Midterm 2
Friday, 11-11-94 9:30-10:20
Math 131
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Name _____ Student ID# _____

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of 400.

Hypotheses for theorems 1–5.
k is a constant.

$\lim_{x \to c} f(x)$ exists.

$\lim_{x \to c} g(x)$ exists.

Conclusions:

1. $\lim_{x \to c} k = k$.
2. $\lim_{x \to c} x = c$.
3. $\lim_{x \to c}(k \cdot f(x)) = k \cdot \lim_{x \to c} f(x)$.
4. $\lim_{x \to c}(f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$.
5. $\lim_{x \to c}(f(x) \cdot g(x)) = (\lim_{x \to c} f(x)) \cdot (\lim_{x \to c} g(x))$.

The Composite Limit Theorem. If $\lim_{x \to c} g(x) = L$ AND f is continuous at L, then

$$\lim_{x \to c}(f \circ g)(x) = f(\lim_{x \to c} g(x))$$

which could also be written as

$$\lim_{x \to c} f(g(x)) = f(L)$$

1. State the "limit" (not the $\epsilon,\ \delta$) definition of "$f(x)$ is continuous at c."
(2 *points*)

2. State the definition of

$$\lim_{z \to b} w(z) = M.$$

(2 *points*)

3. Prove, using the definition,

$$\lim_{x \to 4} -6x + 7 = -17.$$

(6 *points*)

scratchwork:

formal proof:

3. Compute the following limits any way you want. You don't have to explain your reasoning. Write either a real number after the "=," or "**DNE**" if the limit Does Not Exist. The questions with the function $f(x)$ refer to the above graph. $^{(1\ each)}$

- $\lim_{x\to 5} \frac{6}{x} =$
- $\lim_{x\to 5} \frac{6}{x-5} =$
- $\lim_{x\to 5} \frac{x^2-25}{x-5} =$
- $\lim_{x\to 0^+} \sqrt{x} =$
- $\lim_{x\to 1} f(x) =$
- $\lim_{x\to 1^+} f(x) =$
- $\lim_{x\to 2} f(x) =$
- $\lim_{x\to 1} |f(x)| =$
- $\lim_{x\to 0^-} f(x) =$
- $\lim_{x\to -1} f(f(x)) =$

4. For each of the five questions below, give an *example* of a function with all of the required properties. It will be good enough to give a formula, or to sketch a graph that is larger than a quarter. The five functions are unrelated to each other or to any other functions on this test. Try all five; I will count your best four answers. $^{3\ each}$

A. The function $f(x)$

- has domain $\mathbb{R}$,
- AND is continuous at every real number which is not an integer,
- AND is not continuous at n if $n \in \mathbb{Z}$.

B. The function $g(x)$

- has domain $[2, 5]$,
- AND $g(2) = -1$,
- AND $g(5) = 2$,
- AND $\forall x \in [2, 5],\ g(x) \neq 0$.

C. The function $h(x)$

- is a rational function, i.e., $h(x) = \frac{p(x)}{q(x)}$, for polynomials $p(x)$ and $q(x)$.
- AND $h(2) = h(-3) = 0$
- AND $\lim_{x \to 4} h(x)$ does not exist.

(write a formula for this one, not just a graph.)

(problem 4, continued) D. The function $j(x)$

- is not a constant function,
- AND $(j(x))^2$ is the constant function 1.

E. The function $z(x)$

- is a polynomial,
- AND the function $\frac{1}{\sqrt{z(x)}}$ has domain $\mathbb{R}$.

5. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. You need not explain your answer.$^{(1\ each)}$

_____ $f(x)$ continuous on the open interval $(a,\ b) \Rightarrow \lim_{x \to b^-} f(x)$ exists.

_____ $\epsilon > 0$ AND $\delta = \min\{1,\ \epsilon\} \Rightarrow \delta > 0$.

_____ $\epsilon > 0$ AND $\delta = \min\{1,\ \epsilon\} \Rightarrow \delta < 1$.

_____ $\lim_{x \to c} g(x) = 0$ AND $\lim_{x \to c} f(x) = L \Rightarrow \lim_{x \to c} \frac{f(x)}{g(x)}$ cannot possibly exist.

_____ $|x - y| \leq |x| - |y|$.

_____ Given any finite, nonempty set S of positive numbers, there is always some element $p \in S$ such that p is the smallest element.

_____ Given any nonempty set S of positive numbers, there is always some element $p \in S$ such that p is the smallest element.

_____ $\lim_{x \to c} f(x) = L \Rightarrow \lim_{x \to 1+c} f(x) = 1 + L$.

6. Use the Limit Theorems #1–5 from the cover page **one at a time** to carefully calculate the following limits: (Justify each step with the numbered theorem; here's an example of what to do:)$^{(2\ each)}$

a. $\lim_{x\to 2} 2x+2 = \lim_{x\to 2} 2x + \lim_{x\to 2} 2$ (using #4)
$= 2\lim_{x\to 2} x + \lim_{x\to 2} 2$ (#3) $= 2\lim_{x\to 2} x + 2$ (#1)
$= 2\cdot 2 + 2$ (#2) $= 6$.

b. $\lim_{x\to 4} 3x^2 =$

c. $\lim_{x\to -3} x^2(2x-1) =$

7. a. Let $f(x) = \frac{1}{x}$, and let $g(x)$ be some function. What are $(f\circ g)(x)$ and $(g\circ f)(x)$? $^{(2\ pts)}$

b. Use the fact that $f(x) = \frac{1}{x}$ is continuous at L if $L \neq 0$, the Composite Limit Theorem, and Limit Theorem #5 (cover page) to prove the following theorem: $^{(4\ pts)}$

$(\lim_{x\to c} h(x) = N$ AND $\lim_{x\to c} g(x) = L \neq 0) \Rightarrow \lim_{x\to c} \frac{h(x)}{g(x)} = \frac{N}{L}$.