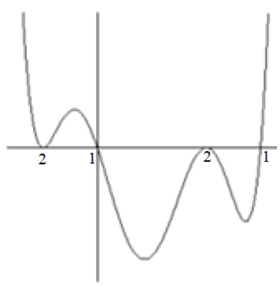


To find when Scooter **C** is cheapest, look for the values of x for which the graph of C is below the graph of A and C.
Scooter **C** is cheapest on the interval $1.25 < x < 11.25$.

Although not needed to answer this question, we can see the following is true:
Scooter **A** is cheapest on the interval $0 < x < 1.25$.
Scooter **B** is cheapest on the interval $x > 11.25$.

We do not use the y-coordinates in reporting the interval. The correct choice is Choice **E**.

65. The end behavior (up-up) indicates the degree of the polynomial is even.
Count the minimum multiplicities of each zero based on the shape of the graph (chair, bounce, line) near the zero.



Lowest multiplicities of each zero

We have 2 + 1 + 2 + 1 = 6. Choice **D**.

66-69. To examine the long run behavior, we use the formula as given.

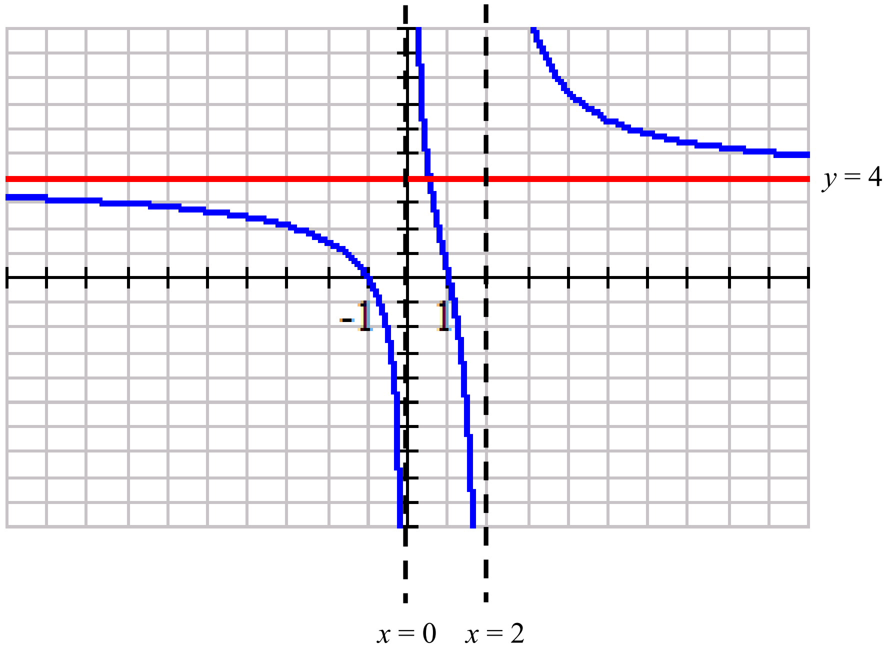
The numerator and denominator is in expanded form, i.e., $y = \dfrac{8x^2 - 8}{2x^2 - 4x}$

To examine the short run behavior, it can be helpful to factor the numerator and the denominator:

$$y = \frac{8x^2 - 8}{2x^2 - 4x} = \frac{8(x^2 - 1)}{2x(x-2)} = \frac{4(x-1)(x+1)}{x(x-2)}$$

We can also use our grapher to check using a graph and a table.

Y1=(8X²-8)/(2X²-4X)
Y2=4



66. To find the zeros of $y=\dfrac{8x^2-8}{2x^2-4x}=\dfrac{8(x^2-1)}{2x(x-2)}=\dfrac{4(x-1)(x+1)}{x(x-2)}$ set the numerator equal to 0.

The zeros are -1 and 1. Choice **C**.

X	Y1
-3	2.1333
-2	1.5
-1	0
0	ERROR
1	0
2	ERROR
3	10.667
4	7.5
5	6.4
6	5.8333
7	5.4857

← There are two zeros at $x=-1$

← and $x=1$.

67. To find any y-intercepts, find the y-value when $x = 0$. However, the function is undefined at $x = 0$ so the graph never crosses the y-axis. Choice **E**.

68. The function has vertical asymptotes when the denominator is zero (and the numerator is not). The denominator $x(x-2)=0$ when $x = 0$ and $x = 2$. This matches the answer to **67**. Choice **E**.

X	Y1
-3	2.1333
-2	1.5
-1	0
0	ERROR
1	0
2	ERROR
3	10.667
4	7.5
5	6.4
6	5.8333
7	5.4857

← There is a vertical asymptote at $x = 0$

← and at $x = 2$.

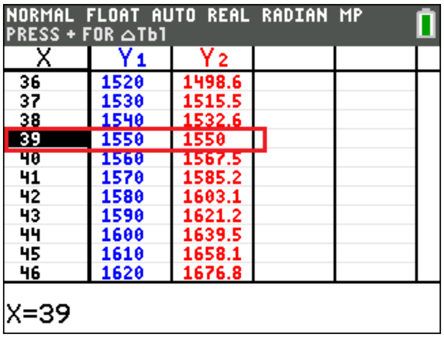
69. To find if there is a horizontal asymptote, examine the long run behavior:

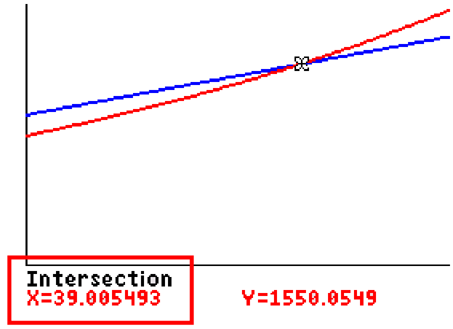
$$y=\frac{8x^2-8}{2x^2-4x}\rightarrow\frac{8x^2}{2x^2}=4\quad\text{as}\quad x\rightarrow\ \pm\infty$$

Since the function looks like the line $y = 4$ for very large values of x, the line $y = 4$ is the horizontal asymptote. Choice **D**.
You can scroll a table with a large Δx to confirm this.

X	Y1
-25000	3.9997
-20000	3.9996
-15000	3.9995
-10000	3.9992
-5000	3.9984
0	ERROR
5000	4.0016
10000	4.0008
15000	4.0005
20000	4.0004
25000	4.0003

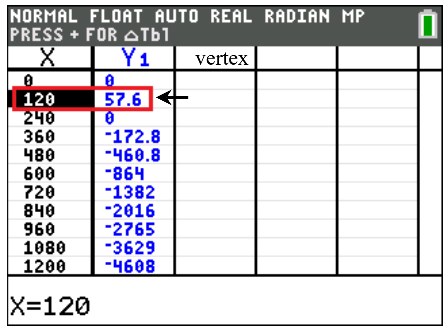
70. $P = 1160 + 10t$ and $Q = 1000(1.0113)^t$
Set the equations equal to each other and solve using technology.
They intersect at $t = 39$ years. Choice **B**.



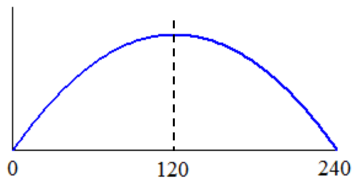


71. The path of an artillery shell, in feet, fired from a military base is given by $h(x) = 0.96x - 0.004x^2$.
Factor $h(x)$ to find the zeros.

$$0.96x - 0.004x^2 = 0$$
$$x(0.96 - 0.004x) = 0$$
$$x = 0 \quad \Big\| \quad 0.96 - 0.004x = 0$$
$$0.96 = 0.004x$$
$$x = \frac{0.96}{0.004} = 240$$



Thus $h(x)$ has zeros at 0 and 240 and is concave down since $a = -0.004$.



Use a table to find the vertex, which is halfway between the zeros at the point (120, 57.6) so the exact maximum height is 57.6 ft. Choice **D**.

72. If a population with initial amount P_0 doubles every 12 years, it is modeled by $P(t) = P_0(2)^{\frac{t}{12}}$.

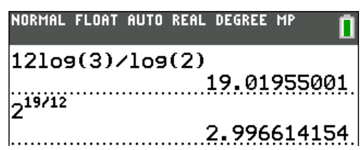
To find the tripling time, solve $P(t) = P_0(2)^{\frac{t}{12}} = 3P_0$.

Divide both sides by P_0 and take logarithms: $(2)^{\frac{t}{12}} = 3$

$$\log(2)^{\frac{t}{12}} = \log 3$$

$$\tfrac{t}{12}\log(2) = \log 3$$

$$t = \frac{12\log 3}{\log 2} \approx 19 \text{ years.}$$



We can check by substituting back into the original equation. $P(19) = P_0(2)^{\frac{19}{12}} \approx 3P_0$.

Choice **C**.