Modeling Laboratory Observations on Stream-Aquifer Interaction

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Abstract: A head loss concept, new to the stream-aquifer interaction, is introduced for modeling laboratory observations. Using this concept, the equation applicable to fully penetrating streams is modified to account for the head loss at the entrance. The modified model is able to explain and fit the observed laboratory data correctly. Equations are proposed to calculate the hydraulic diffusivity of aquifer and head loss at the entrance from the estimate of diffusivity obtained without considering the head loss. An optimization approach is also proposed to estimate the hydraulic diffusivity of the aquifer and head loss from the observed groundwater heads at different sections.

DOI: 10.1061/(ASCE)0733-9437(2007)133:1(79)

CE Database subject headings: Aquifers; Diffusivity; Head loss; Parameters; Optimization; Ground water.

Introduction

Stream-aquifer interaction is important for the assessment and management of groundwater. Although there have been few attempts to experimentally observe the stream-aquifer interaction, several sets of laboratory data are available (see, Rowe 1960; Singh and Sagar 1977). Singh (2006) observed that the application of the theory developed for fully penetrating streams to the laboratory data needs further research. It is observed from Singh (2003) and Srivastava (2003) that the theory developed for fully penetrating streams does not fit correctly to the observed laboratory data, i.e., the estimate of diffusivity is different for different sections. For a homogeneous and isotropic aquifer, the estimates of the hydraulic diffusivity should be the same for different sections. There is a need for an improved theory or model that can explain this discrepancy. This aspect was considered in this technical note. The equation applicable to fully penetrating streams is modified to account for head loss at the entrance so as to have consistent estimates of the hydraulic diffusivity for different sections. The proposed approach is applicable when the direct stream stage is used instead of water level measured in an observation well installed very close to the stream.

Problems with Existing Theory

The estimates of hydraulic diffusivity of soil obtained by Rowe (1960) using the concept of a fully penetrating stream are appreciably different for different sections. Similarly, Singh and Sagar (1977) obtained different estimates for different sections. The explicit equations for aquifer diffusivity, proposed by Singh (2003) for the linear rise (temporal variation) in stream stage are

$$\beta \cong \left[\frac{\pi x E(y_1)}{4(1-E(y_2))}\right]^2 \quad \text{for } \xi \geq 0.4 \tag{1}$$

$$y_1 = \frac{1}{\sqrt{t}} \tag{2}$$

$$y_2 = \left(1 - \frac{\pi\sqrt{\pi}}{4}(1-\xi)\right)^{0.5} \tag{3}$$

$$\xi = \frac{h}{ct} \tag{4}$$

$$E(z) = \frac{1}{n}\sum_{i=1}^{n} z_i \tag{5}$$

where c=coefficient of linear stream-stage (LT^{-1}); h=groundwater head (L); n=number of observations (dimensionless); t=time measured since the commencement of stream stage rise (T); x=distance of observation point from stream-aquifer interface (L); β=hydraulic diffusivity of aquifer ($L^2\,T^{-1}$); and ξ=ratio of groundwater head and stream-stage rise (dimensionless). Using Eq. (1), the hydraulic diffusivity of the soil was estimated by Singh (2003) for the three sets of laboratory data presented in Singh and Sagar (1977); the results are reproduced in Table 1. The estimates of β were obtained using Eq. (1) for the

Table 1. Diffusivity Estimates with No Head Loss (Set 1)

x	β	SEE
0.12	0.1418	0.0025
0.24	0.2116	0.0096
0.36	0.2081	0.0100

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Note. Discussion open until July 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on October 21, 2005; approved on April 27, 2006. This technical note is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 133, No. 1, February 1, 2007. ©ASCE, ISSN 0733-9437/2007/1-79–82/$25.00.

Table 2. Diffusivity Estimates with No Head Loss (Set 2)

x	β	SEE
0.1780	0.2247	0.0004
0.4200	0.1835	0.0021
0.5230	0.1748	0.0035
0.8280	0.1423	0.0058
1.0280	0.1294	0.0080

five sets of laboratory data taken from Rowe (1960). These estimates are presented in Table 2. The SEE is a measure of reliability for the estimated β and is given by

$$\mathrm{SEE} = \left(\frac{1}{(n-p)}\sum_{i=1}^{n}(h_o - h_c)_i^2\right)^{0.5} \tag{6}$$

where SEE=standard error of estimate (L); h_o=observed groundwater head (L); h_c=calculated groundwater head (L); and p=number of estimated parameters (dimensionless). The data presented by both the earlier writers do not fully explain the theory applicable for fully penetrating streams, as the estimates are considerably different for different sections. This condition may also exist for field data observed in a homogeneous and isotropic aquifer.

Concept of Head Loss

Different estimates for different sections as obtained in the previous section may be due to unaccounted head loss at the entrance by the existing model. The head loss at the entrance is due to the flow of water entering in capillary tubes (interstices) of soils. The head loss at the entrance is assumed to be the same for unsteady stream stage and for different sections from the stream-aquifer interface. Since the stream stage is unsteady, the head loss is accounted for by adding it to x, the sum is then termed as the effective distance to the observation point from the stream aquifer interface. Therefore, even if a stream is fully penetrating, there may be certain head loss at the entrance.

Estimating Diffusivity and Head Loss

Let β be the estimate of hydraulic diffusivity of aquifer with unaccounted head loss. If β_l and x_l are the estimates of hydraulic diffusivity and head loss, respectively, the relation between the two estimates of hydraulic diffusivity of aquifer is derived as

$$\beta_l = \beta\left(1 + \frac{x_l}{x}\right)^2 \tag{7}$$

If the estimates of β for different values of x are available, β_l and x_l can be estimated using

$$\beta_l = \left(\frac{E(1/x^2) - [E(1/x)]^2}{E(1/x^2)E(1/\sqrt{\beta}) - E(1/x)E[1/(x\sqrt{\beta})]}\right)^2 \tag{8}$$

$$x_l = \frac{E(1/x)E(1/\sqrt{\beta}) - E[1/(x\sqrt{\beta})]}{E(1/x)E[1/(x\sqrt{\beta})] - E(1/x^2)E[1/\sqrt{\beta}]} \tag{9}$$

Once β_l and x_l are known, the groundwater head at a section due to the linear rise in stream stage can be calculated using the approximation proposed by Singh (2003) for $\xi \geq 0.43$

Table 3. Diffusivity Estimates with No Head Loss (Set 2, Corrected Distances)

x	β	SEE
0.1080	0.0827	0.0004
0.3500	0.1274	0.0021
0.4530	0.1312	0.0035
0.7580	0.1193	0.0058
0.9580	0.1124	0.0096

$$\xi = \frac{h}{ct} \cong 1 - \frac{4}{\pi\sqrt{\pi}}\left[1 - \left(1 - \frac{\pi u}{2}\right)^2\right] \tag{10}$$

where u should be modified to take into account the head loss

$$u = \frac{(x + x_l)}{2\sqrt{\beta_l t}} \tag{11}$$

Once, the head loss and diffusivity are estimated, an estimate of diffusivity from observations at a section can be obtained using Eq. (1) with x replaced by $x + x_l$. This modified theory is equally applicable for partially penetrating and semipervious streams.

Optimization of Diffusivity and Head Loss

The analytical solution for groundwater head due to an unsteady linear stage in a fully penetrating stream, considering the head loss at the entrance is given by

$$h = ct\left[(1 + 2u^2)\mathrm{erfc}(u) - \frac{2}{\sqrt{\pi}}ue^{-u^2}\right] \tag{12}$$

where u is given by Eq. (11). Eqs. (11) and (12) in a function form can be expressed as

$$h = f(c, x, t, x_l, \beta_l) \tag{13}$$

where h=response variable; t=stage variable; x=stage variable; and x_l,β_l=parameters of the model. For known set of input and output, the parameters x_l, and β_l can be estimated by minimizing integral squared error (ISE)

$$\mathrm{ISE} = \sum_{i=1}^{n}(h_o - h_c)_i^2 \tag{14}$$

where ISE=integral squared error (L^2). The ISE can be minimized using a nonlinear optimization approach, such as the Levenberg-Marquardt algorithm (see, Press et al. 1992). The derivatives of h with respect to the parameters that are to be estimated are required for the optimization. These are derived as

$$\frac{\partial h_i}{\partial \beta_l} = -\frac{ct(x + x_l)}{\sqrt{\pi\beta_l^3 t}}(u\,\mathrm{erfc}(u) - e^{-u^2}) \tag{15}$$

$$\frac{\partial h_i}{\partial x_l} = \frac{2ct}{\sqrt{\pi\beta_l t}}(u\,\mathrm{erfc}(u) - e^{-u^2}) = -\frac{2\beta_l}{(x + x_l)}\frac{\partial h_i}{\partial \beta_l} \tag{16}$$

Results and Discussion

The proposed methods were applied on two different sets of laboratory data, viz., three data sets of Singh and Sagar (1977) and five data sets of Rowe (1960), designated as Sets 1 and 2, respec-

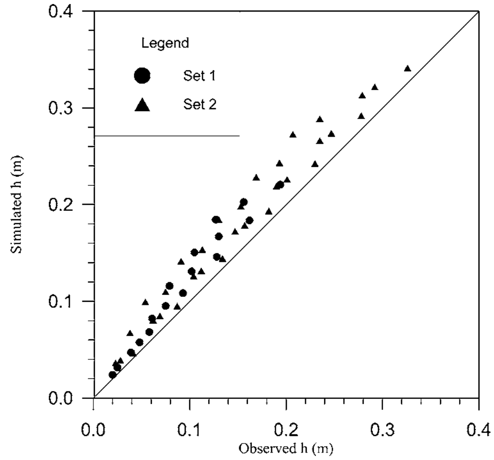

Fig. 1. Comparison of observed and calculated groundwater heads (simple method, sets 1 and 2)

tively. In Set 2, the distance between the first and second observation points was modified to 0.202 from 0.142 to make it consistent with the results of Rowe (1960). Using the estimates of diffusivity (Tables 1 and 2) without considering the head loss, the β_l and x_l, and were estimated using Eqs. (8) and (9); these estimates are 0.2836 and 0.0484, respectively, for Set 1 and are 0.1314 and −0.0458, respectively, for Set 2. The negative value of head loss in the case of Set 2 denotes inconsistency that may be because of additional distances assumed for the observation points. A scrutiny of Fig. 1 of Rowe (1960) suggests that the dimensions shown on it are according to the scale. Hence, besides the earlier modification, the distance to the first observation point was modified to 0.108 from 0.178, in order that the inconsistency is eliminated. With modified values of x, the βs were again estimated using Eq. (1); the results are given in Table 3. With these estimates of β, x_l=0.0274 and β_l=0.1338 are obtained using Eqs. (8) and (9). Now, a positive value of head loss obtained indicates that the inconsistency has been eliminated. Thus, the proposed model is also able to identify the inconsistency in the observed data to some extent. In Set 1, the first value of Subset 3, and in Set 2, the first value each of Subsets 4 and 5 were discarded for they gave $\xi > 0.43$.

With the estimated values of head loss and modified aquifer diffusivity, the groundwater heads were calculated using Eqs. (10) and (11) for all subsets of each set. The calculated groundwater heads were compared to those observed in Fig. 1 for Sets 1 and 2. It is observed that the proposed model is able to satisfactorily model the laboratory data and the model parameters are reliably estimated. The estimates of the parameters obtained using the optimization approach are given in Table 4. The observed and calculated groundwater heads in the case of optimization approach are compared in Fig. 2 for Sets 1 and 2. A comparison of Figs. 1 and 2 shows that the optimization approach yields better estimates of β_l and x_l compared to those obtained using the simple method. Therefore, for accurate results, the optimization approach should be used.

Table 4. Estimated Parameters Using Optimization Approach

Data	x_l	β	SEE
Set 1	0.0316	0.2040	0.0056
Set 2	0.0096	0.1351	0.0037

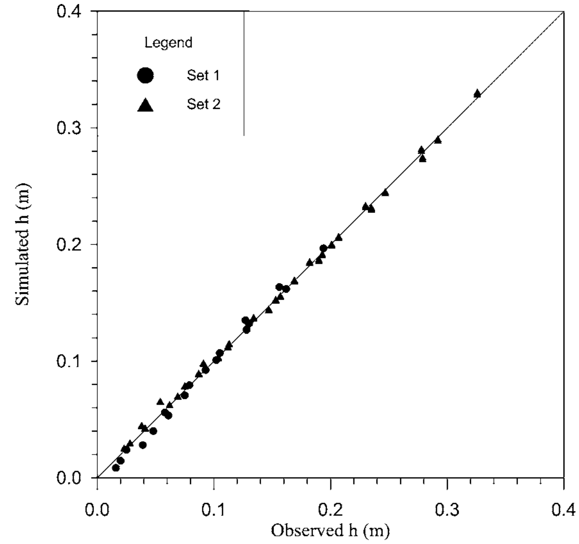

Fig. 2. Comparison of observed and calculated groundwater heads (optimization approach, sets 1 and 2)

Conclusion

The concept of head loss has been proposed for modeling laboratory observations on stream-aquifer interaction. Using this concept, the estimate of aquifer diffusivity is modified. Simple equations have been proposed for estimating modified diffusivity and head loss from the estimates of diffusivity obtained without considering the head loss. An optimization approach has also been proposed for estimating the modified diffusivity and head loss from the observed groundwater heads. The proposed theory has been applied to the observed laboratory data; it correctly models the laboratory data.

Notation

The following symbols are used in this technical note:

c = coefficient of linear stream stage (LT^{-1});
h = groundwater head (L);
h_c = calculated groundwater head (L);
h_o = observed groundwater head (L);
n = number of observations (dimensionless);
p = number of estimated parameters (dimensionless);
t = time measured since the commencement of stream stage rise (T);
x = distance of measurement point from stream aquifer interface (L);
x_l = head loss (L);
β = hydraulic diffusivity of aquifer ($L^2\,T^{-1}$);
β_l = modified hydraulic diffusivity of aquifer ($L^2\,T^{-1}$); and
ξ = ratio of groundwater head and stream-stage rise (dimensionless).

Units

(L) = meter
(T) = minute

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