Name ___________________________ Student ID# _____________

Midterm 3. Wednesday, 3-12-97 12:30-1:20. Math 101. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

There are no questions on this page

(definition of circle): A circle with center (p, q) and radius $r > 0$ is the set of points (x, y) in the plane such that the distance from (p, q) to (x, y) is r.

The "unit circle" is the circle with center $(0, 0)$ and radius 1.

Recall π is the constant $3.14159265\ldots$.

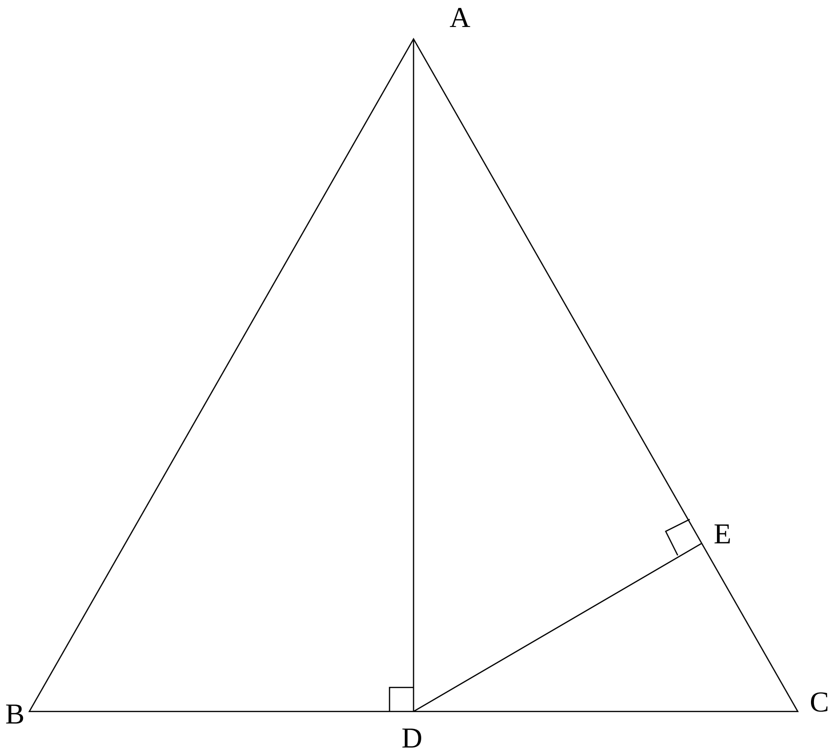
The circumference of a circle with radius r is $2\pi r$.

The letter "theta" is written as θ and I'll use it to denote some angles.

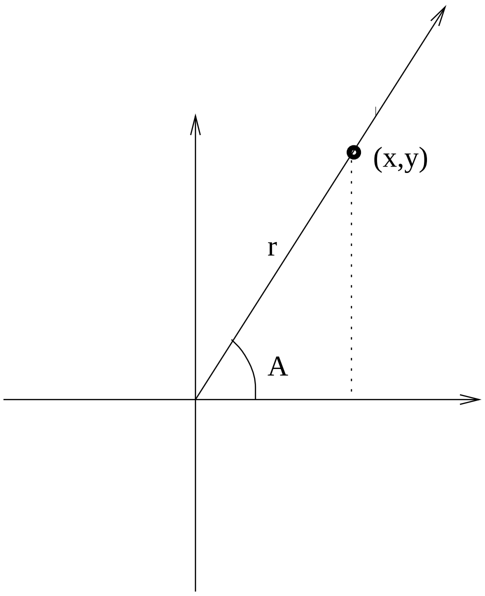
1. (each line 1/2 pt) Fill in the blanks with numbers that measure an angle:

360 degrees	=	__________ radians	=	__________ revolutions,
__________ degrees	=	$\pi/4$ radians	=	__________ revolutions,
__________ degrees	=	__________ radians	=	1/6 revolutions,
30 degrees	=	__________ radians	=	__________ revolutions,
__________ degrees	=	1 radian	=	__________ revolutions,
__________ degrees	=	__________ radians	=	3/4 revolutions.

2. (4 pts) Here's an equilateral triangle $\triangle ABC$, with each side length 2 units. Let D be the midpoint of BC so that AD and BD form a right angle. Calculate the distance between A and D. Also, assuming DE and AE form a right angle, calculate the lengths of segments DE, AE, and CE.



3. For the angle A in standard position through the point (x, y), state a formula for the following quantities in terms of x and y. (1 pt each)



- $r =$ distance from $(0, 0)$ to $(x, y) =$
- $\sin(A) =$
- $\cos(A) =$
- $\tan(A) =$
- $\sec(A) =$
- $\csc(A) =$
- $\cot(A) =$
- $\sin(-A) =$
- $\cos(-A) =$
- $\sin(A + 180°) =$
- $\cos(A + 180°) =$

4. (1 pt) In terms of the quantities r, $\sin(A)$, and $\cos(A)$, state formulas for x and y:

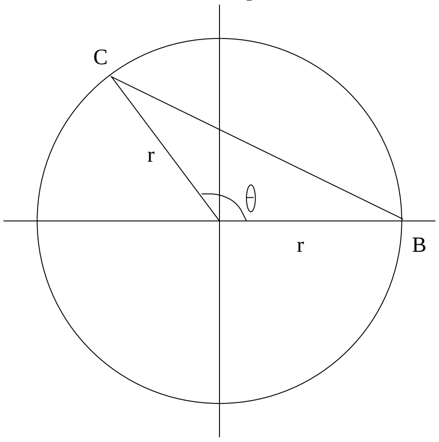
5. (1 pt) Using your definition of $\sin(A)$ and $\cos(A)$, show that

$$(\sin(A))^2 + (\cos(A))^2 = 1.$$

6. (1 pt) Starting with the theorem $(\sin(A))^2 + (\cos(A))^2 = 1$, derive the identity $(\tan(A))^2 + 1 = (\sec(A))^2$.

7. (1 pt) State the definition of "angle A is acute" in terms of radians.

8. (3 pts) A circle of radius r is centered at the origin $(0, 0)$. The point where it intersects the positive x-axis is $B = (r, 0)$. C is some other point on the circle, where the angle θ meets the circle. Calculate the length of the (counterclockwise) arc BC, in terms of θ **degrees**. Also find the arc length if θ is an angle measured in radians. Use Problem #4 and the distance formula to find the length of the straight line segment BC.



9. Here are eleven functions, and on the next two pages are eleven Maple printouts of their graphs, labeled $A(x)$, $B(x)$, ... , $I(x)$. The variable x represents **radians**. Match the function to the graph by writing the corresponding letter next to each function. (There are no repetitions, and don't leave any blank.) (1 pt each)

- ____________ $y = \sin(x)$
- ____________ $y = -\sin(x)$
- ____________ $y = |\sin(x)|$
- ____________ $y = 2\sin(x)$
- ____________ $y = \frac{1}{2}\sin(x)$
- ____________ $y = \frac{1}{5}\sin(x) + 1/2$
- ____________ $y = \sec(x)$
- ____________ $y = \cos(x)$
- ____________ $y = \cos(2x)$
- ____________ $y = \cos(x/2)$
- ____________ $y = \tan(x)$
- ____________ $y = \sec(x)$

11. (1 pt each) Simplify each expression using trigonometric identity theorems. (show work, but you don't have to say which theorem you are using.)

- $\csc^2(\theta) - 1 =$
- $\frac{-\sin(-\theta)}{-\cos(-\theta)} =$
- $1 + \tan^2(-\theta) =$
- $\frac{\tan(\theta)}{\sec(\theta)} =$
- $\sin^4(\theta) + 2\sin^2(\theta)\cos^2(\theta) + \cos^4(\theta) =$

12. (2 pts) Use the trig identities to show that

$$\frac{\cos^2(\theta)}{1 - \sin(\theta)} + \frac{\cos^2(\theta)}{1 + \sin(\theta)}$$

is a constant.

13. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. θ is assumed to be an angle measured in **radians** (1 pt each)

____________ If $\theta > 2\pi$ then $\sin(\theta) > 1$.

____________ $\sin(\theta) = \cos(\theta - \pi/2)$.

____________ $\sin(\theta) = \cos(\pi/2 - \theta)$.

____________ $\frac{\tan(\theta)}{1+\tan^2(\theta)} = \frac{\sin(\theta)}{\sec(\theta)}$.

____________ $\tan(\theta) + \cot(\theta) = \frac{\sin(\theta)+\cos(\theta)}{\sin(\theta)\cos(\theta)}$.

____________ The point $(\frac{9}{41}, \frac{40}{41})$ is on the unit circle. (Hint: $41 \cdot 41 = 1681$)

____________ The point $(\frac{29}{41}, \frac{30}{41})$ is on the unit circle. (Hint: $29 \cdot 29 = 841$)