Why are the logarithmic properties true?

1. Complete the blanks and boxes to show $\log_b QR = \log_b Q + \log_b R$

   a. Let $\log_b Q = x$.

   b. Write the equation in 1a in exponential form: ___________

   c. Let $\log_b R = y$.

   d. Write the equation in 1c in exponential form: ___________

   e. $QR = b^x \cdot \boxed{}$ if we substitute the results from 1b and 1d.

   f. $QR = b^{\boxed{}}$ by properties of exponents.

   g. Now write the equation in 1f in logarithmic form: $(QR) = b^{\boxed{}}$ means $\log_{\bigcirc} \boxed{} = \boxed{}$

   h. Eliminate x and y in the equation in 1g by substituting the equations in 1a and 1c:

   ____

2. Complete the blanks and boxes to show $\log_b Q^k = k \cdot \log_b Q$

   a. Let $\log_b Q = x$.

   b. Write the equation in 2a in exponential form: ___________

   c. $Q^k = \left(\boxed{}\right)^k$ if we substitute 2b.

   d. $Q^k = b^{\boxed{}}$ by properties of exponents

   e. Now write the equation in 2d in logarithmic form: $(Q^k) = b^{\boxed{}}$ means $\log_{\bigcirc} \boxed{} = \boxed{}$

   f. Eliminate x in the equation in 2e by substituting the equation in 2a:

   ____

3. Complete the boxes to show $\log_b \dfrac{Q}{R} = \log_b Q - \log_b R$

Since $\dfrac{1}{R} = R^{\boxed{}}$, we have $\log_b \dfrac{Q}{R} = \log_b \left(Q \cdot \boxed{}\right)$

$= \log_b \left(Q \cdot R^{\boxed{}}\right)$

$=$ ______________________ using the property in 1h above.

$=$ ______________________ using the property in 2f above.