Midterm 2

Friday 2-17-95 9:30-10:20

Math 132

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Name ________________________ Student ID# ____________

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of 400.

Here are some formulas you might need:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{1}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \tag{2}$$

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)g'(x) + f'(x)g(x) \tag{3}$$

$$g(x) \neq 0 \Rightarrow \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \tag{4}$$

$$\frac{d}{dx}((f\circ g)(x)) = f'(g(x))\cdot g'(x) \tag{5}$$

Write your name on each page.

Each question on the attached ditto is worth four points

True or False. Write **T** or **F** next to each sentence. You need not explain your answer.$^{(1\ each)}$

______ $\lim_{x\to\infty} f(x) = L$ AND $\lim_{x\to\infty} g(x) = M \implies \lim_{x\to\infty}(f+g)(x) = L+M$.

______ $\lim_{x\to\infty} f(x) = L$ AND $\lim_{x\to\infty} g(x) = \infty \implies \lim_{x\to\infty}(f+g)(x) = \infty$.

______ $\lim_{x\to\infty} f(x) = \infty$ AND $\lim_{x\to\infty} g(x) = -\infty \implies \lim_{x\to\infty}(f+g)(x) = 0$.

__________ $(\sum_{i=1}^{n} a_i)^2 = \sum_{i=1}^{n} a_i^2$.

__________ $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a_{(n-i+1)}$.

__________ $\sum_{i=1}^{n} a_i^2 = 0 \iff a_1 = a_2 = \ldots = a_n = 0$.

2. Let A be the set $\{(x,y)\in\mathbb{R}^2$ such that $0\le x\le 4$ AND $0\le y\le\sqrt{x}\}$. Sketch this region. Draw inscribed rectangles with base given by the following line segments on the x-axis:

$$[0,\frac{1}{4}],\ [\frac{1}{4},1],\ [1,\frac{9}{4}],\ [\frac{9}{4},4]$$

Compute the total area of these four rectangles. (2 pts)

Find the total area of four circumscribing rectangles with the same bases (but different heights.) (2 pts)

3. (3 pts) Suppose $f(x)$ is differentiable on the open interval (m, n), and that the derivative $f'(c)$ is positive for every c in the interval. Use the Mean Value Theorem to show that if $m < a < b < n$, then $f(a) < f(b)$.

Hint: **Theorem 0.1** (Mean Value Theorem) If $f(x)$ is continuous on a closed interval $[a, b]$ and differentiable on (a, b), then $\exists c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

4. Do the following calculations any way you want. Clearly indicate your answer in terms of x, the sequence a_i, and/or the integer n (assumed ≥ 2). (2 pts each; best 7 of 8)

- $\sum_{i=1}^{n}(3i - 1) =$
- $\sum_{i=1}^{n-1} i =$
- $\sum_{i=-n}^{n} i^2 =$
- $\int x^2 dx =$
- $\int x^{-2} dx =$
- $\int (3x)^{-n} dx =$
- $\int f''(x) dx = \int \frac{d^2}{dx^2} f(x) dx =$
- $\sum_{i=1}^{n}(\sqrt{a_i} - \sqrt{a_{i+1}})(\sqrt{a_i} + \sqrt{a_{i+1}}) =$

In the following two questions, give a formula or a clearly drawn sketch. (2 each)

5. Give an example of a set $S \subseteq \mathbb{R}$ and a function $F(x)$ with domain S such that $\frac{d}{dx}F(x) = 0$ on S, but $F(x)$ is not a constant function on S.

6. Give an example of a set $A \subseteq \mathbb{R}^2$ and another set $B \subseteq \mathbb{R}^2$ such that $A \neq B$, $A \subseteq B$, the Areas of A and B both exist, and $\text{Area}(A) = \text{Area}(B)$.

7. By filling in twelve numbers in the following blanks, give examples of three rectangles A, B, and C such that $A \cup B \subseteq C$ and $\text{Area}(A) + \text{Area}(B) \geq \text{Area}(C)$. (2 pts)

$$A = \{(x, y) \in \mathbb{R}^2 \text{ such that } \_\_\_\_ \leq x \leq \_\_\_\_ \text{ AND } \_\_\_\_ \leq y \leq \_\_\_\_\},$$

$$B = \{(x, y) \in \mathbb{R}^2 \text{ such that } \_\_\_\_ \leq x \leq \_\_\_\_ \text{ AND } \_\_\_\_ \leq y \leq \_\_\_\_\},$$

$$C = \{(x, y) \in \mathbb{R}^2 \text{ such that } \_\_\_\_ \leq x \leq \_\_\_\_ \text{ AND } \_\_\_\_ \leq y \leq \_\_\_\_\}.$$

8. A frog is at the bottom of a well which is thirty feet deep. Each day, it can climb up the vertical wall three feet, but at night, it slides back down two feet. How many days will it take the frog to escape the well? (1 pt)