Name ________________________ Student ID# ____________

Midterm 3. Wednesday, 11-27-96 12:30-1:20. Math 100. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

Definition 2.1 $\mathbb{R}^2$ is the set $\{(x,y) : x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$, of "ordered pairs" of real numbers.

Definition 2.2 The x-axis is a subset of $\mathbb{R}^2$, defined by

$$\{(x,y) \in \mathbb{R}^2 : y = 0\}.$$

The y-axis is the set

$$\{(x,y) \in \mathbb{R}^2 : x = 0\}.$$

Definition 2.3 A subset $G \subseteq \mathbb{R}^2$ is "the graph of a function" if the following implication is true for any points (x_1, y_1) and (x_2, y_2) in the set G:

$$x_1 = x_2 \implies y_1 = y_2.$$

Definition 2.4 A "function" is the rule that assigns to the number x the number y such that (x,y) is on the graph.

There can't be two numbers y that get assigned to x by the definition of "graph of a function." However, there may be some $x \in \mathbb{R}$ that are not assigned a number y.

Definition 2.5 The "domain" of a function with graph G is the set $\{x \in \mathbb{R} : \text{there is a point } (x,y) \in G\}$.

Questions #1–#10 one point each. Questions #1–#7 refer to points $P = (a, b)$ and $Q = (c, d)$ in $\mathbb{R}^2$ such that $a \neq c$. Let $\overline{PQ}$ be the line segment connecting them. State the formulas:

1. The length of $\overline{PQ} =$

2. The slope of $\overline{PQ} =$

3. The slope of any line perpendicular to $\overline{PQ} =$

4. The midpoint of $\overline{PQ} =$

5. The line through P and Q has equation:

6. The circle C with center P and radius 4 has equation:

7. The circle D with center P such that $Q \in D$ has equation:

8. Assume $B \neq 0$. State the formula for the slope of the line

$$\{(x, y) \in \mathbb{R}^2 : Ax + By + C = 0\},$$

and calculate the intersection of this line with the y-axis.

9. State the definition of "the function $f(x)$ is increasing on the set I."

10. State the Triangle Inequality for points K, L, M in $\mathbb{R}^2$.

Graph the following functions. Each graph should be larger than a quarter. (#11–#17 3 pts each)

For #11 and #12, indicate where the graph intersects the x-axis and the y-axis. (plug in $x = 0$ and $y = 0$ and plot these ordered pairs on your graph)

11. $y = \frac{x}{2} + 6$.

12. $y = -6x + 12$.

13. $y = (x - 2)^2 + 1$. (Hint: expanding the polynomial won't help.)

14. On the same coordinate axes, sketch both $y = |x| - 1$ and $y = |x - 1|$. (Label each graph—they will overlap for some x)

15. $y = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ 3x+5 & \text{if } x \leq 0 \end{cases}$

16. $y = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

17. On the same coordinate axes, graph both $y = \sqrt{1-x^2}$ and $y = \frac{1}{\sqrt{1-x^2}}$. (Label each graph—they will overlap for some x) (Hint:The first function has domain $[-1,1]$)

(#18–#20 two pts each)

18. Graph the circle $(x-2)^2+(y-5)^2=16$. Sketch all the lines through the origin that are tangent to the circle. (But you don't have to state any equations)

19. The function $f(x)=x^2-5x+2$ is a parabola, and I've graphed it. Calculate exactly the intersection of the parabola with the x-axis and with the y-axis.

20. Let S be the half-plane $\{(x,y)\in\mathbb{R}^2 : x>1\}$, and let T be the half-plane $\{(x,y)\in\mathbb{R}^2 : y\geq 1\}$. (Note that S does not contain its boundary line, but T does.) Sketch, on separate coordinate axes, the sets S, T, $S\cup T$, $S\cap T$, and the set $\{(x,y)\in\mathbb{R}^2 : (x,y)\in S\cup T \text{ AND } (x,y)\notin S\cap T\}$.

21. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. The capital letters A, B, C refer to points in $\mathbb{R}^2$. You need not explain your answer.$^{(1\ each)}$

____________ If two circles intersect in exactly one point, then they can't have the same area.

____________ If the distance from A to C is equal to the distance from A to B plus the distance from B to C, then A, B, and C lie on the same line.

____________ If the distance from A to C is not equal to the distance from A to B plus the distance from B to C, then A, B, and C do not lie on the same line.

____________ A subset containing only one point, $\{(a, b)\} \subseteq \mathbb{R}^2$, is the graph of a function.

____________ Every polynomial function has range $\mathbb{R}$.

____________ If $f(x)$ is increasing on a set I, and $I \subseteq J$, then $f(x)$ is increasing on J.

____________ If $f(x)$ is increasing on a set I, and $J \subseteq I$, then $f(x)$ is increasing on J.

____________ If $f(x)$ is increasing on a set I then all scalar multiples $k \cdot f(x)$ are increasing functions on I.

____________ If line L is perpendicular to line M, and line M is perpendicular to line N, then line L is perpendicular to line N.

____________ If line L is parallel to line M, and line M is perpendicular to line N, then line L is perpendicular to line N.

____________ If line L is the set $\{(x, y) : 2x + 3y + 4 = 0\}$, and line M is the set $\{(x, y) : 4x + 6y + 8 = 0\}$, then $L \neq M$.

22. (2 pts) A piece of chalk starts at length $3\frac{1}{4}$ inches, with diameter $\frac{3}{8}$ inches. Assume the shape of the chalk is cylindrical and that every ten minutes that Adam writes with it, its length shrinks one inch as a linear function of time.

a. Write a function describing the length $L(t)$ of the chalk after t minutes, and calculate how long it will take to shrink to length 0. Also sketch a graph of $L(t)$.

b. The volume of a cylinder of length L and radius R is $\pi \cdot R^2 \cdot L$. Write the volume $V(t)$ of the chalk as a function of time t.