Adaptive Energy-Efficient Spectrum Probing in Cognitive Radio Networks

Zesheng Chen and Chao Chen
Department of Engineering
Indiana University - Purdue University Fort Wayne, Indiana 46805
Email: {zchen, chen}@engr.ipfw.edu

***Abstract*—In cognitive radio networks, secondary users must constantly probe the spectrum to promptly detect the arrival and the departure of primary users (PUs). However, spectrum probing is an energy-consuming process. This indicates the tradeoff between the frequency of spectrum probing and the delay of detecting the PU state change, and highlights the need for energy-conscious spectrum-probing strategies. In this paper, we provide a theoretical framework to find the optimal spectrum-probing methods that minimize the probing delay under a constraint on energy consumption in real stochastic environments. Specifically, we find that the most widely used spectrum-probing scheme, *i.e.,* periodic probing, is not optimal when the arrival rate of the PU state change is not constant or when the distribution of PU channel occupancy/vacancy is not uniform. On the other hand, the derived optimal strategies can adapt to the dynamics of PUs and choose the probing intervals based on the time-varying arrival rate of the PU state change or the non-uniform distribution of PU channel occupancy/vacancy. Our simulation results show that the optimal spectrum-probing strategies perform much better and consume much less energy than periodic probing in realistic environments.**

I. Introduction

In a cognitive radio network, licensed users or primary users (PUs) allow unlicensed users or secondary users (SUs) to access the licensed spectrum opportunistically when PUs are not transmitting. To allow for the coexistence of PUs and SUs, an important condition is that SUs can monitor channels and promptly detect the arrival of PUs, avoiding the harmful interference to the transmission of PUs. Moreover, when PUs transit from active to dormant states, SUs should be able to sense the availability of the spectrum as soon as possible to effectively use the precious channel resource. Therefore, how fast a SU can respond to the arrival and the departure of PUs is an important metric to the design of cognitive radio networks. For example, IEEE 802.22 specifies that SUs should be able to detect the appearance of PUs within 2 seconds with low probabilities of misdetection and false alarm [19].

This work focuses on when a SU should schedule its channel scans to detect the PU state change in a timely manner. Traditionally, the term “spectrum sensing” is used to specify the underlying physical and MAC techniques that detect channel opportunities [1], [18], [8]. To avoid confusion, in this paper we apply the term “spectrum probing” to specifically refer to a scheduling strategy that a SU uses to constantly probe the spectrum to discover transmission opportunities.

Spectrum probing is an energy-consuming process. Each probe consumes a certain amount of energy [1]. As a result, it is not desired to have a SU keep probing the spectrum all the time when PUs are present. One way to conserve energy is to prolong the time interval between spectrum probes. However, this would introduce latency for SUs to discover channel opportunities. Therefore, frequent spectrum probing can be energy inefficient, whereas infrequent spectrum probing can lead to the long delay to respond to the PU state change. That is, there is obviously a tradeoff between the frequency of spectrum probing (*i.e.*, energy consumption) and the delay of detecting the arrival or the departure of PUs.

In cognitive radio networks, the most widely used probing method is periodic probing that SUs scan the spectrum at a constant interval. In our previous work [3], we have shown that for a single SU, periodic probing is an optimal method. That is, given the same power budget on spectrum probing, periodic probing can achieve the minimum delay to detect the PU state change. The optimality of periodic probing, however, relies on two key assumptions: 1) The arrival rate of the PU state change is constant, and 2) the distribution of PU channel occupancy or vacancy is uniform. It is clearly that these two assumptions are not valid in real environments [17], [4]. First, the arrival rate of the PU state change is time-varying [17]. Taking TV channels as an example, PUs are much less active late at night than in the daytime. Second, real measurements demonstrate that the PU channel occupancy/vacancy distribution is not uniform [4]. Hence, questions arise: In real stochastic environments, is periodic probing still the optimal energy-efficient scheme? If not, what is the optimal spectrum-probing strategy?

The goal of this work is to provide a theoretical framework on the optimal spectrum-probing strategy that a SU can use to effectively and efficiently detect the PU state change in real environments. To achieve this goal, we formulate an optimization problem that minimizes the delay to detect the PU state change given a constraint on energy consumption. Equivalently, the derived optimal strategy minimizes energy consumption given the delay performance constraint. We discover that under realistic conditions, the periodic probing scheme is not optimal and cannot adapt to the dynamic behaviors of PUs. On the other hand, our proposed optimal strategy adaptively chooses the probing intervals based on the time-varying arrival rate of the PU state change or the non-uniform distribution of PU channel occupancy/vacancy. That is, a smart SU should probe the channels more frequently for the time slots when the PU changes its state more frequently or with a higher probability. For example, a SU can probe the spectrum at large intervals late at night and use more energy for probing in the daytime to catch the dynamics of TV channels. Through simulation study, we find that the optimal spectrum-probing scheme performs much better than periodic probing. For example, our simulation results show that depending on the environment conditions and the parameter setting, the energy saving in spectrum probing can be as much as 29% to 82% if we switch the spectrum-probing method from periodic probing to the optimal scheme. Although the optimal scheme requires the knowledge on the arrival rate of the PU state change or the distribution of PU channel occupancy/vacancy, which may not be available or accurate in practice, our theoretical results can provide the performance bounds and aid in the design of practical adaptive energy-efficient spectrum-probing strategies.

The remainder of this paper is structured as follows. Section II introduces the system model and problem definition. Section III revisits the optimality of periodic probing under certain conditions. Sections IV and V derive and evaluate the optimal probing scheme when the arrival rate of the PU state change is not constant, and when the distribution of PU channel occupancy/vacancy is not uniform, respectively. Finally, Section VI discusses the related work, and Section VII concludes this paper.

II. Spectrum Probing in Cognitive Radio Networks

A. System Model

In a cognitive radio network, a SU senses the spectrum to detect the presence/absence of PUs. The detection methods of PUs' signals include energy detection, matched filter detection, and cyclostationary feature detection [1], [18]. In this work, we are not interested in *how* a SU can detect PUs' signals, but *when* a SU should sense channels to promptly find the arrival/departure of PUs and efficiently save the probing energy. Therefore, the optimal spectrum-probing schemes proposed in this paper can complement and apply to any physical and MAC spectrum-sensing mechanisms.

Spectrum probing aims to detect the availability of a channel in a timely manner. Similar to other works [7], [8], [9], [4], we model a channel as a renewal process alternating between ON (*i.e.*, occupancy) and OFF (*i.e.,* vacancy) states. As shown in Figure 1, we use T_V and T_O to represent the sojourn times of ON and OFF states. When a PU changes from ON to OFF at time t_0, if a SU probes the spectrum successfully at time t_1, then the SU can detect the PU state change with the delay $D = t_1 - t_0$. Since the behavior of the PU is not deterministic, *i.e.,* T_V and T_O are random variables, D is also a random variable. Hence, a good metric for evaluating a spectrum-probing scheme is the average of D, *i.e.,* $\mathrm{E}[D]$. In this work, we call such a mean delay the "probing delay," which characterizes how fast a probing method can respond to the PU state change (*i.e.,* ON to OFF or OFF to ON).

To shorten the probing delay, one way is to reduce the time interval between spectrum probes. However, probes consume energy. In this work, we assume that each spectrum probe uses the same amount of energy. With the same constraint on the probing delay, the spectrum-probing strategy that uses fewer probes (*i.e.,* a longer average spectrum-probing interval) is more energy efficient. Moreover, similar to the work [8], we assume that each spectrum probe consumes only a small amount of time and can be regarded as an impulse. Furthermore, we assume that compared with T_V or T_O, the spectrum-probing interval is much smaller so that a PU state change is never missed.

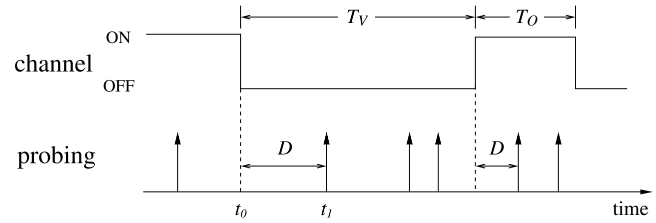


Fig. 1. Illustration of spectrum probing.

To model the sojourn times of a channel, we assume that T_V and T_O are independent and identically distributed (i.i.d.), and use random variable Y to represent them. However, the optimization framework proposed in this paper can easily be extended to the case when T_V and T_O have different distributions. There are commonly two ways to characterize Y. One concise way is to use the average, *i.e.*, $\mathrm{E}[Y]$, representing the expected inter-change time of PU states. If we set λ as the arrival rate of the PU state change, then $\mathrm{E}[Y] = 1/\lambda$. Note that if λ is constant, the distribution of PU state change times is stationary; otherwise, it is nonstationary. Another way to describe Y is based on $f_Y(y)$, *i.e.,* the probability density function of PU channel occupancy/vacancy. Such a distribution can be uniform, exponential, Pareto, or other distributions. In this paper, we attempt to derive the optimal spectrum-probing strategies when λ is either constant or time-varying, and when $f_Y(y)$ follows an arbitrary distribution.

B. Problem Definition

In cognitive radio networks, the default spectrum-probing scheme is periodic probing, which sends out probes at a constant interval. Given a constraint on energy consumption, would periodic probing be the optimal scheme that minimizes the probing delay? Specifically, we study the following problems in this paper:

- Under what conditions is periodic probing the optimal scheme, and why?
- If $\mathrm{E}[Y]$ (*i.e.,* λ) varies with time, what is the optimal spectrum-probing scheme?
- If $f_Y(y)$ follows an arbitrary distribution, what is the optimal spectrum-probing scheme?

III. Optimality of Periodic Probing

In this section, we revisit the optimality of periodic probing. Different from our previous work in [3], here we emphasize the conditions under which periodic probing is optimal, and use a more intuitive and general method to prove the optimality of periodic probing under such conditions.

A. Periodic Probing

Periodic probing senses the spectrum once every T seconds periodically, *i.e.*, the spectrum is probed at time instants $0,\ T,\ 2T,\ \cdots$. We first consider when spectrum sensing is reliable. That is, every probe can successfully detect the presence/absence of the PU. We use A to denote the time when the PU changes its state. Analytically, we assume that random variable A is uniform over $[0, T]$, *i.e.*,

$$f_A(x) = \begin{cases} \frac{1}{T}, & 0 \leq x \leq T \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Since periodic probing takes $D_P = T - A$ seconds to detect the PU state change, the probing delay is

$$\mathrm{E}[D_P] = \mathrm{E}[T - A] = \int_0^T \frac{1}{T}(T - x)dx = \frac{T}{2}. \quad (2)$$

That is, on average periodic probing takes $T/2$ seconds to find the arrival or the departure of the PU.

Next, we study the probing delay of periodic probing when spectrum sensing is unreliable due to various reasons such as wireless channel path loss or shadowing [5]. Specifically, we assume that spectrum sensing can correctly detect the presence/absence of the PU with probability p $(0 < p \leq 1)$ and spectrum probes are independent. Since sensing is unreliable, a SU may need to use multiple probes to detect the PU state change, as shown in Figure 2. We use X to denote whether the first probe can successfully detect the PU state change, *i.e.*,

$$X = \begin{cases} 1, & \text{first probe is successful with probability } p \\ 0, & \text{otherwise with probability } 1 - p. \end{cases} \quad (3)$$

Using the law of total expectation, we obtain a relationship about the probing delay:

$$\mathrm{E}[D_P] = p\mathrm{E}[D_P|X = 1] + (1 - p)\mathrm{E}[D_P|X = 0]. \quad (4)$$

Note that $\mathrm{E}[D_P|X = 1] = T/2$ from Equation (2) and $\mathrm{E}[D_P|X = 0] = \mathrm{E}[D_P] + T$. Hence, the probing delay is

$$\mathrm{E}[D_P] = \frac{T}{2} + \frac{1 - p}{p}T. \quad (5)$$

When $p = 1$, the above equation is reduced to Equation (2). If p becomes smaller, on average a SU takes a longer time to find the arrival/departure of the PU. Moreover, the probing delay is proportional to probing interval T. A smaller value of T leads to the faster detection of the PU state change, but costs more energy. This indicates the tradeoff between probing performance and energy consumption. Since the observations from unreliable sensing are similar to those from reliable sensing, in the rest of this paper we only consider the cases when all spectrum probes are reliable.

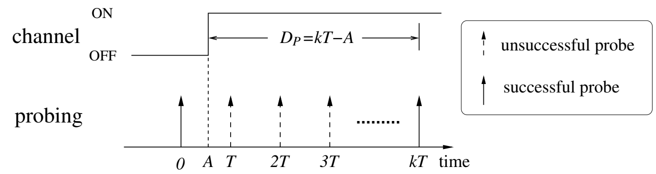


Fig. 2. Periodic probing with unreliable sensing.

B. Optimality and Conditions

Periodic probing is optimal when the following two conditions are satisfied: (1) The distribution of PU state change times is **stationary**, and (2) the distribution of the PU state change time is **uniform**. Specifically, we prove the following theorem on the optimality of periodic probing.

Theorem 1: Consider a PU that has a stationary distribution of state change times over a large time interval of L, with an expected inter-change time of $1/\lambda$. Assume that the PU state change time has a uniform distribution. Among all spectrum-probing strategies that do not exploit the knowledge of the PU and have the same average spectrum-probing interval of T, periodic probing is the best strategy that minimizes the probing delay.

Proof: Consider all strategies that probe the spectrum n times in the interval $[0, L]$. Thus, $L = nT$. A periodic-probing scheme and an arbitrary probing scheme are shown in Figure 3. For periodic probing, the probing delay is $\mathrm{E}[D_P] = T/2$ from Equation (2). For an arbitrary probing scheme, the probes are sent at times $t_1, t_2, \cdots, t_n$. To guarantee that all PU state changes can be detected, $t_n = L$. Set $t_0 = 0$, and $I_i = t_i - t_{i-1}$, where $i = 1, 2, \cdots, n$. Note that $\sum_{i=1}^{n} I_i = L$. For interval $[t_{i-1}, t_i]$, there are λI_i PU state changes. Moreover, since the arrival of the PU state change is uniform, it takes on average $I_i/2$ to detect each state change. Therefore, the sum of probing delays in interval $[t_{i-1}, t_i]$ is $\lambda I_i \cdot I_i/2$. As there are totally λL PU state changes, the probing delay over $[0, L]$ is

$$\mathrm{E}[D_A] = \frac{1}{\lambda L} \sum_{i=1}^{n} \frac{1}{2} \lambda I_i^2 = \frac{1}{2L} \sum_{i=1}^{n} I_i^2. \quad (6)$$

Using the Cauchy-Schwarz inequality, we obtain

$$\sum_{i=1}^{n} I_i^2 \cdot \sum_{i=1}^{n} 1^2 \geq \left(\sum_{i=1}^{n} I_i \cdot 1 \right)^2 = L^2. \quad (7)$$

Hence, we have

$$\mathrm{E}[D_A] \geq \frac{1}{2L} \cdot \frac{L^2}{n} = \frac{T}{2} = \mathrm{E}[D_P]. \quad (8)$$

■

We emphasize that periodic probing is optimal when both "stationary" and "uniform" conditions are satisfied. If one of these two conditions is not met, periodic probing may not be the best strategy that minimizes the probing delay. We demonstrate this and derive optimal probing methods in Sections IV and V when these two conditions are violated.

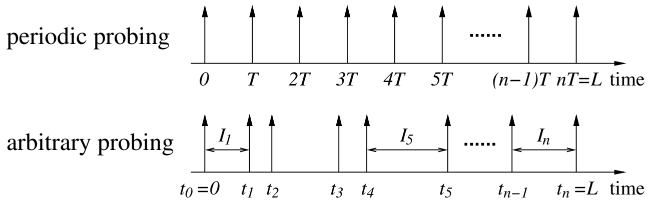


Fig. 3. A periodic-probing scheme versus an arbitrary probing scheme.

IV. Nonstationary PU State Change Times

The assumption that the distribution of PU state change times is stationary is clearly not valid. For example, a large-scale spectrum measurement study has shown that the usage of channels by PUs has an obvious diurnal pattern [4]. The PUs are much more active during the daytime than late at night. This implies that an optimal spectrum-probing strategy should vary with time and adapt to how active PUs are. In this section, we first derive an optimal spectrum-probing scheme when the distribution of PU state change times is nonstationary, *i.e.,* the PU state change rate varies with time. We then apply simulations to study the performance of the optimal spectrum-probing scheme.

A. An Optimal Spectrum-Probing Method

We assume that the "uniform" condition still holds. That is, the arrival of PU state changes is uniform over a large interval of L. Moreover, the time interval $[0, L]$ is divided into m time slots. For the i-th time slot, the time duration is L_i, and thus $\sum_{i=1}^{m} L_i = L$. Meanwhile, the arrival rate of PU state changes during the i-th time slot is λ_i. Here, L_i's can be small so that λ_i's are constant. If λ_i's are all equal, the distribution of PU state change times is stationary; otherwise, it is *nonstationary*. Since for the i-th time slot both "stationary" and "uniform" conditions hold, periodic probing with a constant spectrum-probing interval of T_i is optimal from Theorem 1. As a result, the probing delay in i-th time slot is $T_i/2$. Moreover, as there are $\lambda_i L_i$ PU state changes, the sum of probing delays in this time slot is $\lambda_i L_i T_i/2$. For the entire time interval $[0, L]$, the total number of PU state changes is $\sum_{i=1}^{m} \lambda_i L_i$, and therefore, the overall probing delay is $\mathrm{E}[D_N] = (\sum_{i=1}^{m} \lambda_i L_i T_i/2)/(\sum_{i=1}^{m} \lambda_i L_i)$. To minimize this overall probing delay, T_i should be different for different time slots that have different λ_i. Intuitively, when λ_i is larger, T_i should be smaller to reduce the delay in detecting the arrival/departure of the PU. That is, more energy should be used in time slots where the PU changes its state more often.

To compare different spectrum-probing schemes, we assume that the total energy used in $[0, L]$ is constant. Since each spectrum probe consumes equal energy, the constraint on total energy consumption is equivalent to a constraint on the total number of spectrum probes. Hence, we assume that there are at most n probes used in time interval $[0, L]$. As there are L_i/T_i probes used in the i-th time slot, $\sum_{i=1}^{m} L_i/T_i \leq n$.

Summarizing the above problem description and constraint, we can then formulate the following optimization problem that finds the optimal spectrum-probing interval T_i and minimizes the overall probing delay:

$$\text{Minimize} \qquad \mathrm{E}[D_N] = \frac{\sum_{i=1}^{m} \lambda_i L_i T_i}{2\sum_{i=1}^{m} \lambda_i L_i} \tag{9}$$

$$\text{s.t.} \qquad \sum_{i=1}^{m} \frac{L_i}{T_i} \leq n, \quad T_i \geq 0, \quad \forall i. \tag{10}$$

The solution of this optimization problem can be found using the theorem of Kuhn and Tucker [14], as shown in the following theorem.

Theorem 2: Among all possible spectrum-probing strategies, T_j^* is the strategy that minimizes $\mathrm{E}[D_N]$ subject to $\sum_{i=1}^{m} L_i/T_i \leq n$, where

$$T_j^* = \frac{1}{n\sqrt{\lambda_j}} \sum_{i=1}^{m} \sqrt{\lambda_i} L_i, \quad j = 1, 2, \cdots, m. \tag{11}$$

Proof: We construct the Lagrangian function

$$\mathcal{L}(T_j, \forall j; \alpha) = \frac{\sum_{i=1}^{m} \lambda_i L_i T_i}{2\sum_{i=1}^{m} \lambda_i L_i} - \alpha \left(n - \sum_{i=1}^{m} \frac{L_i}{T_i} \right). \tag{12}$$

Using the theorem of Kuhn and Tucker, we differentiate function $\mathcal{L}(T_j, \forall j; \alpha)$ with respect to T_j, set the result equal to zero, and obtain

$$T_j = \sqrt{\frac{2\alpha \sum_{i=1}^{m} \lambda_i L_i}{\lambda_j}}. \tag{13}$$

Similarly, the theorem of Kuhn and Tucker yields $\alpha \frac{\partial \mathcal{L}(T_j, \forall j; \alpha)}{\partial \alpha} = 0$, which leads to $\alpha = 0$ or $\sum_{i=1}^{m} L_i/T_i = n$. Note that α cannot be zero. Otherwise, $T_j = 0$, and $\sum_{i=1}^{m} L_i/T_i$ is unbounded. Therefore, putting Equation (13) into $\sum_{i=1}^{m} L_i/T_i = n$, we find

$$\alpha = \frac{\left(\sum_{i=1}^{m} \sqrt{\lambda_i} L_i\right)^2}{2n^2 \sum_{i=1}^{m} \lambda_i L_i}, \tag{14}$$

which leads to Equation (11).

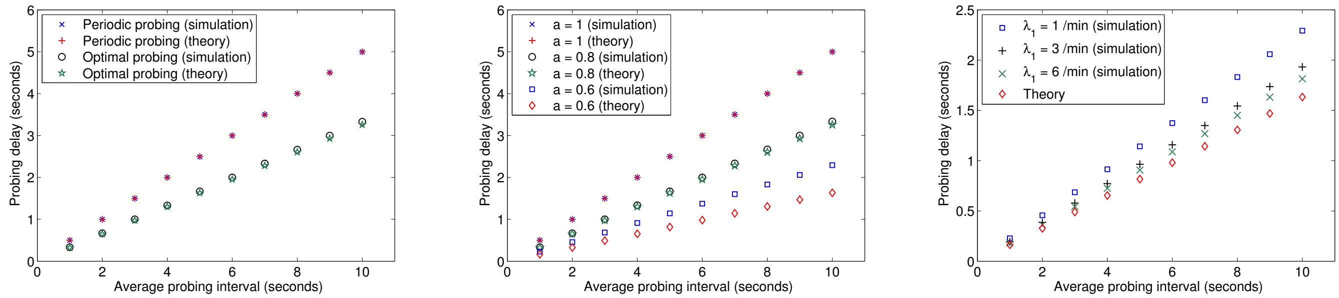
Since

$$\frac{\partial^2 \mathcal{L}}{\partial T_j^2}(T_j^*, \forall j; \alpha) = 2\alpha \frac{L_j}{T_j^{*3}} \geq 0, \tag{15}$$

T_j^* is the optimal strategy that minimizes $\mathrm{E}[D_N]$. ■

From Theorem 2, we have the following observations:

- When the distribution of PU state change times is stationary, *i.e.*, $\lambda_1 = \lambda_2 = \cdots = \lambda_m$, according to Equation (11), $T_1^* = T_2^* = \cdots = T_m^* = L/n$, *i.e.*, periodic probing is the optimal scheme.
- When $L_1 = L_2 = \cdots = L_m = L/m$, the optimal probing strategy is $T_j^* = \frac{L\sum_{i=1}^{m} \sqrt{\lambda_i}}{mn\sqrt{\lambda_j}}$.
- If $\lambda_i > \lambda_j$, from Equation (11) we find that $T_i^* < T_j^*$. That is, the optimal probing scheme probes the spectrum more frequently for the time intervals with higher PU state change rates.
- If periodic probing is applied, *i.e.*, $T_i = L/n$, $\forall i$, the probing delay is $\mathrm{E}[D_P] = L/(2n)$. On the other hand, if the optimal probing strategy is applied, we derive the



(a) Periodic probing and optimal probing schemes ($\lambda_1 = 1$ /min and $a = 0.8$)

(b) Effect of parameter a on the optimal probing scheme ($\lambda_1 = 1$ /min)

(c) Effect of parameter λ_1 on the optimal probing scheme ($a = 0.6$)

Fig. 4. Probing delay when the distribution of PU state change times is nonstationary ($L = 1$ day and $m = 24$).

probing delay by putting Equation (11) into Equation (9) and find

$$\mathrm{E}[D_N] = \frac{\left(\sum_{i=1}^{m} \sqrt{\lambda_i} L_i\right)^2}{2n \sum_{i=1}^{m} \lambda_i L_i}. \tag{16}$$

Note that the Cauchy-Schwarz inequality yields

$$\left(\sum_{i=1}^{m} \sqrt{\lambda_i} L_i\right)^2 \leq \left(\sum_{i=1}^{m} \lambda_i L_i\right) \left(\sum_{i=1}^{m} L_i\right). \tag{17}$$

Hence, we find a relationship between $\mathrm{E}[D_N]$ and $\mathrm{E}[D_P]$:

$$\mathrm{E}[D_N] \leq \frac{L}{2n} = \mathrm{E}[D_P]. \tag{18}$$

This indicates that the optimal probing scheme can achieve a shorter probing delay than periodic probing.

B. Performance Evaluation

We apply simulations to evaluate the performance of periodic probing and optimal probing schemes. In our simulations, we assume the total duration of one day (*i.e.,* $L = 1$ day) and 24 time slots of equal duration (*i.e.,* $m = 24$ and $L_1 = L_2 = \cdots = L_{24} = 1$ hour). To study λ_i's with both stationary and nonstationary distributions, we assume

$$\lambda_i = \lambda_1 a^{i-1}, \quad i = 1, 2, \cdots, 24. \tag{19}$$

When $a = 1$, all λ_i's are equal, and the distribution of PU state change times is stationary. Otherwise, the distribution is nonstationary. Moreover, when $a < 1$ and a is smaller, the distribution is more skewed. Parameter λ_1 is used to control the average arrival rate over $[0, L]$. In this way, we can use λ_1 and a to design different distributions of arrival rates of PU state changes. To generate the events of PU state changes, we assume that the PU arrival process is Poisson. That is, the inter-arrival times of the PU in time slot i follow an exponential distribution with mean $1/\lambda_i$. Specifically, we apply the inverse transform method to generate a nonhomogeneous Poisson process for PU states [12]. If a PU state change occurs at time t in the time slot i, then the next PU state change will happen at $t + X$, where $X = -\ln(U)/\lambda_i$ and U is a uniform random variable over $[0, 1]$. According to Theorem 2.3.1 in [11], if there are k arrivals in a time slot, these k arrival times have the same distribution as the order statistics corresponding to k independent random variables uniformly distributed on this time slot. Therefore, the "uniform" condition holds for the Poisson process in each time slot. For periodic probing with n samples, the spectrum is probed at times $i \cdot (L/n)$, $i = 1, 2, \cdots, n$. For optimal probing, the probing interval in each time slot is obtained from Equation (11).

Figure 4 shows the probing delay when the PU state change times are nonstationary. Each point of simulation results is the average of 10^5 independent runs. From the theoretical analysis, the probing delay of periodic probing is $\mathrm{E}[D_P] = L/(2n)$, whereas the probing delay of optimal probing is obtained from Equation (16). Figure 4(a) compares the probing delay of optimal probing with that of periodic probing when $\lambda_1 = 1$ /min, $a = 0.8$, and the average probing interval varies from 1 s to 10 s. It can be seen that the theoretical and simulation results for both probing schemes are (almost) identical. Moreover, the optimal probing scheme performs much better than the periodic probing method. For example, to achieve a probing delay of 2 s, periodic probing uses 21,603 samples over one day, whereas optimal probing needs only 14,396 probes. This means that compared with periodic probing, the optimal scheme can have an energy saving of 33%.

Next, we study the effect of parameters λ_1 and a on both probing schemes. Through extensive simulations, we find that λ_1 and a do not affect the probing delay of periodic probing. In all cases, both theoretical and simulation results of periodic probing are equal to $L/(2n)$. Figure 4(b) shows the probing delay of the optimal probing scheme when $\lambda_1 = 1$ /min and $a = 1$, 0.8, or 0.6. It can be seen that when $a = 1$, the distribution of PU state change times becomes stationary. Thus, the probing delay is $L/(2n)$, and the optimal scheme is indeed periodic probing. Moreover, when a decreases, the distribution of PU state change times is more skewed, and optimal probing with the same number of samples can further reduce the probing delay. Figure 4(b) also demonstrates that when $a = 0.6$, the simulation results have a slightly higher probing delay than the theoretical results. This is because in this case (*i.e.,* $\lambda_1 = 1$ /min and $a = 0.6$), the generated arrival rates of PU state changes in some time slots are so small in simulations that some generated arrival rates may deviate from the predetermined arrival rates in Equation (19). To verify our explanation, we keep $a = 0.6$ and vary λ_1 (*i.e.,* $\lambda_1 = 1$, 3, and 6 /min) in Figure 4(c). From Equation (16), it can be seen that λ_1 does not affect the probing delay of optimal probing in theory. When λ_1 increases, however, the generated arrival rates in simulations better match the predetermined ones, and the simulation results are closer to the theoretical results.

V. Nonuniform PU Channel Occupancy/Vacancy Distributions

The assumption that the PU channel occupancy/vancancy distribution (*i.e.,* the PU state change time) is uniform may not be valid. To simplify the analysis, many works have assumed that such a distribution is exponential [8], [9]. On the other hand, measurement studies have demonstrated that the real distribution does not have the memoryless property and may not be exponential [17], [4]. Therefore, in this section we study the optimal spectrum-probing schemes when the PU channel occupancy/vacancy distribution is either exponential or non-exponential (*e.g.*, Pareto-distributed). From the view of information obtained by a SU, the PU channel occupancy/vacancy distribution contains much more information than the arrival rates of PU state changes. Hence, the optimal scheme should be very different from that proposed in the previous section. However, the underlying principle for optimal probing is the same, *i.e.*, to reduce the probing delay a smart SU should probe the spectrum more frequently when the PU changes its state with a higher probability.

A. Problem Formulation

We use $f_Y(y)$ to denote the PU channel occupancy/vacancy distribution and assume that $f_Y(y) \neq 0$ only for $0 \leq y \leq L$. That is, a PU state change would occur during time interval $[0, L]$, and $\int_0^L f_Y(y)\mathrm{d}y = 1$. We assume that a SU has learned $f_Y(y)$ beforehand and known the starting time of the last PU state change. The problem is how the SU can efficiently probe the spectrum to reduce the delay of detecting the next PU state change. As in Section IV, we assume that the total energy for spectrum probing in $[0, L]$ is constant, and thus, the total number of spectrum probes is at most n. Figure 5 shows an example of $f_Y(y)$ and a spectrum-probing scheme, where probes are sent at time instants $t_1, t_2, \cdots, t_n$ and $t_n = L$. Set $t_0 = 0$. If the PU state change occurs at time y, where $y \in (t_{i-1}, t_i]$ ($i = 1, 2, \cdots, n$), then the delay for the probing scheme to detect the change is $t_i - y$. Therefore, the overall probing delay over $[0, L]$ is $\mathrm{E}[D_Y] = \sum_{i=1}^n \int_{t_{i-1}}^{t_i} (t_i - y) f_Y(y)\mathrm{d}y$. Summarizing the above problem description and constraint, we formulate the following optimization problem that finds the optimal spectrum-probing scheme (*i.e.*, t_i's):

$$\text{Minimize} \quad \mathrm{E}[D_Y] = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} (t_i - y) f_Y(y)\mathrm{d}y \tag{20}$$

$$\text{s.t.} \quad \int_0^L f_Y(y)\mathrm{d}y = 1, \tag{21}$$

where $0 = t_0 \leq t_1 \leq \cdots \leq t_n = L$.

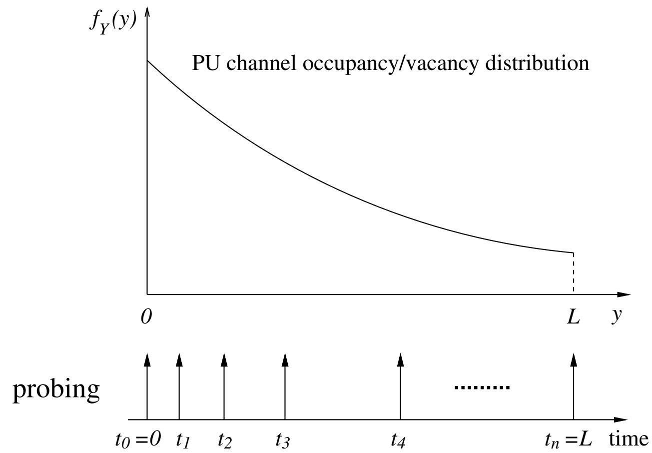


Fig. 5. An example of $f_Y(y)$ and a spectrum-probing scheme.

Using the theorem of Lagrange [14], we find a recursive relationship of the optimal t_i's in the following theorem:

Theorem 3: Among all possible spectrum-probing strategies, t_i^* is the strategy that minimizes $\mathrm{E}[D_Y]$, where

$$\sum_{i=1}^{n} \int_{t_{i-1}^*}^{t_i^*} f_Y(y)\mathrm{d}y = 1 \tag{22}$$

$$t_{i+1}^* = t_i^* + \frac{1}{f_Y(t_i^*)} \int_{t_{i-1}^*}^{t_i^*} f_Y(y)\mathrm{d}y, \tag{23}$$

where $i = 1, 2, \cdots, n-1$.

Proof: Note that $[0, L] = [t_0, t_1] \cup [t_1, t_2] \cup \cdots \cup [t_{n-1}, t_n]$ and $[t_{i-1}, t_i] \cap [t_{j-1}, t_j] = \emptyset$ for $i \neq j$ and $1 \leq i, j \leq n$. Thus, Equation (22) is the direct extension from Equation (21). Next, we construct the Lagrangian function

$$\mathcal{L}(t_i, \forall i; \alpha) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} (t_i - y) f_Y(y)\mathrm{d}y + \alpha \left(\sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} f_Y(y)\mathrm{d}y - 1 \right). \tag{24}$$

Using the theorem of Lagrange, we differentiate function $\mathcal{L}(t_i, \forall i; \alpha)$ with respect to t_i, set the result equal to zero, and thus obtain Equation (23). ■

Although Theorem 3 does not provide a closed-form expression for the optimal solution, it gives ways to calculate t_i^*'s numerically to find the optimal probing scheme. Specifically, one way is that from Equation (23), $t_2^*, t_3^*, \cdots, t_n^*$ can be expressed by t_1^* recursively. Since $t_n^* = L$, t_1^* can be found, and so can $t_2^*, t_3^*, \cdots, t_{n-1}^*$. The following example shows another way to derive the optimal probing scheme.

Example (Uniform Distribution): If the PU channel occupancy/vacancy distribution is uniform over $[0, L]$,

$$f_Y(y) = \begin{cases} \frac{1}{L}, & 0 \leq y \leq L \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

Putting $f_Y(y)$ into Equation (23), we find

$$t_{i+1}^* - t_i^* = t_i^* - t_{i-1}^*, \; 1 \leq i \leq n-1. \tag{26}$$

Since $\sum_{i=1}^{n}(t_i^* - t_{i-1}^*) = L$,

$$t_i^* - t_{i-1}^* = \frac{L}{n}, \quad i = 1, 2, \cdots, n. \tag{27}$$

That is, the periodic probing scheme is the best strategy that minimizes the probing delay, which confirms Theorem 1.

On the other hand, if the PU channel occupancy/vacancy is not uniform, it is clear that the optimal spectrum-probing method is not periodic. In the following, we specifically study the optimal probing scheme when the PU channel occupancy/vacancy distribution is a truncated exponential distribution or a truncated Pareto distribution over $[0, L]$ by applying Theorem 3.

B. Truncated Exponential Distribution

If a PU channel occupancy/vacancy distribution is a truncated exponential distribution with parameter λ over $[0, L]$,

$$f_Y(y) = \begin{cases} \frac{\lambda e^{-\lambda y}}{1-e^{-\lambda L}}, & 0 \le y \le L \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Applying Theorem 3, we have that when $1 \le i \le n-1$,

$$t_{i+1}^* - t_i^* = \frac{1}{\lambda}\left[e^{\lambda(t_i^* - t_{i-1}^*)} - 1\right]. \tag{29}$$

Putting Equations (28) and (29) into Equation (20), we can obtain the probing delay of the optimal probing scheme

$$\mathrm{E}[D_Y] = \frac{t_1^*}{1-e^{-\lambda L}} - \frac{e^{-\lambda L}}{\lambda(1-e^{-\lambda L})}\left[e^{\lambda(L-t_{n-1}^*)} - 1\right]. \tag{30}$$

Setting $x_i^* = \lambda(t_i^* - t_{i-1}^*)$, we find from Equation (29) that

$$\begin{cases} x_i^* = \ln(1 + x_{i+1}^*), \quad i = 1, 2, \cdots, n-1 \\ \sum_{i=1}^{n} x_i^* = \lambda L. \end{cases} \tag{31}$$

By solving x_i^*'s numerically, we can obtain the optimal spectrum-probing scheme (*i.e.*, t_i^*'s). Note that $x_i^* = \ln(1 + x_{i+1}^*) \le x_{i+1}^*$, *i.e.*,

$$t_i^* - t_{i-1}^* \le t_{i+1}^* - t_i^*. \tag{32}$$

Hence, the optimal solution requires a SU to probe more frequently at the early stage of $[0, L]$.

To numerically find the solution to Equation (31), we apply the binary search method. The algorithm is given in Algorithm 1, where Δ is a small number that controls the accuracy of the solution.

Algorithm 1 Finding x_i^*'s in Equation (31)

```
Input: λ, L, n, and Δ
Output: x_i^*, i = 1, 2, ··· , n

Set found = 0; lower = 0; upper = λL/n
while found = 0 do
  x_1^* = (lower + upper)/2
  for i = 2 to n do
    x_i^* = exp(x_{i-1}^*) − 1
  end for
  if |Σ_{i=1}^n x_i^* − λL| < Δ then
    found = 1
  else if Σ_{i=1}^n x_i^* > λL then
    upper = x_1^*
  else
    lower = x_1^*
  end if
end while
```

By applying Algorithm 1, we can find the optimal probing strategy. Figure 6 shows the index of probing intervals (*i.e.,* i) versus the probing interval (*i.e.,* $t_i - t_{i-1}$) for both the optimal probing scheme and the periodic probing scheme. In this experiment, we use a total duration of 1,500 seconds (*i.e.,* $L = 1,500$ s) and a total of 100 probes (*i.e.,* $n = 100$). For the optimal scheme, we set $\Delta = 0.001$. To study the effect of λ on the optimal probing method, we construct two cases based on the probability of the original exponential distribution over $[0, L]$, *i.e.,* $1 - e^{-\lambda L}$. Specifically, we consider when such a probability is 0.99 and 0.99999, *i.e.,* $\lambda L = 2\ln(10)$ and $\lambda L = 5\ln(10)$. From Figure 6, we can see that whereas periodic probing uses a constant probing interval, the optimal scheme increases the probing interval with time. Moreover, if λ increases, the optimal probing strategy uses a more skewed distribution of probing intervals, and samples the spectrum more frequently at the early stage of $[0, L]$ and less frequently at the late stage.

We further apply simulations to calculate the probing delay of both periodic probing and optimal probing schemes. Specifically, we use the inverse transform method and the rejection method to generate the truncated exponential random variable [12]. That is, first generate a uniform random variable among $[0, 1]$, which is denoted by U. Then, $X = -\ln(U)/\lambda$ is an exponential random variable with rate λ. If $X \le L$, the value of X is accepted (*i.e.,* $Y = X$); otherwise, it is rejected, and the procedure repeats. For each generated Y, if a probing scheme samples the spectrum at time instant t_i ($i = 1, 2, \cdots, n$) and $Y \in (t_i, t_{i+1}]$, then the probing delay is calculated as $t_{i+1} - Y$. To obtain an accurate estimate of the probing delay, we independently run the simulation 10^5 times for each probing scheme and obtain the average. Figure 7 compares the simulated probing delay of periodic probing with that of the optimal scheme when $L = 1,500$ s, $\Delta = 0.001$, $\lambda = 2\ln(10)/L$ or $\lambda = 5\ln(10)/L$, and n varies from 100 to 1,500. This figure also plots the theoretical probing delay of the optimal scheme from Equation (30), which is shown to overlap with the simulated probing delay. Since the average probing interval is equal to L/n, the average probing interval varies from 1 s to 15 s. Figure 7 shows some interesting results. First, for both probing schemes, the probing delay increases almost linearly with the average probing interval. This again indicates the tradeoff between energy consumption and probing performance. Second, while λ affects the periodic probing scheme little, it has a significant effect on the optimal probing scheme. Specifically, if λ is larger, *i.e.*, the PU changes its state faster, the reduction in the probing delay by the optimal scheme is more significant.

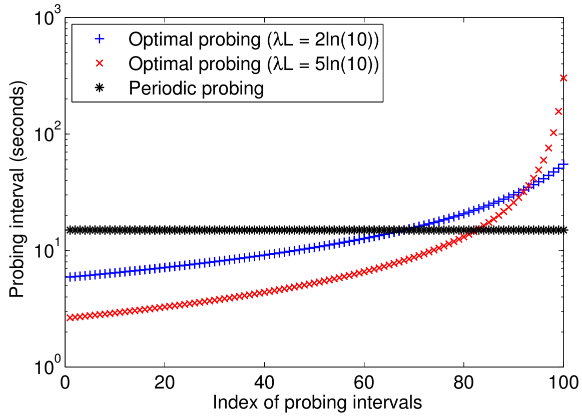


Fig. 6. Distribution of probing intervals when a PU channel occupancy/vacancy distribution is a truncated exponential distribution ($L = 1,500$ s, $n = 100$, and $\Delta = 0.001$).

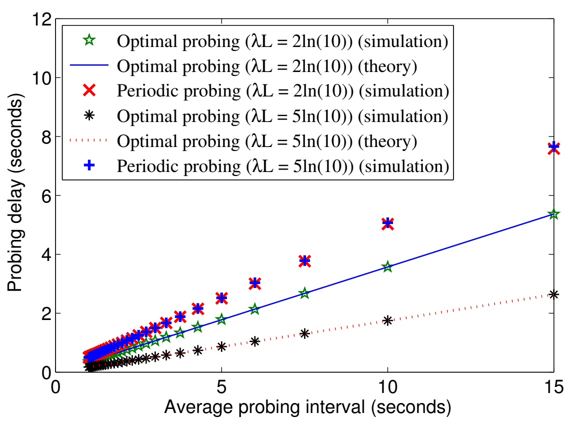


Fig. 7. Probing delay when a PU channel occupancy/vacancy distribution is a truncated exponential distribution ($L = 1,500$ s and $\Delta = 0.001$).

Third, the optimal probing scheme performs much better than the periodic probing scheme. For example, to achieve a probing delay of 2 s, periodic probing requires about 377 samples over $[0, L]$, whereas optimal probing uses only 268 and 133 samples for $\lambda = 2\ln(10)/L$ and $\lambda = 5\ln(10)/L$, respectively. This means that the energy savings are 29% and 65% if we switch the spectrum-probing method from the periodic probing scheme to the optimal probing scheme.

C. Truncated Pareto Distribution

If a PU channel occupancy/vacancy distribution is a truncated Pareto distribution with parameter β over $[0, L]$,

$$f_Y(y) = \begin{cases} \frac{\beta y_m^\beta}{\gamma y^{\beta+1}}, & y_m \le y \le L \\ 0, & \text{otherwise.} \end{cases} \tag{33}$$

where $\gamma = 1 - (y_m/L)^\beta$. Since $f_Y(y) = 0$ for time interval $[0, y_m]$, a SU only probes during $[y_m, L]$, and thus $t_0^* = y_m$. Applying Theorem 3, we have

$$t_{i+1}^* = t_i^* + \frac{t_i^*}{\beta}\left[\left(\frac{t_i^*}{t_{i-1}^*}\right)^\beta - 1\right], \tag{34}$$

where $i = 1, 2, \cdots, n-1$. Since $t_0^* = y_m$ and $t_n^* = L$, t_i^*'s can be obtained numerically. Similar to Algorithm 1, we can also apply the binary search method to numerically find the value of t_1^* and then all t_i^*'s. Moreover, putting Equations (33) and (34) into Equation (20), we can obtain the theoretical probing delay of the optimal probing scheme

$$\mathrm{E}[D_Y] = \frac{y_m^\beta}{\gamma}\sum_{i=1}^{n} t_i^*\left(t_{i-1}^{*-\beta} - t_i^{*-\beta}\right) - \frac{\beta y_m\left[1-(y_m/L)^{\beta-1}\right]}{\gamma(\beta-1)}. \tag{35}$$

Figure 8 shows an example on the probing intervals for both periodic probing and optimal probing schemes when $L = 1,500$ s, $y_m = 50$ s, $n = 100$, and $\Delta = 0.001$. Since the probability of the original Pareto distribution over $[0, L]$ is $\gamma = 1 - (y_m/L)^\beta$, we consider two cases of β: $\gamma = 0.9$ or 0.999, *i.e.*, $\beta = 0.677$ or $\beta = 2.031$. From Figure 8, it can be seen that when β increases (*i.e.,* the truncated Pareto distribution is more skewed), the optimal scheme has also a more skewed distribution of probing intervals, and probes the spectrum more frequently at the early stage of $[y_m, L]$ and less frequently at the late stage.

Next, we apply simulations to study the probing delay when the PU channel occupancy/vacancy distribution is a truncated Pareto distribution. We again use the inverse transform method and the rejection method to generate the truncated Pareto distribution. That is, $X = y_m/(U^{1/\beta})$, where U is a uniform random variable over $[0, 1]$. Then, X is a Pareto random variable with parameter β. If $X \le L$, then $Y = X$; otherwise, X is rejected, and the new value of X is generated. Figure 9 compares the performance of optimal probing with that of periodic probing when $L = 1,500$ s, $y_m = 50$ s, $\Delta = 0.001$, and the average probing interval varies from 1 s to 29 s. Each point is the average of 10^5 independent runs. It can be seen that the simulation results overlap with the theoretical results from Equation (35). It can also be seen that the optimal scheme performs much better than periodic probing. For example, to achieve a probing delay of 2 s, periodic probing uses about 372 samples over $[y_m, L]$, whereas optimal probing requires only 200 and 67 probes for $\beta = 0.677$ or $\beta = 2.031$, respectively. It means that the energy can be significantly saved by 46% and 82%.

VI. Related Work

Spectrum sensing in cognitive radio networks is an active research topic. For example, many physical and MAC spectrum-sensing techniques have been designed, such as energy detection, matched filter detection, and cyclostationary feature detection [1], [18]. Some mechanisms have been proposed to optimize the tradeoff between sensing and transmission [15], [2], [6]. Moreover, efficient methods of sensing multiple channels have been studied in [7], [8]. Different from the previous works, in this paper we focus on the metric of the

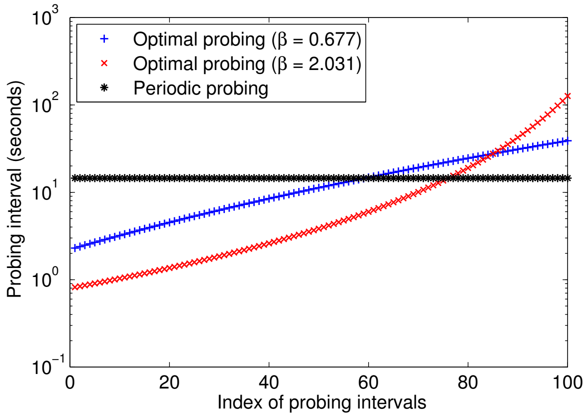


Fig. 8. Distribution of probing intervals when a PU channel occupancy/vacancy distribution is a truncated Pareto distribution ($L = 1,500$ s, $y_m = 50$ s, $n = 100$, and $\Delta = 0.001$).

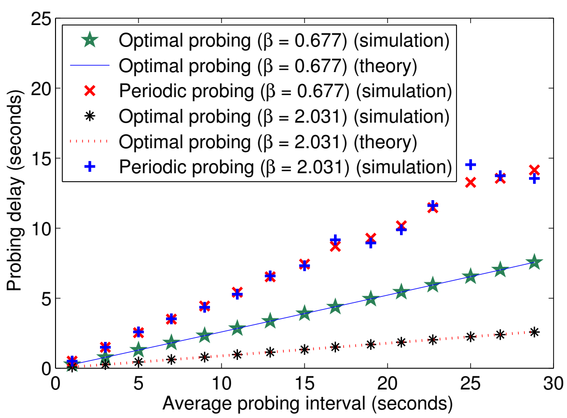


Fig. 9. Probing delay when a PU channel occupancy/vacancy distribution is a truncated Pareto distribution ($L = 1,500$ s, $y_m = 50$ s, and $\Delta = 0.001$).

probing delay, *i.e.,* the average delay with which a SU can detect a PU channel state change. The study of this metric can potentially make SUs more sensitive to dynamic environments, actively avoiding the harmful interference to the transmission of PUs and effectively exploiting the available channels.

Probing techniques have been widely used in wireline and wireless networks. For example, different probing methods have been applied in estimating the performance of the Internet [10], [13]. Moreover, optimal contact-probing strategies have been designed to minimize the contact missing probability in delay-tolerant networks [16]. Inspired by these works, we study the optimal spectrum-probing mechanisms that minimize the probing delay in cognitive radio networks.

VII. Conclusions

In this paper, we attempt to provide a theoretical framework on the optimal spectrum-probing strategy that minimizes the probing delay given a constraint on energy consumption. We have emphasized the conditions under which the widely used periodic probing is optimal. That is, if the "stationary" condition or the "uniform" condition is not met, periodic probing may not be the best probing strategy. We have then derived optimal probing methods when the distribution of PU state change times is nonstationary or the PU channel occupancy/vacancy follows an arbitrary distribution. Our simulation results have shown that given a constraint on the probing delay, the proposed optimal spectrum-probing strategies perform much better than periodic probing in saving energy.

As part of our ongoing work, we are studying how a SU can accurately estimate $\mathrm{E}[Y]$ or $f_Y(y)$, while probing the spectrum using periodic probing or optimal probing strategies.

References

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Computer Networks*, vol. 50, no. 13, pp. 2127-2159, Sept. 2006.

[2] N. Chang and M. Liu, "Optimal channel probing and transmission scheduling for opportunistic spectrum access," in *Proc. of ACM International Conference on Mobile Computing and Networking (MobiCom'07)*, Montreal, Canada, Sept. 2007.

[3] C. Chen, Z. Chen, T. Cooklev, and C. Pomalaza-Ráez, "On spectrum probing in cognitive radio networks: Does randomization matter?" in *Proc. of IEEE International Conference on Communications (ICC'10)*, Cape Town, South Africa, May 2010.

[4] D. Chen, S. Yin, Q. Zhang, M. Liu, and S. Li, "Mining spectrum usage data: A large-scale spectrum measurement study," in *Proc. of ACM International Conference on Mobile Computing and Networking (MobiCom'09)*, Beijing, China, Sept. 2009.

[5] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[6] S. Huang, X. Liu, and Z. Ding, "Optimization of transmission strategies for opportunistic access in cognitive radio networks," *IEEE Transactions on Mobile Computing*, vol. 8, no. 12, Dec. 2009.

[7] H. Kim and K. G. Shin, "Fast discovery of spectrum opportunities in cognitive radio networks," in *Proc. of the 3rd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'08)*, Chicago, IL, Oct. 2008.

[8] H. Kim and K. G. Shin, "Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks," *IEEE Transaction on Mobile Computing*, vol. 7, no. 5, pp. 533-545, May 2008.

[9] Q. Liang, M. Liu, and D. Yuan, "Channel estimation for opportunistic spectrum sensing: uniform and random sensing," in *Information Theory and Applications Workshop (ITA'10)*, University of California, San Diego, Feb. 2010.

[10] V. Paxson, "End-to-end routing behavior in the Internet," *IEEE/ACM Transactions on Networking*, vol.5, no.5, pp. 601-615, Oct. 1997.

[11] S. M. Ross, *Stochastic Processes*, Second Edition. John Wiley & Sons, Inc., 1996.

[12] S. M. Ross, *Simulation*, Third Edition. Academic Press, 2002.

[13] M. Roughan, "A comparison of Poisson and uniform sampling for active measurements," *IEEE Journal on Selected Areas in Communication*, vol. 24, no. 12, pp. 2299-2312, Dec. 2006.

[14] R. K. Sundaram, *A First Course in Optimization Theory*. Cambridge University Press, 1996.

[15] P. Wang, L. Xiao, S. Zhou, and J. Wang, "Optimization of detection time for channel efficiency in cognitive radio systems," in *Wireless Communications and Networking Conference (WCNC'07)*, Hong Kong, China, March 2007.

[16] W. Wang, M. Motani, and V. Srinivasan, "Opportunistic energy-efficient contact probing in delay-tolerant applications," *IEEE/ACM Transactions on Networking*, vol. 17, no. 5, pp. 1592-1605, Oct. 2009.

[17] D. Willkomm, S. Machiraju, J. Bolot, and A. Wolisz, "Primary users in cellular networks: A large-scale measurement study," in *Proc. of the 3rd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'08)*, Chicago, IL, Oct. 2008.

[18] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys and Tutorials*, vol. 11, no. 1, pp. 116-130, 2009.

[19] IEEE 802.22, Working Group on Wireless Regional Area Networks. [Online]. Available: http://www.ieee802.org/22/.