Midterm 3
Monday, 3-6-95 9:30-10:20
Math 132
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Name ______________________ Student ID# ____________

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 35 points.

Here are some formulas you might need:

$$\llbracket t \rrbracket = \text{greatest integer less than } t \tag{1}$$

$$n \neq 0 \Rightarrow \frac{d}{dx}x^n = nx^{n-1} \tag{2}$$

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)g'(x) + f'(x)g(x) \tag{3}$$

$$g(x) \neq 0 \Rightarrow \frac{d}{dx}(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \tag{4}$$

$$\frac{d}{dx}((f\circ g)(x)) = f'(g(x))\cdot g'(x) \tag{5}$$

$$\int_a^b k\cdot f(x)dx = k\int_a^b f(x)dx \tag{6}$$

$$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx \tag{7}$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx \tag{8}$$

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du \tag{9}$$

Theorem 0.1 (Mean Value Theorem) If $F(x)$ is continuous on a closed interval $[x_{i-1}, x_i]$ and differentiable on (x_{i-1}, x_i), then $\exists \bar{x}_i \in (x_{i-1}, x_i)$ such that

$$\frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}} = F'(\bar{x}_i).$$

Write your name on each page.

Consider the interval $[a, b]$. State the definition of a partition P of this interval. State the definition of the norm of P and the definition of a set of sample points for P. (4)

If $f(x)$ is defined on the interval $[a, b]$, use the partition P you just described in the previous question to state the formula for the Riemann sum of $f(x)$:(1)

$$R_P(f) = \underline{\hspace{6cm}}.$$

State the definition of $f(x)$ is integrable on $[a, b]$.(2)

State the Fundamental Theorem of Calculus.(3)

Prove the Fundamental Theorem of Calculus.(6)

Give an example of a function $f(x)$ defined on $[0,1]$ which is integrable on $[0,1]$ and positive ($f(x) > 0$ on $[0,1]$) but not continuous on $[0,1]$. Then compute the definite integral $\int_0^1 f(x)dx$. (Hint: don't pick a difficult example.)$^{(2)}$

Give an example of an integrable function $g(x)$ on $[1,2]$ such that $g(x) \geq 0$ on $[1,2]$, $g(x)$ is not a constant function on $[1,2]$, and $\int_1^2 g(x)dx = 0$.$^{(2)}$

What's wrong with the following calculation? (hint: $\frac{1}{x^2}$ is never negative, but a negative area seems to follow from using FTC.)$^{(2)}$

$$\int_{-2}^{1} \frac{1}{x^2} dx = \frac{-1}{1} - \frac{-1}{-2} = -\frac{3}{2}.$$

Let $h(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$. Define $A(x) = \displaystyle\int_0^x h(t)dt$. Find a simple expression for $A(x)$, for all points $x \in \mathbb{R}$, and sketch a graph of $A(x)$. Why doesn't $A'(x) = h(x)$?$^{(4)}$

4. Do the following calculations any way you want, using a different page for scratchwork. Clearly indicate your answer in terms of the positive constants a, b, c, d if they appear; assume $f(x)$ is continuous on $\mathbb{R}$, with antiderivative $F(x)$ (i.e., $F'(x) = f(x)$). (1.5 pts each; best 6 of 9)

- $\displaystyle\int_0^1 (x^2 - 5)dx$
- $\displaystyle\int_a^b \frac{f(\sqrt{x})}{2\sqrt{x}}dx$
- $\displaystyle\int_c^d \left(\int_a^b (3t^2x^2 + 2t)dx\right) dt$
- $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{4i}{n}}\frac{4}{n}$
- $\displaystyle\int_1^6 |x - 3|dx$
- $\displaystyle\int_1^6 (f(x))^2 f'(x)dx$
- $\displaystyle\int_0^2 [\![2x]\!]\, dx$
- $\displaystyle\int_0^9 \sqrt{[\![x]\!]}\, dx$
- $\displaystyle\int_{-4}^{-9} \frac{-1}{\sqrt{-x}}dx$

Optional Extra Credit$^{(5)}$: Use the Fundamental Theorem of Calculus to prove the Monotonicity Theorem.