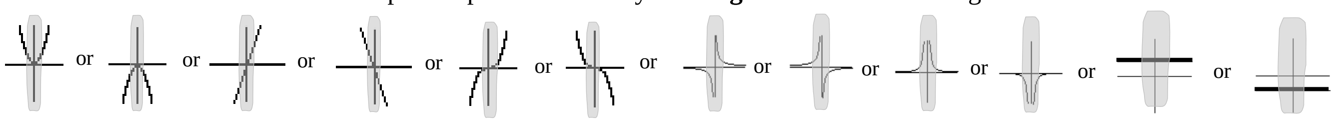
Long Run Behavior and Horizontal Asymptotes

For each rational function in 1-5,

a. Find the power function it most closely resembles for very large values of x.

b. Describe the **long run** behavior by completing the boxes:
As $x \to -\infty$, then $y \to \boxed{}$; as $x \to \infty$, then $y \to \boxed{}$

c. Sketch the power function which has the same **long run** behavior. Pick from these choices:
The *short run* behavior is covered up to emphasize that only the **long run** behavior is being mirrored.



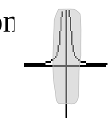
d. Find the horizontal asymptote, if there is one. If none, state so.

1. $f(x) = \dfrac{8x^3 + 5x - 9}{4x^7 + 200x^2 - 6}$

a. Power function model: $y = \dfrac{8x^3}{4x^7} = \dfrac{2}{x^4}$ (simplify)

b. As $x \to -\infty$, then $y \to \boxed{0}$; as $x \to \infty$, then $y \to \boxed{0}$

c. Long run behavior looks like this power function



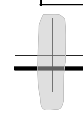
d. horizontal asymptote: $y = 0$

2. $f(x) = \dfrac{36x^3 + 3x - 7}{x^2 - 4x^3}$

a. Power function model: $y = \dfrac{36x^3}{-4x^3} = -9$ (simplify)

b. As $x \to -\infty$, then $y \to \boxed{-9}$; as $x \to -\infty$, then $y \to \boxed{-9}$

c. Long run behavior looks like this power function:



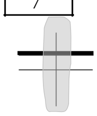
d. horizontal asymptote: $y = -9$

3. $f(x) = \dfrac{3 + 4x}{2 + 7x}$

a. Power function model: $y = \dfrac{4x}{7x} = \dfrac{4}{7}$ (simplify)

b. As $x \to -\infty$, then $y \to \boxed{\tfrac{4}{7}}$; as $x \to \infty$, then $y \to \boxed{\tfrac{4}{7}}$

c. Long run behavior looks like this power function:



d. horizontal asymptote: $y = \tfrac{4}{7}$

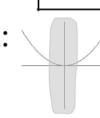
4. $f(x) = \dfrac{10x^6 - 4x}{(x-3)(x-4)}$

$= \dfrac{10x^6 - 4x}{x^2 + \text{remaining terms}}$

a. Power function model: $y = \dfrac{10x^6}{x^2} = 10x^4$ (simplify)

b. As $x \to -\infty$, then $y \to \boxed{\infty}$; as $x \to \infty$, then $y \to \boxed{\infty}$

c. Long run behavior looks like this power function:



d. horizontal asymptote: None

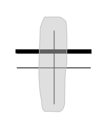
5. $f(x) = \dfrac{2(x-2)^2(x-6)}{9(x-5)^3}$

$= \dfrac{2x^3 + \text{remaining terms}}{9x^3 + \text{remaining terms}}$

a. Power function model: $y = \dfrac{2x^3}{9x^3} = \dfrac{2}{9}$ (simplify)

b. As $x \to -\infty$, then $y \to \boxed{\tfrac{2}{9}}$; as $x \to \infty$, then $y \to \boxed{\tfrac{2}{9}}$

c. Long run behavior looks like this power function:



d. horizontal asymptote: $y = \tfrac{2}{9}$

For the functions below, report the horizontal asymptote, if there is one. If none, state so.

6. $f(x) = \dfrac{7(x+2)(x+5)}{11(x-5)}$

7. $f(x) = \dfrac{12x^2 + 1}{3x^2 + 2} + 3$

8. $f(x) = \dfrac{25x^2 + 38}{x(1 + 0.02x)}$

9. $f(x) = \dfrac{6x}{3x^2 + 10} + 2$

10. $f(x) = \dfrac{3(x+5)^2(x-4)}{4(x-6)^3(x-1)}$

Answers:
6. None
7. $y = 7$
8. $y = 1250$
9. $y = 2$
10. $y = 0$

DETAILED SOLUTIONS

6. $f(x) = \dfrac{7(x+2)(x+5)}{11(x-5)}$ looks like the power function $\dfrac{7x^2}{11x} = \dfrac{7x}{11}$ for very large values of *x*.
This is a line of positive slope. There is no horizontal line this function approaches.
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to \infty$.
So, no horizontal asymptote.

7. $f(x) = \dfrac{12x^2+1}{3x^2+2} + 3$ is a vertical shift of the function $y = \dfrac{12x^2+1}{3x^2+2}$ up 3.
Since $y = \dfrac{12x^2+1}{3x^2+2}$ looks like the power function $\dfrac{12x^2}{3x^2} = 4$ for very large values of *x*, the graph of $y = \dfrac{12x^2+1}{3x^2+2}$ would have a horizontal asymptote of *y* = 4. Therefore, the shifted function $f(x) = \dfrac{12x^2+1}{3x^2+2} + 3$ would have a horizontal asymptote of *y* = 7. Note that for very large values of *x*, $f(x) = \dfrac{12x^2+1}{3x^2+2} + 3 \longrightarrow \dfrac{12x^2}{3x^2} + 3 = 4+3 = 7$.

8. $f(x) = \dfrac{25x^2+38}{x(1+0.02x)}$ looks like the power function $\dfrac{25x^2}{0.02x^2} = \dfrac{25}{0.02} = 1250$ for very large values of *x*. Therefore, it has a horizontal asymptote of *y* = 1250.

9. $f(x) = \dfrac{6x}{3x^2+10} + 2$ is a vertical shift of the function $y = \dfrac{6x}{3x^2+10}$ up 2.
Since $y = \dfrac{6x}{3x^2+10}$ looks like the power function $\dfrac{6x}{3x^2} = \dfrac{2}{x}$ for very large values of *x*, the graph of $y = \dfrac{6x}{3x^2+10}$ would have a horizontal asymptote of *y* = 0. Therefore, the shifted function $f(x) = \dfrac{6x}{3x^2+10} + 2$ would have a horizontal asymptote of *y* = 2. Note that for very large values of *x*, $\dfrac{6x}{3x^2} + 2 = \dfrac{2}{x} + 2 \to 0+2$

10. $f(x) = \dfrac{3(x+5)^2(x-4)}{4(x-6)^3(x-1)} = \dfrac{3x^3 + \text{remaining terms}}{4x^4 + \text{remaining terms}}$, so it looks like the power function $\dfrac{3x^3}{4x^4}$ for very large values of *x*. Note that $\dfrac{3x^3}{4x^4} = \dfrac{3}{4x}$ approaches 0 as *x* increases without bound, so $f(x) = \dfrac{3(x+5)^2(x-4)}{4(x-6)^3(x-1)}$ has a horizontal asymptote of *y* = 0.