# Comments on “Modeling TCP Reno Performance: A Simple Model and Its Empirical Validation”

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**Abstract**

In this Comments, several errors in [1] are pointed out. The more serious of these errors result in an over prediction of the send rate. The expression obtained for send rate in this Comments leads to greater accuracy when compared with the measurement data than the original send rate expression in [1].

## I. Correction to the Derivation of the TCP Send Rate

In Section II.A (loss indications are exclusively triple-duplicate ACK’s) of [1], Equation (7) is incorrect. The window size increase between $W_{i-1}/2$ and $W_i$ should be

$$W_i = \frac{W_{i-1}}{2} + \frac{X_i}{b} - 1, \qquad i = 1, 2, 3, \cdots. \tag{1}$$

The above equation can be verified by looking at Figure 2 in [1], where $W_{i-1}/2 = 3$, $b = 2$, $X_i = 8$, and $W_i = 6$. Thus, Equations (10) and (11) in [1] change to

$$\begin{aligned} Y_i &= \sum_{k=0}^{X_i/b-1} \left( \frac{W_{i-1}}{2} + k \right) b + \beta_i \qquad (2)\\ &= \frac{X_i W_{i-1}}{2} + \frac{X_i}{2} \left( \frac{X_i}{b} - 1 \right) + \beta_i \qquad (3)\\ &= \frac{X_i}{2} \left( \frac{W_{i-1}}{2} + W_i \right) + \beta_i \qquad (4) \end{aligned}$$

and

$$E[W] = \frac{2}{b} E[X] - 2. \tag{5}$$

Recalling Equation (5) in [1], the same arguments lead to the following replacement of Equation (12) in [1] by

$$\frac{1-p}{p} + E[W] = \frac{E[X]}{2} \left( \frac{E[W]}{2} + E[W] \right) + E[\beta]. \tag{6}$$

Assuming as in [1] that $E[\beta] = E[W]/2$, coupled with the above two equations yields

$$E[W] = \sqrt{\frac{8(1-p)}{3bp} + \frac{(3b-2)^2}{9b^2}} - \frac{3b-2}{3b}, \tag{7}$$

which is different from Equation (13) in [1]. Observe that $E[W] \approx \sqrt{8/3bp}$ for small values of $p$. Furthermore, unlike the expression for $E[W]$ in [1], Equation (7) yields $E[W] = 0$ as $p \to 1$, matching one’s intuition. As all packets sent are lost, the congestion window is forcibly reduced to 0 in the end. With this new representation of $E[W]$, Equations (15) and (16) in [1] are revised to

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$$E[X] = \sqrt{\frac{2b(1-p)}{3p} + \frac{(3b-2)^2}{36}} + \frac{3b+2}{6} \tag{8}$$

$$E[A] = RTT\left(\sqrt{\frac{2b(1-p)}{3p} + \frac{(3b-2)^2}{36}} + \frac{3b+8}{6}\right). \tag{9}$$

From the above equations, the send rate (Equation (19) in [1]) should be

$$B(p) = \frac{\frac{1-p}{p} + E[W]}{E[A]} = \frac{\frac{1-p}{p} + \sqrt{\frac{8(1-p)}{3bp} + \frac{(3b-2)^2}{9b^2}} - \frac{3b-2}{3b}}{\mathrm{RTT}\left(\sqrt{\frac{2b(1-p)}{3p} + \frac{(3b-2)^2}{36}} + \frac{3b+8}{6}\right)}. \tag{10}$$

Observe that the above expression yields the expression in Equation (20) in [1] for small values of $p$. When $p$ equals to 1, however, Equation (19) in [1] gives $B(p) > 0$. Intuitively, if all packets are lost, the congestion window reduces to 0 and thus the sender cannot send any more packets. That is, $B(p) = 0$ in the case that loss indications are exclusively triple-duplicate ACK's. The above equation verifies this.

In Section II.C (impact of window limitation) of [1], the equation for the window size increase between $W_m/2$ and $W_m$ is also incorrect. It should be

$$W_m = \frac{W_m}{2} + \frac{U_i}{b} - 1, \quad \forall i \geq 2, \tag{11}$$

which implies $E[U] = (b/2)W_m + b$. Considering the number of packets sent in the *i*th TDP, we have

$$Y_i = \frac{U_i}{2}\left(\frac{W_m}{2} + W_m\right) + V_i W_m + \beta_i, \tag{12}$$

where the last item ($\beta_i$) is ignored in [1]. From the above equation, we have

$$E[Y] = \frac{3}{4}W_m E[U] + W_m E[V] + E[\beta] = \frac{3b}{8}W_m^2 + W_m E[V] + \frac{3b}{4}W_m + \frac{W_m}{2}, \tag{13}$$

where $E[\beta] = W_m/2$. Combining Equation (5) in [1], yields the following equation describing $E[Y]$

$$E[Y] = \frac{1-p}{p} + W_m = \frac{3b}{8}W_m^2 + W_m E[V] + \frac{3b}{4}W_m + \frac{W_m}{2}, \tag{14}$$

which gives

$$E[V] = \frac{1-p}{pW_m} + 1 - \frac{3b}{8}W_m - \frac{3b+2}{4}. \tag{15}$$

Finally, since $X_i = U_i + V_i$, we have

$$E[X] = E[U] + E[V] = \frac{b}{8}W_m + \frac{1-p}{pW_m} + \frac{b+2}{4}. \tag{16}$$

By substituting this result in Equation (28) in [1], we obtain the following expression for the TCP send rate, $B(p)$, when the window is limited

$$B(p) = \frac{\frac{1-p}{p} + W_m + \hat{Q}(W_m)\frac{1}{1-p}}{\mathrm{RTT}(\frac{b}{8}W_m + \frac{1-p}{pW_m} + \frac{b+6}{4}) + \hat{Q}(W_m)T_0\frac{f(p)}{1-p}}, \tag{17}$$

which differs from the one in [1].

Combining all the above corrections, the complete characterization of TCP send rate, $B(p)$, is

$$B(p) = \begin{cases} \frac{\frac{1-p}{p} + E[W_u] + \hat{Q}(E[W_u])\frac{1}{1-p}}{\mathrm{RTT}(\frac{b}{2}E[W_u] + b + 1) + \hat{Q}(E[W_u])T_0\frac{f(p)}{1-p}}, & E[W_u] < W_m; \\ \frac{\frac{1-p}{p} + W_m + \hat{Q}(W_m)\frac{1}{1-p}}{\mathrm{RTT}(\frac{b}{8}W_m + \frac{1-p}{pW_m} + \frac{b+6}{4}) + \hat{Q}(W_m)T_0\frac{f(p)}{1-p}}, & otherwise, \end{cases} \tag{18}$$

where $f(p)$ and $\hat{Q}$ are given in Equations (29) and (24) in [1], respectively; while $E[W_u]$ is given in Equation (7) in this Comments. We refer to Equation (18) as the "corrected model". Observe that when $E[W_u] = W_m$,

$$\frac{b}{2}E[W_u] + b + 1 = \frac{b}{8}W_m + \frac{1-p}{pW_m} + \frac{b+6}{4},$$

which leads to Equation (7) in this Comments. If we set the boundary condition, $E[W_u] = W_m$, in Equation (32) in [1], however, $W_m$ does not satisfy Equation (13) in [1].

The graphs in Figure 1 compare the predictions of the full model in [1] and the predictions of the corrected model with the measurement data for 1 h-long traces from Figure 7 in [1]. The average error of the full model in [1] and the corrected model are shown in Figure 2, following the visual representation of the data from Figure 9 in [1]. The average error is given by:

$$\frac{\sum_{\text{observations}} |N_{\text{predicted}} - N_{\text{observed}}|/N_{\text{observed}}}{\text{number of observations}},$$

where $N_{\text{observed}}$ is the number of packets observed during 100 s intervals and $N_{\text{predicted}}$ is the number of packets predicted by the models. The smaller average error indicates better model accuracy. The detailed explanation of the traces and average error can be found in [1]. Figures 1 and 2 show that the full model in [1] overestimates the send rate, and the corrected model is more accurate than the full model.

## II. Correction to $A(w,k)$

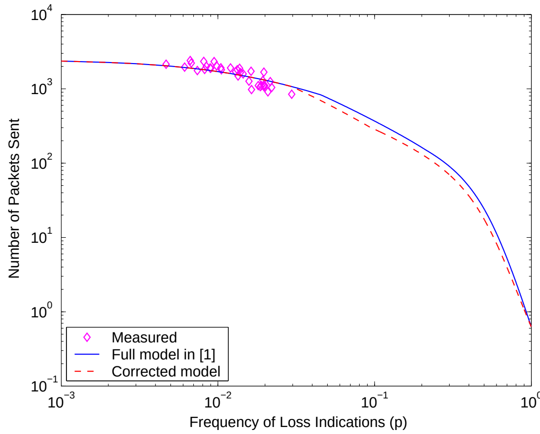
$A(w,k)$ should be expressed as

$$A(w,k) = \begin{cases} \frac{(1-p)^k p}{1-(1-p)^w}, & 0 \le k < w; \\ 0, & otherwise. \end{cases} \tag{19}$$

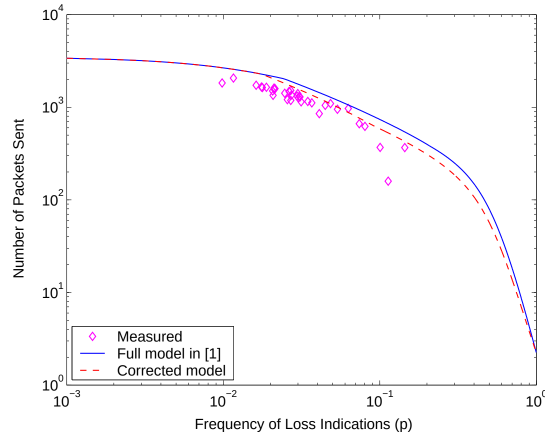
It is suggested that $A(w,k) \neq 0$ for $k = w$ in [1], which is not the case. This correction does not affect the final result for $\hat{Q}(w)$ (Equation (24) in [1]).

## References

[1] J. Padhye, V. Firoiu, D. Towsley and J. Kurose, "Modeling TCP Reno Performance: A Simple Model and Its Empirical Validation", *IEEE/ACM Transactions on Networking*, vol. 8, no. 2, pp. 133-145, April 2000.




(a)

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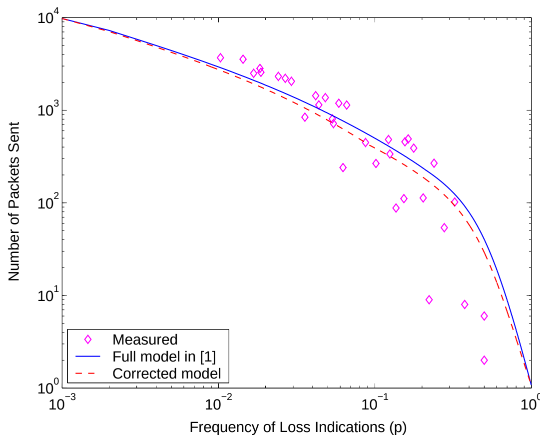


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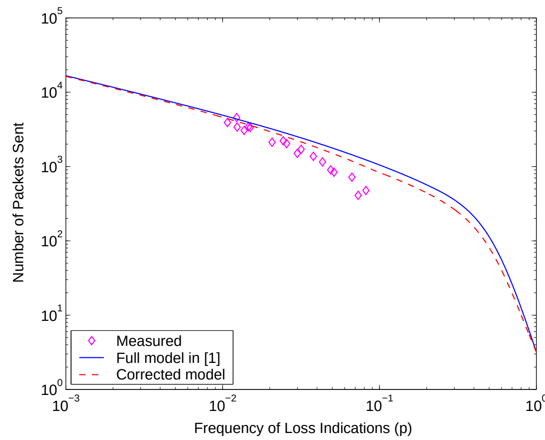


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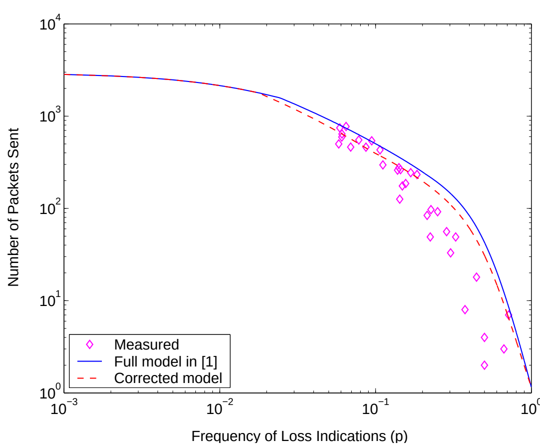


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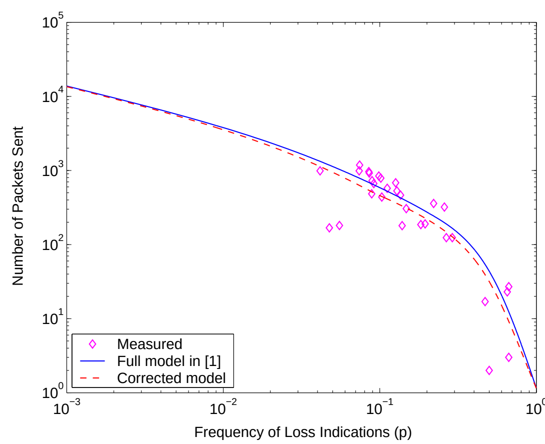


(f)

Fig. 1. (a)Manic to baskerville, RTT = 0.243, $T_0$ = 2.495, $W_m$ = 6. (b)Pif to imagine, RTT = 0.229, $T_0$ = 0.700, $W_m$ = 8. (c)Pif to manic, RTT = 0.257, $T_0$ = 1.454, $W_m$ = 33. (d)Void to alps, RTT = 0.162, $T_0$ = 0.489, $W_m$ = 48. (e)Void to tove, RTT = 0.272, $T_0$ = 1.356, $W_m$ = 8. (f)Babel to alps, RTT = 0.194, $T_0$ = 1.359, $W_m$ = 48.

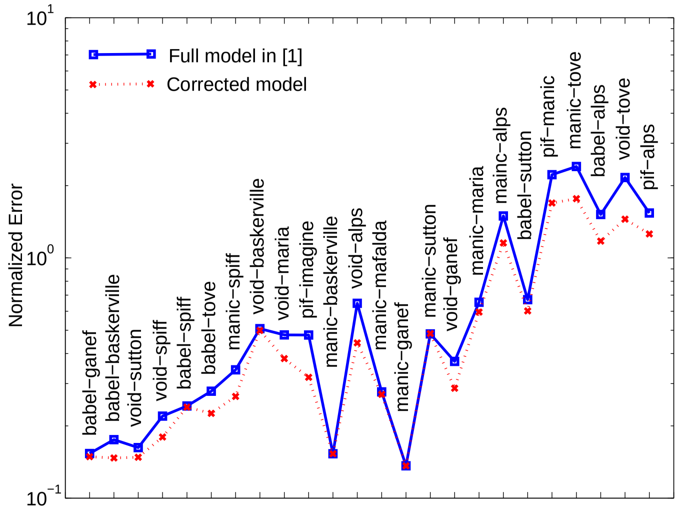


Fig. 2. Comparison of the models for 1 h traces.