

## The three cases for end behavior of rational functions

Assume  $a$  and  $b$  are any constants, and  $m, n$  and  $p$  are positive integers.

Then as  $x \rightarrow \pm\infty$ ,  $f(x) = \frac{ax^m + \text{remaining terms of lower degree}}{bx^n + \text{remaining terms of lower degree}}$  has the same **end behavior** as  $y = \frac{ax^m}{bx^n}$ .

This simplifies to three cases:

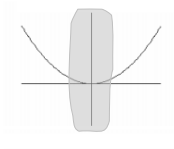
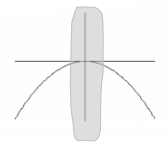
$$m < n: y = \frac{ax^m}{bx^n} = \frac{a}{bx^p}$$

$$m > n: y = \frac{ax^m}{bx^n} = \frac{ax^p}{b}$$

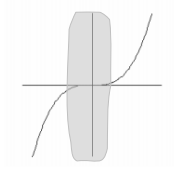
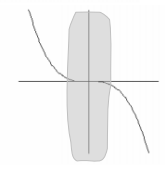
$$m = n: y = \frac{ax^m}{bx^n} = \frac{ax^p}{bx^p} = \frac{a}{b}$$

Match the equation in each of the three boxes to the set of possible graphs which have the same end behavior. The *short run* behavior is covered up to emphasize that only the **end** behavior is being mirrored.

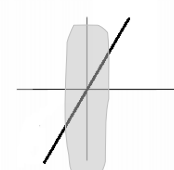
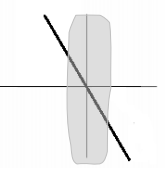
I

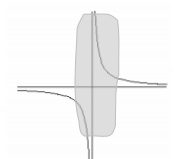
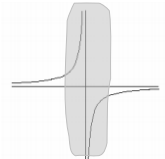

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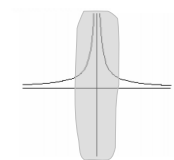
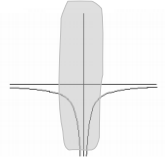

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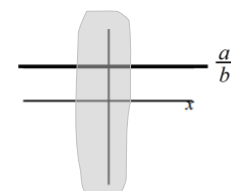
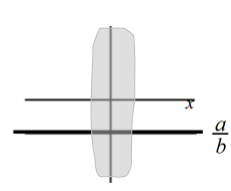
II


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III

Report the horizontal asymptote, if any, for each of the above cases. If none exists, state so.