Limiting risk in environmental problems: Corporate budget constraints and minimum involvement

I. Lerche and E. Paleologos

ABSTRACT

For a given opportunity in which a company can invest to perform environmental remediation for profit, the influences of value, cost, success probability, and corporate risk tolerance provide an optimal working interest (OWI) that should be taken to maximize the risk-adjusted value (RAV). When several opportunities are available, but when the total budget is insufficient to take OWI in each, an analytical procedure is undertaken for optimizing the RAV of the total portfolio; the relevant working interests are also derived based on a cost-exposure constraint. Several numerical illustrations will exhibit the use of the method under different budget conditions and with different numbers of available opportunities. The result is that the computations of portfolio balancing can be done quickly using the analytical expressions presented here, thereby providing rapid assessments of environmental opportunities and their worth.

INTRODUCTION

Environmental remediation projects are characterized by cost overruns, uncertainties in terms of technological or fiscal performance, and the potential of high liability costs that may put at risk a corporation's resources and reputation (Lerche and Paleologos, 2001). Procedures for limiting corporate economic exposure are always involved in the decision to pursue particular available opportunities. Decisions commonly revolve around the fractional involvement (the working interest fraction, W) that a particular corporation would prefer to take in a given opportunity. When involvement in a single project is only considered, Paleologos and Lerche (2000) have provided a procedure that allows the calculation of the fractional involvement that would maximize corporate profits for different contract prices and costs, probabilities of success (or failure), and various levels of corporate tolerance to risk.

DOI:10.1306/eg.04040302009

AUTHORS

I. Lerche ~ *Department of Geological Sciences, University of South Carolina, Columbia, South Carolina 29208*

Ian Lerche is a professor in the Department of Geological Sciences at the University of South Carolina, Columbia. He specializes in physical and economic aspects of gas and oil exploration and has coauthored over 18 books. He is a recipient of, among others, the Levorsen Award from AAPG, the 1991 Nordic Professor of Petroleum Geology Award, and the 1996–1997 French Academy of Science Professor of Geology Award.

E. Paleologos ~ *Department of Geological Sciences, University of South Carolina, Columbia, South Carolina 29208; epal@geol.sc.edu*

Evan K. Paleologos is an associate professor in the Department of Geological Sciences at the University of South Carolina, Columbia. He specializes in ground-water hydrology and decision-making problems. He has coauthored two monographs on environmental risk analysis topics. The author of more than 40 publications, he is serving on the editorial board of two scientific journals and has organized several international conferences.

ACKNOWLEDGEMENTS

The authors would like to thank E. D. Guy for his helpful suggestions that improved our manuscript. Particular thanks go to the Alexander von Humboldt Stiftung and the University of Halle-Wittenberg for their generous support during a sabbatical year at Halle for I. Lerche, which enabled this paper to be completed.

These authors used exponential and parabolic utility models (measures of managerial attitude toward risk) to illustrate how decision makers can evaluate projects of transport and burial of hazardous wastes (Paleologos and Lerche, 1999; 2002) and assess, for a given contract price, whether involvement in a project is desirable, what fraction of the contract should be accepted, and how acceptance could influence corporate profits. For many corporations, it is not easy to assess the accuracy of such a decision made in the absence of some corporate internal group performing an extensive risk-analysis investigation. This luxury is commonly not available to many small and midsized environmental corporations who must make do either with estimates based on guesswork or on practical experience. The problem is exacerbated when several environmental opportunities are available, and the corporation must split its limited budget to undertake some fraction of the projects or to become involved with partners in all of the available opportunities. A corporation commonly needs a quick procedure for determining the worth of becoming involved and the extent to which it should be involved. The objectives of this paper are twofold: first, to provide analytic expressions that would allow, in the face of constrained budget, for the calculation of the working interest of each environmental opportunity so that the total profit returned by all projects is maximized; second, to determine the effect of minimum required involvement for each opportunity. Thus, the analytic method and solution to the problem provided here are such that a simple calculation based on a spreadsheet program such as Excel can easily be constructed well within the financial reach of almost all environmental corporations. Additionally, the ability to provide a rapid investigation of working interest limits for each opportunity means that one can quickly focus on the parameters that, for a range of possible budgets, control the problem.

Although each opportunity presented to, or available to, a corporation can be evaluated in isolation, the difficulty is that most corporations do not have an unlimited budget, so that they cannot take the optimal working interest (OWI) in each opportunity. An OWI is that working interest returning the largest risk-adjusted value (RAV) to the corporation. The RAV is defined as follows. Consider a two-branch decision situation (or lottery L) that can lead either to monetary outcome A or monetary outcome B with equal probabilities ($p = 0.5$). The expected value of this gamble or the value that will be returned if one is involved in a large number of these situations, is $E = (A + B)/2$. An amount CE is termed the certainty equivalent of L (in decision theory terminology; Raiffa, 1997), or RAV (in oil-industry terminology; Lerche and MacKay, 1996) if an individual is willing to exchange the rights to the above lottery for the certain amount CE (Appendix A provides details on the calculation of RAV for the simple case of a two-node decision tree diagram). The RAV provides a measure of the value the corporation can expect from a project when there is corporate reluctance to expose capital to the chance of loss and, as will be discussed later, can be quantified in terms of a risk tolerance, RT, monetary amount for each project.

The reason behind the acceptance of nonoptimal working interest in each project is related to corporate liability. If each project were to fail with probability p_{fi} for the ith project, with a total cost (including not only the cost of undertaking the remediation venture, but also liability costs and insurance costs) of C_i, and if OWI_i were to be taken in each opportunity, then for N such projects, the corporate liability on cost exposure is $CO = \sum_{i=1}^{N} C_i OWI_i$. The amount CO is the total at risk should the corporation have to pay all liability costs on all the N projects with which it is involved. Large corporations with many projects in different phases of development and operating on a continuing, steady, cash-flow basis commonly consider it appropriate to limit corporate liability to cost expenditure, given as $CE = \sum_{i=1}^{N} C_i p_{fi} OWI_i$. The amount CE measures what a corporation should, on average, have to commit, given some estimate of the probability of failure of each remediation project. Most corporations are relatively conservative, however, and commonly insist that working interests W_i be taken such that $CO(W) = \sum_{i=1}^{N} C_i W_i$ be less than or equal to the available budget, B. In such cases, as we shall demonstrate, commonly, one cannot take the OWI in each opportunity, particularly when the budget is small. The problem then is to find some procedure to optimize the total RAV from the N opportunities while staying within the mandated corporate budget B, i.e., the portfolio of opportunities is balanced (Lerche and Paleologos, 2000).

In addition to the limitations imposed by a finite corporate budget, there is also commonly a limit imposed by potential partners in an environmental opportunity. From the partner's side, it is commonly the case that an offer for joint participation is made to a corporation, but with the caveat that a minimum working interest, $W_{i,\min}$, shall be undertaken in the ith project.

Thus, the corporation cannot arbitrarily discard those projects that do not meet its internal criteria for involvement; it must temper those criteria to allow any involvement at all. It can happen that some of the projects offered are not considered to be profitable by the corporation, but involvement in such projects is a necessary evil to participate in the more lucrative deals offered.

In such a situation, for N projects, one has a secondary requirement in place that

$$\sum_{i=1}^{N} W_i C_i \geq \sum_{i=1}^{N} W_{i,\min} C_i = M \tag{1a}$$

and

$$1 \geq W_i \geq W_{i,\min};\; i = 1, \ldots N \tag{1b}$$

where M is cost involvement for minimum working interest for all projects.

Thus, the corporation has to determine the best working interest to take in each of the N projects subject to equations 1a and b and also subject to the budget limitation of

$$B \geq \sum_{i=1}^{N} W_i C_i \tag{2}$$

For later convenience, it is best to combine equations 1a and 2 in the form

$$\sum_{i=1}^{N} W_i C_i = M + (B - M)\sin^2\theta = M\cos^2\theta + B\sin^2\theta \tag{3}$$

where θ is an angle in the range $0 \leq \theta \leq \pi/2$. (More will be said later in this paper about constraints on the angle θ arising when a minimum working interest involvement is required.) Equation 3 automatically ensures that both inequalities 1a and 2 are satisfied. Also assumed is that $B \geq M$, or else there is no involvement possible that will satisfy the corporate budget limitations.

The primary purpose of this paper is to provide an analytic technique for determining the portfolio balancing with a fixed total cost exposure (or cost expenditure) limit for determined input parameter values for each opportunity. The advantages to such an analytic technique are that one can see immediately which factors are playing dominant roles in controlling estimates of worth (something that is difficult to do with a complex computer program); one can perform the relevant estimates quickly on a hand calculator or a simple spreadsheet program and also plot results; one does not need access to a large computer to undertake the estimates; the effect of anthropogenic bias and guesswork are removed in favor of a more rational approach; and the procedure is not dependent on having a few experts in a corporation, who may leave or retire, but instead can be undertaken by almost anyone, thus preserving the knowledge base and also providing an open forum for assessment instead of a black box approach in which one does not know or cannot find out very easily what a particular procedure does to evaluate risk of involvement. For all these reasons, it is appropriate to describe how one does such assessments of risk for environmental projects in a simple manner.

PORTFOLIO BALANCING WITH STATISTICALLY SHARP PARAMETERS

Consider a sequence of opportunities, $i = 1, 2, \ldots, N$, in which RAVs (i, W_i) have been computed for each opportunity as a function of working interest, W_i for a corporate risk tolerance, RT (Appendix A). Parameters related to the value, costs, and success chance of each environmental opportunity are also shown in Figure 1 for ease of presentation of the remainder of the argument given here. The optimum working interest, OWI(i), for each opportunity is calculated so that the maximum RAV, $\mathrm{RAV}_{\max}(i, \mathrm{OWI})$, is also known. In making these estimates, it is taken that value V_i, costs C_i, success probability p_{si}, and risk tolerance RT are all statistically sharp (precisely known). The expected worth of an opportunity is then $E_i = p_{si}V_i - p_{fi}C_i$ (with $p_{fi} = 1 - p_{si}$), whereas the standard error, σ_i, of the expected worth is $\sigma_i = |(V_i + C_i)|\,(p_{si}p_{fi})^{1/2}$ so that the volatility, v_i, of the expected worth is $v_i = \sigma_i/|E_i|$ (see Appendix A).

Relative Importance

The relative importance, RI_i, of the ith opportunity in relation to all other opportunities can be defined either as

$$\mathrm{RI}_i(W_i) = \mathrm{RAV}(i, W_i) \Big/ \sum_{j=1}^{N} \mathrm{RAV}(j, W_j), \quad \text{(unweighted)} \tag{4a}$$

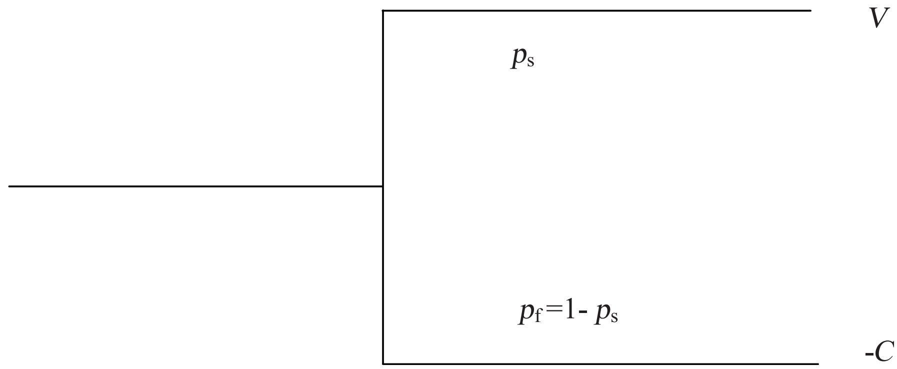

Figure 1. Sketch of a decision tree diagram with notations of value V, costs C, and success and failure probabilities (p_s and $p_f = 1 - p_s$).

or, if weighted inversely with respect to volatility for each project, as

$$\mathrm{RI}_i(W_i) = (\mathrm{RAV}(i, W_i)/\nu_i)/\sum_{j=1}^{N}(\mathrm{RAV}(j, W_j)/\nu_j), \quad \text{(weighted)} \qquad (4b)$$

Here, RAV(i, W) is positive or zero only. In either situation, the relative importance of each opportunity is available.

Profitability

The contribution of each opportunity to the total estimated profitability, P, is then commonly defined through

$$P_i = \mathrm{RAV}(i, W_i) \quad \text{(unweighted)} \qquad (5a)$$

or

$$P_i = (\mathrm{RAV}(i, W_i)/\nu_l)/\sum_{j=1}^{N} 1/\nu_j, \quad \text{(weighted)} \qquad (5b)$$

with the total estimated profit, P, then being given by

$$P = \sum_{j=1}^{N} P_j \qquad (5c)$$

Costs

Maximum costs of each opportunity, for a fractional working interest W_i, are C_iW_i and have to be borne by the corporation. If the opportunity does not succeed, then no future revenues are ever available against which to replenish corporate resources, so a net cost of C_iW_i is the fiscal drain against the corporation. If the opportunity succeeds, then future revenues allow later replenishment of the corporate outlay. However, at the time of committing to the project, the corporation only has a budget B to use with the estimated costs that it knows it must bear. Two limiting cases are available:

1. Cost exposure, defined as total costs

$$\mathrm{CO}(W) = \sum_{i=1}^{N} C_i W_i \qquad (6a)$$

measures the authorization for expenditure and participation in each and every opportunity.

2. Cost expenditure, defined as total probable costs

$$\mathrm{CE} = \sum_{i=1}^{N} C_i\, W_i p_{fi} \qquad (6b)$$

measures the likely amount of cost that one would have to bear on a long-term continuing cash-flow basis once the failure probability of each opportunity is allowed for. Computations will be carried through here on a cost-exposure basis.

BUDGET CONSTRAINTS

A total budget B is available. The problem is to figure out what fraction of the budget should go to each opportunity to maximize the total portfolio of profitability from each. Two cases are analyzed here.

High Budget

Suppose first that the budget is sufficiently high that participation in each opportunity could be at the OWI_i for each opportunity; each project is then maximized in terms of its RAV. The total cost exposure is

$$C_0(\max) = \sum_{i=1}^{N} C_i \mathrm{OWI}_i \qquad (7a)$$

so the budget should be at

$$B \geq C_0(\max) \qquad (7b)$$

The development of RAVs have depended, to a large extent, on exponential utility models of the form $U(x) = 1 - \exp(-x/\mathrm{RT})$ to represent risk-averse preferences (Krzysztofowicz, 1986; Keeney and Raiffa, 1993; Lerche and Paleologos, 2001). A utility function, U, is a way to standardize monetary outcomes and consequences of a decision that are not measurable in monetary terms in the same units, and it encodes the strength of an individual's preferences for specific outcomes and risk attitudes toward the uncertainty of such outcomes. As the dollar amount x becomes large, $U(x)$ tends to 1, $U(0)$ equals 0, and for negative x (losses), the exponential utility function becomes negative. RT is denoted as risk tolerance, and it determines how risk averse a utility function is. Appendix A provides a physical analog of the exponential utility function for a simple two-branch decision-making situation. Based on the exponential model (also denoted here as the Cozzolino (1977a, b) formula) the RAV is given by

$$\mathrm{RAV} = -\mathrm{RT}\ \ln\{p_s \exp(-WV/\mathrm{RT}) + p_f \exp(WC/\mathrm{RT})\} \qquad (8a)$$

Not all corporations model risk attitudes with a risk aversion which is exponentially weighted; alternative formulae for risk aversion are as many and as varied as the individual corporations involved. The parabolic approximation for RAV is more risk averse at high working interest values than the exponential formula 8a, but is almost identical at low working interests. This property of the parabolic RAV formula parallels decisions in the oil industry, in which managers are commonly willing to accept an opportunity to about 50% involvement, even when the risk tolerance is high, but are less eager to accept smaller working interests. The idea behind a parabolic rule is that there is greater stability in the management of high-loss scenarios than there is with the exponential rule. Here, the parabolic rule is investigated corresponding to a risk-aversion factor that can be determined simply by expanding the Cozzolino risk-aversion formula above to quadratic order in the working interest W. In this case, the equivalent to the Cozzolino formula (equation 8a) is the RAV, given through

$$\mathrm{RAV}(W) = WE[1 - \frac{1}{2}\nu^2 W(E/\mathrm{RT})] \qquad (8b)$$

which is equivalent to Cozzolino's formula, but for parabolic weighting of risk aversion instead of exponential. A full proof of the development of equation 8b from expression 8a is provided in Paleologos and Lerche (2000) (Appendix B). Equation 8b can be expressed in a standard parabolic form as

$$\mathrm{RAV}(W) = aW^2 + bW; \quad a = -\nu^2E^2/(2\mathrm{RT}), \quad b = E \qquad (8c)$$

Low Budget

If the total budget, B, is less than total cost exposure, $C_0(\max)$, then OWI cannot be taken in each opportunity. One has to settle for less than optimal profitability. The question is to figure out a procedure for balancing the portfolio of opportunities so as to maximize profitability, recognizing that the profitability will be less than optimal. It is this question that is now addressed subject to the constraints of equation 1b for greater than minimum required working interest, and equation 3 for corporate budget limitations.

FINDING THE BEST WORKING INTERESTS UNDER CONSTRAINED CONDITIONS FOR INVESTMENT

To show how the procedure works at maximizing portfolio balancing, in the next section of this paper, several numerical illustrations of increasing complexity are considered. For the exponential RAV (equation 8a), the maximum value of RAV with respect to working interest, W, is given by

$$\mathrm{OWI} = \frac{\mathrm{RT}}{(C+V)} \ln\left(\frac{p_s V}{p_f C}\right) > 0 \quad \text{if } E_1 > 0 \qquad (9a)$$

and at this value of OWI, the maximum value of RAV is

$$\mathrm{RAV}_{\max} = V\,\mathrm{OWI} - \mathrm{RT}\ln[p_s(1 + V/C)] \qquad (9b)$$

Equations 9a and 9b are derived by setting the derivative of expression 8a equal to zero, solving the resulting equation to obtain the OWI, and then substituting this OWI into equation 8a.

From the properties of parabolic functions (see any standard text on mathematical analysis), using equation 8c, one obtains $W_{\max}$ (for $E > 0$) as

$$\mathrm{OWI} = W_{\max} = -b/2a = \mathrm{RT}/(E\, v^2) \qquad (10a)$$

and on $W = W_{\max}$, the maximum RAV is attained at

$$\mathrm{RAV}_{\max} = -b^2/4a = \mathrm{RT}/(2v^2) \qquad (10b)$$

In this paper, the unweighted procedure will be used, together with the parabolic RAV formula for illustrative purposes only, because analytically exact expressions can be written down for the working interests and the total RAV.

For each opportunity $i = 1, \ldots, N$, there is some best W_i. The task is to obtain an explicit formula describing W_i. The procedure for so doing is as follows. The total RAV of the N projects is

$$\mathrm{RAV} = \sum_{i=1}^{N} \mathrm{RAV}_i(W_i) \qquad (11)$$

where $W_i \geq 0$ and $W_i \leq W_{\max,i}$. It is also taken that the total budget is constrained, and for a parabolic risk-aversion formula, substitution of equation 10b into equation 7a yields

$$B < \mathrm{C}_0(\max) = \mathrm{RT} \sum_{i=1}^{N} (\mathrm{C}_i/(v_i^2 E_i)) \qquad (12)$$

so that optimum working interest in each and every opportunity cannot be taken.

There are N values, $W_1, \ldots, W_N$, to obtain from equation 11. If no constraints were in place, then each W_i would be at its optimum of OWI_i. However, equation 3 poses the constraint of

$$b = \sum_{k=1}^{N} \mathrm{C}_k W_k \qquad (13)$$

where $b = M + (B - M)\sin^2\theta$ (equation 3), together with the constraint that each W shall be less than unity and greater than the required involvement minimum. Equation 13 can be used to write the jth opportunity working interest as

$$W_j = \mathrm{C}_j^{-1} \left\{ b - \sum_{\substack{k=1 \\ k \neq j}}^{N} \mathrm{C}_k W_k \right\} \qquad (14)$$

where, in the summation, the term $k = j$ is to be omitted. Then, rewrite equation 11 as

$$\mathrm{RAV} = \sum_{\substack{m=1 \\ m \neq i,j}}^{N} \mathrm{RAV}_m(W_m) + \mathrm{RAV}_i(W_i) + \mathrm{RAV}_j \left\{ \mathrm{C}_j^{-1}(b - \sum_{\substack{k=1 \\ k \neq j}}^{N} \mathrm{C}_k W_k) \right\} \qquad (15)$$

where, in the first summation, the terms $m = i$ and $m = j$ are to be omitted, and in the second summation, the term $k = j$ is to be omitted.

Inspection of equation 15 shows that only the two end terms involve W_i. Then, the maximum of RAV with respect to W_i occurs when

$$\mathrm{C}_i^{-1} \frac{\partial \mathrm{RAV}_i(W_i)}{\partial W_i} = \mathrm{C}_j^{-1} \frac{\partial \mathrm{RAV}_j(x)}{\partial x} \qquad (16)$$

where $x = \mathrm{C}_j^{-1}(b - \sum_{\substack{k=1 \\ k \neq j}}^{N} \mathrm{C}_k W_k)$. For the parabolic RAV formula, one has from equation 8b

$$\partial \mathrm{RAV}_i(W_i)/\partial W_i = E_i[1 - v_i{}^2 W_i(E_i/\mathrm{RT})] \qquad (17)$$

Hence, equation 16 becomes

$$E_i \mathrm{C}_i^{-1}(1 - W_i v_i^2 E_i/\mathrm{RT}) = E_j \mathrm{C}_j^{-1} \times \left[1 - (v_j^2 E_j/\mathrm{RT}) \mathrm{C}_j^{-1}(b - \sum_{\substack{k=1 \\ k \neq j}}^{N} \mathrm{C}_k W_k) \right] \qquad (18a)$$

i.e.,

$$E_i \mathrm{C}_i^{-1}(1 - W_i v_i^2 E_i/\mathrm{RT}) = E_j \mathrm{C}_j^{-1}(1 - W_j v_j^2 E_j/\mathrm{RT}) \qquad (18b)$$

However, i and j are arbitrary choices; hence, each side of equation 18b must equal a constant H, independent of i or j. Solving equation 18b for W_i yields

$$W_i = \mathrm{RT}(1 - \mathrm{C}_i H E_i^{-1})/(v_i^2 E_i), \qquad i = 1, \ldots, N \quad (19)$$

Then, substituting equation 19 into equation 13 and solving for the constant H yields

$$H = \left[\sum_{j=1}^{N} \frac{C_j}{v_j^2 E_j} - b/\mathrm{RT}\right] \Big/ \left[\sum_{j=1}^{N} \frac{C_j^2}{v_j^2 E_j^2}\right] \qquad (20)$$

Substituting the above expression for H in equation 19 allows rewriting equation 19 for W_i as a function of the financial and statistical parameters of the projects only

$$W_i = \mathrm{RT}(E_i v_i^2)^{-1}\left\{1 - (C_i/E_i)\left[\sum_{j=1}^{N}(C_j/(v_j^2 E_j)) - b/\mathrm{RT}\right] \Big/ \sum_{j=1}^{N}(C_j^2/(v_j^2 E_j^2))\right\} \qquad (21)$$

provided $W_{i,\min} \leq W_i \leq \min\{2\mathrm{OWI}_i, 1\}$ and under the low budget constraint. Inspection of equation 21 shows that the budget constraint, equation 3, is just the requirement that

$$\sum_{j=1}^{N}(C_j/v_j^2 E_j) > b/\mathrm{RT} \qquad (22)$$

and, when this inequality is in force, then each $W_i \leq \mathrm{OWI}_i$.

Substituting equation 21 for W_i into the requirement that $W_i \geq W_{i,\min}$ and rearranging terms, one obtains the inequality

$$b/\mathrm{RT} \geq \alpha_i \equiv \sum_{j=1}^{N}(C_j/(v_j^2 E_j)) - \left(\sum_{j=1}^{N} C_j^2/(v_j^2 E_j^2)\right) \times (E_i/C_i)\,(1 - W_{i,\min} E_i v_i^2/\mathrm{RT}) \qquad (23)$$

Now remember that $b = M\cos^2\theta + B\sin^2\theta$ (equation 3), and that M is the sum of $W_{i,\min}C_i$ (equation 1a) over all the i opportunities. Hence, by rearrangement, one can sum equation 23 over all projects and so provide the global constraint on the angle θ that

$$1 \geq \sin^2\theta \geq \frac{a(1 - 1/d)\mathrm{RT}}{(B - M)} \qquad (24a)$$

where

$$a = \sum_{i=1}^{N} C_i/(v_i^2 E_i), \quad \text{and} \quad d = \sum_{i=1}^{N}(C_i/v_i E_i)^2 \qquad (24b)$$

The outer inequalities of equation 24a then set a limit on the sum of the minimum working interests to make the projects at all palatable to the corporation. One has

$$B - a(1 - 1/d)\mathrm{RT} \geq M \qquad (25)$$

If inequality 25 cannot be satisfied, then there is no way that the sum of all projects can be at all profitable with the minimum working interest constraint in place. Hence, order the summations with respect to α_i (defined through equation 23) with $\alpha_1 < \alpha_2 < \alpha_3 \ldots < \alpha_N$. Then, as the budget, B, is systematically decreased, $W_N = W_{N,\min}$ when $B = \mathrm{RT}\alpha_N$, and the Nth opportunity is removed from consideration. As B systematically decreases, in turn, W_{N-1}, W_{N-2} etc. reach their minimum values, and those opportunities are discarded. Thus, at any given budget, it is relatively simple to determine which opportunities should be invested in and also the working interest that should be taken.

The analytical exact formula for determining W_i, as given through equation 21, then maximizes the total RAV of all the N opportunities under the fixed budget constraint. Thus, as the budget decreases from the maximum of $B_{\max} = \sum_{i=1}^{N} C_i\mathrm{OWI}_i$ (at which OWI can be taken in each and every opportunity) to $B = 0$, the various opportunities are steadily discarded in order of their values of α. The general argument, although explicitly developed here for unweighted RAV formulae, is also appropriate if any weighting is done. The logic proceeds as above mutatis mutandis (see Appendix B).

NUMERICAL ILLUSTRATIONS

Optimization with Zero Working Interest Minimum

Consider three opportunities A, B and C with the following characteristic parameters:

Opportunity A: V = \$110 million; $p_s = 0.5 = p_f$; C C = \$10 million; RT = \$30 million.
Opportunity B: V = \$200 million; p_s = 0.5; p_f = 0.5; C = \$100 million; RT = \$30 million.
Opportunity C: V = \$300 million; p_s = 0.4; p_f = 0.6; C = \$120 million; RT = \$30 million.

Opportunities A and B each have a mean value of E = \$50 million, whereas opportunity C has E = \$48 million. Using the parabolic formula, opportunity A has $RAV(A)_{max}$ = \$10.417 million at an OWI(A) = 0.417, whereas opportunity B has $RAV(B)_{max}$ = \$1.667 million at OWI(B) = 0.067, whereas opportunity C has $RAV(C)_{max}$ = \$0.816 million at OWI(C) = 0.034, for a total possible maximum RAV of \$12.9 million.

Note that the working interest minimum is greater than the OWI for projects B and C. Then, one has to ask whether project A will provide sufficient return on its own that its potential gains will more than offset potential losses from projects B and C. Accordingly, it is a good strategy to carry through the analysis at first, as though the minimum working interest offered is zero. This way, one can see where a particular offer would lie relative to the available budget, risk tolerance, and intrinsic worth of each available project.

Optimization of the total RAV at different budgets has been carried out under two conditions: (i) using the analytic formula for parabolic RAV given in the preceding section and (ii) comparing the results obtained from the analytic formula with those arising from a Simplex solution solver (Hillier and Lieberman, 1980), which has been the standard tool for optimization operations in the oil industry. Both procedures yielded identical results under all conditions addressed, but the analytic method is much faster numerically than the Simplex search method.

A Budget of \$20 Million

The value of C × OWI is \$4.16 million for opportunity A, \$6.667 million for opportunity B, and \$4.082 million for opportunity C, for a total cost of $C_i OWI_i$ = \$14.915 million. Thus, the budget of \$20 million exceeds the total costs at the optimum working interest for each opportunity, so that OWI is taken in each at a cost of \$14.915 million; a budget return of \$5.085 million to the corporate coffers is then made.

A Budget of \$11 Million

In this case, the budget is less than the total needed to invest in each opportunity at its OWI, but is large enough so that no single opportunity should be discounted. Both the parabolic analytic formula and the Simplex method return the optimum values of

W(A) = 0.4033	RAV(A) = \$10.406 million	C(A)*W*(A) = \$4.0328 million
W(B) = 0.0452	RAV(B) = \$1.4946 million	C(B)*W*(B) = \$4.5248 million
W(C) = 0.0204	RAV(C) = \$0.6847 million	C(C)*W*(C) = \$2.4424 million

for a total RAV of \$12.585 million. In this case, note that the optimization of total RAV has kept the working interest in opportunity A very close to its OWI and has kept the involvement in opportunity B at about 3/4 of the OWI, but has dropped the involvement in opportunity C to about 2/3 of its OWI, in line with *E*/C being smallest for opportunity C, so that the lower budget is forcing the working interest in opportunity C closer to zero first.

A Budget of \$4 Million

In this case, the threshold value of *E*/C = 0.3 has been crossed for opportunity C, which is therefore discounted completely. The budget is then split between opportunities A and B in the proportions

W(A) = 0.3765	RAV(A) = \$10.3 million	C(A)*W*(A) = \$3.7647 million
W(B) = 0.0024	RAV(B) = \$0.115 million	C(B)*W*(B) = \$0.2353 million

for a total RAV of \$10.415 million. In this case, because opportunity B has a lower *E*/C (= 0.5) than opportunity A (= 5), the lower budget forces less involvement in opportunity B. The total maximum RAV for both opportunities is \$12.9 million, so that portfolio balancing yields 97.6% (at *B* = \$11 million) and 80.9% (at *B* = \$4 million) of the maximum by optimizing the budget fractions allocated to each opportunity.

Optimization with a Finite Working Interest Minimum

Although the above illustration indicates the best allocation of funds that a corporation could make between the three projects under different budgets, it does not include the limitation imposed by the requirement of minimum working interest involvement.

Suppose then that all parameters for each of the above projects are kept as above, with the sole exception of a required minimum working interest of 5%. Thus, if the corporation wishes to become involved in the more lucrative opportunity A at greater than 5%, then the corporation must also take at least a 5% minimum interest in the less lucrative opportunities B and C. Because the optimum working interest for opportunity C is only 3.4%, it follows that a minimum 5% requirement will lower the RAV for opportunity C from its maximum possible. At 5% working interest, the cash involvement in opportunity C is then \$6 million, which is well in excess of the \$4.082 million that would have been committed at the optimum working interest. Thus, for a budget of \$17 million, there would then be only \$11 million remaining to be spent on opportunities A and B. If one was to ignore the minimum working interest requirement again for the moment, then a residual budget of $11 million split between opportunities A and B means that one would take only 4.25% of opportunity B as shown in subcase c of the previous section. However, such a low fractional interest is not permitted, and one must take at least 5%. Hence, from the residual budget of $11 million, one must spend at least $5 million on opportunity B, leaving a residual budget of $6 million that can be spent on opportunity A. Now, the optimum working interest for opportunity A is 41.7%, and the total project cost is estimated at $10 million, so that a maximum of $4.17 million should be committed to opportunity A, leaving a residual budget of $1.83 million uncommitted. The result is that for the budget of $17 million, one is forced to take a minimum position of 5% in projects B and C, whereas an optimal position of 41.7% can still be taken in opportunity A.

The simple example given here indicates sharply how the extra requirement of a minimum position in each opportunity alters the distribution of budget between the three projects. The result is not the optimal choice that the corporation would like to make in the sense of returning the largest potential RAV to the corporation from all three projects, but it is the best that can be achieved with the minimum working interest requirement in force. Other budget values can be worked through in similar vein, so that a simple spreadsheet calculation will quickly generate an evaluation of all projects under any constraint for each project.

CONCLUSIONS

This paper has served two purposes: first was the need to obtain analytic expressions for optimizing total RAV for a portfolio of environmental opportunities in the face of a constrained budget; second was the need to determine the effect of minimum required involvement for each opportunity.

In respect of the first purpose, for a parabolic profile of RAV versus working interest for each opportunity, it was shown that a closed form expression could be written down exactly for the working interest that should be taken in each opportunity to maximize the total RAV under a constrained budget condition. The procedure developed operates with any functional form chosen for RAV versus working interest. Numerical illustrations of the procedure indicated the pragmatic operation of the method (which is extremely fast numerically compared to the more conventional Simplex solution solver methods that have been used to date).

In respect of the second purpose, the ability to provide a rapid investigation of working interest limits for each opportunity means that one can quickly focus on which parameters need to be addressed if one is to narrow the contribution to the total RAV for a given budget or, indeed, for a range of possible budgets.

The methods presented here can be used with any functional form of RAV dependence on working interest to provide simple expressions for the working interest to be taken to optimize a portfolio of opportunities. It is this fact that the numerical illustrations have been designed to exhibit.

APPENDIX A

Cozzolino (1977a,b; 1978) constructed his RAV formula based on utility theory applied to a chance node decision tree diagram, such as that given in Figure 1. The notation of Figure 1 is as follows: V = value (positive); p_s = probability of success; C = cost of failure (positive); p_f = probability of the project not succeeding ($p_f = 1 - p_s$). At the chance node point of Figure 1, the expected value or weighted average of the two possible outcomes of the project is

$$E_1 = p_s V - p_f C \tag{A1}$$

which is positive, provided that $p_s V > p_f C$. The second moment of the project value is $E_2 = p_s V^2 + p_f C^2$, so that a measure of the uncertainty in the outcome is provided by the variance

$$\sigma^2 = E_2 - E_1^2 = p_s p_f (V + C)^2 \tag{A2}$$

A measure of risk is commonly assigned by the volatility, ν, defined by

$$\nu = \sigma/E_1 = (V + C)(p_s p_f)^{1/2}/(p_s V - p_f C) \tag{A3}$$

which evaluates the stability of the estimated mean value, E_1, relative to the fluctuations about the mean. A small volatility ($\nu << 1$) implies that there is little uncertainty in the expected value, whereas a large volatility ($\nu >> 1$) implies a considerable uncertainty in the expected value.

Although the expected value, E_1, of a project may be high, and the volatility small, nevertheless, it can be the case that if failure does occur, then the total project costs, C, may be so large as to bankrupt or cause serious financial damage to the corporation. Under such conditions, it makes corporate sense to take less than 100% working interest in the project. A smaller fraction of the project will cut potential gains, but will also cut catastrophic potential losses. Thus, with a working interest fraction, W, the expected value to the corporation at the chance node is

$$E_1(W) = p_s(WV) - p_f(WC) \tag{A4}$$

on the assumption that fractional working interest does not change the probabilities of success or failure of the project. The effective corporate value is reduced from V to WV, whereas the potential losses are reduced from C to WC.

Cozzolino's (1977a,b; 1978) determination of RAV in relation to the risk tolerance (= risk threshold), RT, of a corporation can be given simply using an analogy from geochemistry. Imagine two energy states that exist with activation energies $E_1 = WV$ and $E_2 = -W\mathrm{C}$. At a given temperature, T, the rate at which a compound can be lost by decay to the state E_1 is given by the Arrhenius formula

$$p_s \exp(-E_1/RT) \quad \text{(A5a)}$$

where R is the gas constant, and p_s is the probability of the reaction pathway, being along the path determined by energy state E_1. Decay along the path of E_2 is then proportional to

$$p_f \exp(-E_2/RT) \quad \text{(A5b)}$$

and because only two paths exist, $p_s + p_f = 1$. Thus, the total decay is then proportional to

$$p_s \exp(-E_1/RT) + p_f \exp(-E_2/RT) \quad \text{(A5c)}$$

If one were to represent the total decay of equation A5c by an equivalent activation energy, RAV, through a single equivalent pathway, then based on the Arrhenius formula, one obtains

$$\exp(-\mathrm{RAV}/\mathrm{RT}) = p_s \exp(-E_1/\mathrm{RT}) + p_f \exp(-E_2/\mathrm{RT}) \quad \text{(A6)}$$

Hence,

$$\mathrm{RAV} = -\mathrm{RT} \ln\{p_s \exp(-WV/\mathrm{RT}) + p_f \exp(W\mathrm{C}/\mathrm{RT})\} \quad \text{(A7)}$$

The intrinsic assumption here is that the activation energy rates of conversion are controlled by the exponential Arrhenius formula. With RT understood as risk tolerance, equation A7 is Cozzolino's formula relating estimated RAV to risk tolerance, RT, working interest, W, and the value, V, cost, C, and probabilities of success/failure, with $p_s + p_f = 1$.

Maximum Working Interest

Note that for a given value of RT, equation A7 has a maximum value of RAV with respect to working interest, W, when W takes on the value

$$W_{\max} = \frac{\mathrm{RT}}{\mathrm{C}+V} \ln\left(\frac{p_s V}{p_f \mathrm{C}}\right) > 0 \quad \text{if } E_1 > 0 \quad \text{(A8)}$$

and at this value of $W_{\max}$, the maximum value of RAV is

$$\mathrm{RAV(max)} = -\mathrm{RT} \ln\left\{p_s \left(\frac{p_f \mathrm{C}}{p_s V}\right)^{V/(V+\mathrm{C})} + p_f \left(\frac{p_s V}{p_f \mathrm{C}}\right)^{\mathrm{C}/(V+\mathrm{C})}\right\}$$
$$= V W_{\max} - \mathrm{RT} \ln[p_s(1 + V/\mathrm{C})] \quad \text{(A9)}$$

Note that the requirement $0 \le W \le 1$ (no less than 0% or more than 100% working interest can be taken in a project) then implies, from equation A8, that

$$p_s \mathrm{C} < p_s V < p_f \mathrm{C} \exp((\mathrm{C}+V)/\mathrm{RT}) \quad \text{(A10)}$$

If inequality A10 is not satisfied, then RAV does not have a maximum in the range $0 \le W \le 1$, so that RAV is then either monotonically increasing or decreasing as W increases from zero to unity.

As W tends to zero, then, from equation A7, one has RAV(W = 0) = 0 and

$$\left.\frac{\mathrm{dRAV}}{\mathrm{d}W}\right|_{W=0} = p_s V - p_f \mathrm{C} > 0 \quad \text{(A11)}$$

Thus, RAV is positive, increasing at small W provided $p_s V > p_f \mathrm{C}$, i.e., the expected value at the chance node of Figure 1 is positive. In such a case, because there is no maximum in the range requirement $0 \le W \le 1$, the largest positive value of RAV occurs at the maximum $W = 1$, indicating that 100% interest in the project should be taken. Equally, if $p_s V < p_f \mathrm{C}$, then RAV is negative, decreasing throughout $0 \le W \le 1$, indicating that the project should not be invested in at all. When inequality A10 is satisfied, then there is a range of values of W around $W_{\max}$ where RAV can be positive, so that some positive risk-adjusted return is likely even if a working interest is taken other than that which maximizes the RAV.

Apparent Risk Tolerance

By rearrangement, equation A8 can be used in a different manner in the form

$$\mathrm{RT} = W_{\max}(\mathrm{C}+V)/\ln(p_s V/p_f \mathrm{C}) \quad \text{(A12)}$$

If an arbitrary working interest value, W, is used in equation A12 to replace the optimum value $W_{\max}$, the apparent risk tolerance, $\mathrm{RT_A}$, is then given by the left side of equation A12. This apparent risk tolerance expresses the ability to see to what extent a corporate mandate of risk tolerance has over-risked or under-risked a particular project, or the extent to which a particular working interest choice permits the apparent risk tolerance to be in reasonable accord with the corporate-mandated value. It is particularly useful in determining what the corporate attitude toward risk tolerance is based on prior working interest decisions.

Break-Even Working Interest

The break-even value of RAV is conventionally set to zero, which, for the Cozzolino formula A7, occurs at a working interest W_o, determined from

$$p_f \exp(W_o(\mathrm{C}+V)/\mathrm{RT}) + p_s - \exp(W_o V/\mathrm{RT}) = 0 \quad \text{(A13a)}$$

provided $0 \le W_o \le 1$.

For $V >> \mathrm{C}$, equation A13a has the approximate solution

$$W_o = 2\mathrm{RT}\, V^{-1} \ln\{1 + (3\mathrm{C}p_f)/Vp_s\} \quad \text{(A13b)}$$

so that $W_o < 1$ in $\mathrm{RT} \le 0.5V[\ln\{1 + (3\mathrm{C}p_f/Vp_s)\}]^{-1}$.

Maximum Risk Tolerance

Occasionally, a corporation requests that a particular fixed working interest be taken. The question then is how does the RAV relate to the risk tolerance. In this situation, RAV has a maximum, RAV(max), with respect to RT at a value of $\mathrm{RT_m}$ given through

$$\mathrm{RT_m} = \frac{W(-p_s V \exp(-WV/\mathrm{RT_m}) + p_f \mathrm{C} \exp(W\mathrm{C}/\mathrm{RT_m}))}{\ln\{p_s \exp(-WV/\mathrm{RT_m}) + p_f \exp(W\mathrm{C}/\mathrm{RT_m})\}} \quad \text{(A14)}$$

with

$$\mathrm{RAV(max)} = W[p_s V \exp(-WV/\mathrm{RT_m}) - p_f C \exp(WC/\mathrm{RT_m})] \quad \text{(A15)}$$

The nonlinearity of equation A14 with respect to $\mathrm{RT_m}$ precludes an analytic expression being available, expressing $\mathrm{RT_m}$ in terms of W, V, C, p_s, and p_f, but simple values for RAV versus RT (for fixed values of the remaining parameters) programmed in a simple spreadsheet program can be used to estimate quickly whether the risk tolerance for a required working interest is less than the corporation limit.

APPENDIX B: WEIGHTED RAV OPTIMIZATION

In the body of the paper, the total RAV was maximized with each opportunity having its RAV added arithmetically to the total. However, corporations commonly weigh the relative RAV contributions. The purpose of this appendix is to show that the optimization of the portfolio can be carried through equally with weighting factors.

Thus, if the weights assigned are $\sigma_i > 0$ ($i = 1, \dots N$) for $\mathrm{RAV}_i(W_i)$, then the total weighted RAV is

$$\mathrm{RAV} = \sum_{i=1}^{N} \sigma_i \mathrm{RAV}_i(W_i) / \sum_{j=1}^{N} \sigma_j \quad \text{(B1)}$$

The budget constraint could also be weighted if done more on a cash-flow basis instead of a fixed budget basis. Let that weighting be $\rho_i > 0$ ($i = 1, \dots, N$) in the sense that the constraint to be applied is

$$\sum_{i=1}^{N} C_i \rho_i W_i = b \quad \text{(B2)}$$

Then, following the procedure of the main body of the text (see equation 16), the optimum RAV occurs when

$$\frac{\sigma_i}{\rho_i C_i} \frac{\partial \mathrm{RAV}_i(W_i)}{\partial W_i} = H = \text{constant}, \quad \text{all } i \quad \text{(B3)}$$

For the parabolic formula 8b using equation 10a, for OWI, one has

$$\frac{\partial \mathrm{RAV}_i(W_i)}{\partial W_i} = E_i[1 - W_i/\mathrm{OWI}_i] \quad \text{(B4)}$$

Substituting equation B4 into equation B3 yields

$$W_i = \mathrm{OWI}_i \left(1 - \frac{\rho_i C_i}{\sigma_i E_i} H\right) \quad \text{(B5)}$$

Substituting equation B5 into equation B2 gives

$$\sum_{i=1}^{N} C_i \rho_i \mathrm{OWI}_i - H \sum_{i=1}^{N} \frac{C_i^2 \rho_i^2}{\sigma_i E_i} \mathrm{OWI}_i = b \quad \text{(B6)}$$

which determines H in terms of the budget B and the minimum working interest requirement M of the text. A similar analysis can be carried through for any weighting applied to any functional form chosen for the dependence of RAV on working interest.

REFERENCES CITED

Cozzolino, J. M., 1977a, A simplified utility framework for the analysis of financial risk, *in* Economics and Evaluation Symposium of the Society of Petroleum Engineers, Dallas, Texas, February 21, 1977: Society of Petroleum Engineers no. 6359, 161 p.

Cozzolino, J. M., 1977b, Management of oil and gas exploration risk: West Berlin, New Jersey, Cozzolino Associates, 155 p.

Cozzolino, J. M., 1978, A new method for measurement and control of exploration risk, *in* Society of Petroleum Engineers of the American Institute of Mining, Metallurgical and Petroleum Engineers, March 1978, Society of Petroleum Engineers no. 6632, p. 72–81.

Hillier, F. S., and G. J. Lieberman, 1980, Introduction to operations research, 3d ed.: Oakland, Holden-Day, 829 p.

Keeney, R. L., and H. Raiffa, 1993, Decisions with multiple objectives: Preferences and value trade-offs: Cambridge, Cambridge University Press, 569 p.

Krzysztofowicz, R., 1986, Expected utility, benefit, and loss criteria for seasonal water supply planning: Water Resources Research, v. 22, p. 303–312.

Lerche, I., and J. A. MacKay, 1996, Portfolio balancing and risk adjusted values under constrained budget conditions: Energy Exploration Exploitation, v. 14, p. 197–225.

Lerche, I., and E. K. Paleologos, 2000, Optimal involvement in multiple environmental projects under budgetary constraints: Journal Stochastic Environmental Research & Risk Assessment, v. 14, p. 371–383.

Lerche, I., and E. K. Paleologos, 2001, Environmental risk analysis: New York, McGraw-Hill, 437 p.

Paleologos, E. K., and I. Lerche, 1999, Multiple decision-making criteria in the transport and burial of hazardous and radioactive wastes: Journal Stochastic Environmental Research & Risk Assessment, v. 13, p. 381–395.

Paleologos, E. K., and I. Lerche, 2000, Working interest optimization in the transport and burial of hazardous wastes: Journal Environmental Geosciences, v. 7, no. 2, p. 106–114.

Paleologos, E. K., and I. Lerche, 2002, Option coverage techniques for environmental projects: Journal Management in Engineering, American Association of Civil Engineers, v. 18, p. 3–6.

Raiffa, H., 1997, Decision analysis: Introductory lectures on choices under uncertainty: New York, McGraw-Hill, 307 p.