Name ________________________ Student ID# ____________

Midterm 2. Wednesday, 5-14-97 12:30-1:20. Math 102. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

There are no questions on this page

θ radians	θ degrees	$\sin(\theta)$
0	0	0
$\pi/6$	30	$1/2$
$\pi/4$	45	$\sqrt{2}/2$
$\pi/3$	60	$\sqrt{3}/2$
$\pi/2$	90	1
$3\pi/4$	135	$\sqrt{2}/2$
π	180	0

Definition 1.1 $\sin^2(\theta)$ is an abbreviation for $(\sin(\theta))^2$, and there are similar abbreviations for the other trigonometric functions: cos, tan, cot, sec, csc.

Theorem 1.2 $\sin^2(\theta) + \cos^2(\theta) = 1$.

Theorem 1.3 $\cos(\theta) = \sin(90^\circ - \theta)$. ($\theta$ degrees)

Theorem 1.4 $\sin(\theta) = \cos(90^\circ - \theta)$. ($\theta$ degrees)

Theorem 1.5 $\sin(-\theta) = -\sin(\theta)$.

Theorem 1.6 $\cos(-\theta) = \cos(\theta)$.

Theorem 1.7 $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

Theorem 1.8 $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.

Questions 1–8 refer to triangle $\triangle ABC$ with (positive) side lengths a, b, c. (2 pts each)

1. State the definition of the "semiperimeter" s.

2. State Heron's formula for the area of $\triangle ABC$ in terms of s, a, b, c.

3. If γ is the angle $\angle ACB$, state the "Law of Cosines" formula for c^2 in terms of a, b, and γ.

$c^2 =$

4. Solve the "Law of Cosines" formula for γ in terms of a, b, and c.

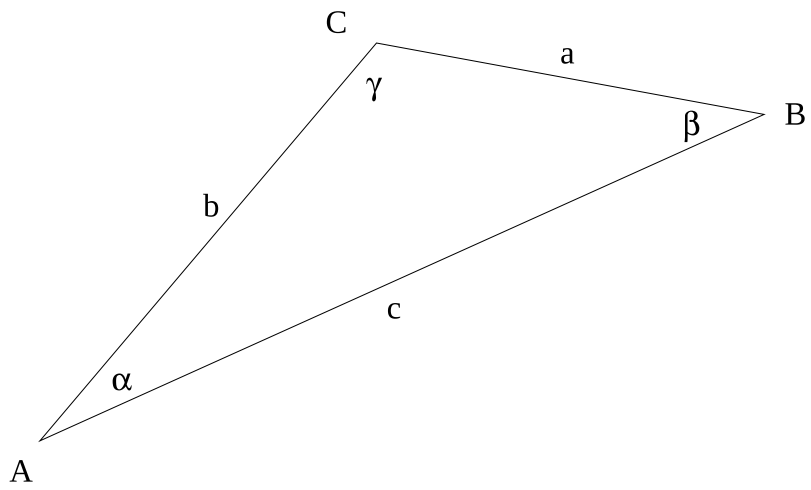
$\gamma =$

5. $\triangle XYZ$ is called "similar to $\triangle ABC$" if its side lengths x, y, z are constant multiples (k) of the side lengths of $\triangle ABC$, that is: $x = ka$, $y = kb$, and $z = kc$.

Calculate and simplify:

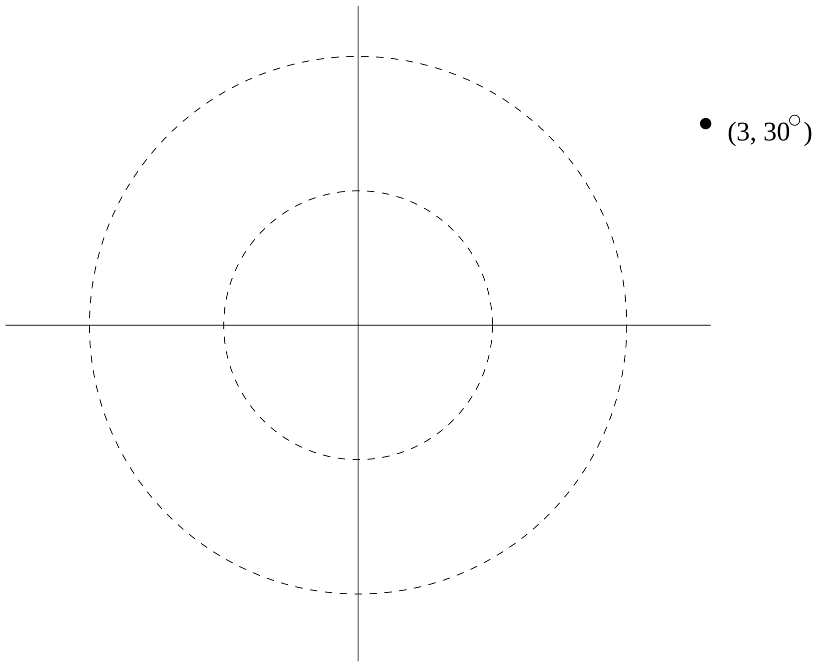
- $\frac{\text{perimeter } _{XYZ}}{\text{perimeter } _{ABC}} =$
- $\frac{\text{area } _{XYZ}}{\text{area } _{ABC}} =$

6. Use #4 to show that $\angle ACB = \angle XZY$.



7. Suppose, in the above triangle, $\gamma = 135°$, $a = 10$, and $b = 8$. Calculate side length c exactly (not necessarily simplifying fractions or radicals. Hint: for obtuse γ, $\cos\gamma < 0$).

8. Suppose that side length c in #7 shrinks to length 9, but a and b are still 10 and 8. Calculate γ.



9. On the above picture are the circles of radius 1 and 2 centered at the origin of the polar coordinate system. I've plotted the point $(r, \theta) = (3, 30°)$. Plot and label the following points. Also, below, convert to rectangular coordinates. (1.5 pts each)

- $(r, \theta) = (2, -60°)$. $(x, y) =$
- $(r, \theta) = (2.5, 0°)$. $(x, y) =$
- $(r, \theta) = (1, 210°)$. $(x, y) =$
- $(r, \theta) = (\sqrt{2}, 5\pi/4$ radians). $(x, y) =$
- $(r, \theta) = (1.5, 5\pi$ radians). $(x, y) =$
- $(r, \theta) = (0, \pi/7$ radians). $(x, y) =$

10. (1 pt) If the rectangular coordinates (x, y) are given, state the formula for conversion to polar coordinates:

$r =$

$\theta =$

Questions 11–15 refer to the number $s \in \mathbb{R}$ and two vectors $\vec{v}$, $\vec{w} \in \mathbb{R}^2$ with rectangular coordinates $\vec{v} = (x, y)$ and $\vec{w} = (a, b)$.

11. (1 pt) If the angle between $\vec{v}$ and $\vec{w}$ is θ, state the definition of the dot product $\vec{v} \cdot \vec{w}$.

12. (1 pt) State the definition of "$\vec{v}$ and $\vec{w}$ are orthogonal."

13. (1 pt each) In terms of the numbers x, y, a, b, s, state formulas for:

- additive inverse: $-\vec{v} =$
- vector sum: $\vec{v} + \vec{w} =$
- scalar multiplication: $s\vec{v} =$
- dot product: $\vec{v} \cdot \vec{w} =$
- complex product: $(x + iy)(a + ib) =$

14. (1.5 pts) Use the radius formula (#10) and the definition of scalar multiplication to prove $|s\vec{v}| = |s||\vec{v}|$. ($|s|$ is the usual absolute value of the real number s, and $|\vec{v}|$ is the "length" of $\vec{v}$.)

15. (1.5 pts) If s and t are scalars (that is, real numbers, not vectors), use either formula for the dot product to prove $(s\vec{v}) \cdot (t\vec{w}) = (st)(\vec{v} \cdot \vec{w})$.

16. (2 pts) The complex number i has rectangular coordinates $0 + 1i$ and polar coordinates $(r = 1, \theta = 90°)$. It has two square roots (which are also complex numbers). Calculate both, in polar and rectangular coordinates.

17. (1 pt) The complex number z can be written in complex polar coordinates $z = (r, \theta)$, and the conversion to complex rectangular coordinates is $z = r(\cos\theta + i\sin\theta)$. Suppose n is a positive integer.

- State the polar form of the nth power of $z = (r, \theta)$: $z^n =$
- State De Moivre's theorem on the rectangular form: $z^n =$

18. True or False. Write **T** or **F** next to each sentence. You need not explain your answer.$^{(1\ each)}$

The first five questions are about polar coordinates with radius r and polar angle θ.

_________ $(r, \theta°) = (-r, -\theta°)$.

_________ $(r, \theta°) = (-r, \theta° + 180°)$.

_________ The scalar multiplication formula is $s(r, \theta) = (sr, s\theta)$.

_________ The scalar multiplication formula is $s(r, \theta) = (sr, \theta)$.

_________ The scalar multiplication formula is $s(r, \theta) = (sr, s + \theta)$.

_________ Every complex number has two different square roots which are also complex numbers.

_________ Every nonzero complex number has two different square roots which are also complex numbers.

_________ (scalars s, t and vectors $\vec{v}$, $\vec{w}$): $s\vec{v} + t\vec{w} = (s + t)(\vec{v} + \vec{w})$.

19. (3 pts) Consider the vector $\vec{B} \neq \vec{0}$. Recall that the "projection onto $\vec{B}$" is defined as a function P whose input is a vector $\vec{A}$, and whose output is another vector $P(\vec{A})$. The formula for the function P is

$$P(\vec{A}) = \left(\frac{\vec{A} \cdot \vec{B}}{\vec{B} \cdot \vec{B}} \right) \vec{B}.$$

Note that this is a scalar multiple of $\vec{B}$, where the scalar is the quotient of two dot products.

Easy: Show that $P(\vec{B}) = \vec{B}$.

Easy: Show that if $\vec{A}$ is orthogonal to $\vec{B}$ then $P(\vec{A}) = \vec{0}$.

Use problem #15 with $t = 1$ to show that $P(s\vec{A}) = sP(\vec{A})$.

Show that $P(P(\vec{A})) = P(\vec{A})$.