Name ________________________ Student ID# ____________

Midterm 2. Friday, 2-14-97 12:30-1:20. Math 101. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

There are no questions on this page

Recall the positive integers, or natural numbers, are elements of the set $\{1, 2, 3, 4, 5, \dots\}$.

1. (1 pt) If b is a positive integer, state the definition of x^b.

2. (1 pt) State the definition of "$\log_b x = y$."

3. (2 pts) State the definition of the function "$\ln(x) = y$." Also state its domain.

4. (1 pt) State the "base-changing formula."

5. (1 pt) In the compound interest formula, $S = P(1 + \frac{r}{n})^{nt}$, S is the "final value" of an initial investment after some period of time. What do the other symbols mean?

- interest rate=
- number of interest calculations per year=
- initial investment=
- time in years=

6. (3 pts) Using this formula for compound interest, and assuming that interest is calculated only once per year, how long will it take to double the investment? (Solve for the time in years in terms of the unknown interest rate)

7. (1 pt) Write these five numbers in increasing order: 10^{-6}, 5×10^{-7}, 5×10^{-5}, 0.2×10^{-6}, 30×10^{-7}:

__________ < __________ < __________ < __________ < __________.

8. (1 pt) Give an example of two positive numbers x, y such that $x^y < y^x$.

- $x =$
- $y =$

9. (1 pt) Give an example of two positive numbers x, y such that $x \cdot y = x^y$.

- $x =$
- $y =$

10. Here are eight functions, and on the next page are eight Maple printouts of their graphs, labeled $A(x)$, $B(x)$, ... , $H(x)$. Match the function to the graph by writing the corresponding letter next to each function. (There are no repetitions, and don't leave any blank.) (1 pt each)

- __________ $y = 4^x$
- __________ $y = x \cdot \ln(x)$
- __________ $y = x + 1$
- __________ $y = e^{-x^2}$
- __________ $y = \ln(x)$
- __________ $y = \log_{10}(x)$
- __________ $y = e^x$
- __________ $y = (1.7)^x$

11. Calculate the following numbers: (assume $b > 1$) ($\frac{1}{2}$ pt each)

- $\log_b 1 =$
- $\log_b b =$
- $\log_2 16 =$
- $\log_3 \frac{1}{3} =$
- $\ln(e^e) =$
- $\log_5 \sqrt{5} =$
- $e^{2\ln 7} =$
- $e^{(\frac{1}{2}\ln 4 + 2\ln 5)} =$
- $\log_{10}(4) + \log_{10}(25) =$
- $\log_{10}(2) - \log_{10}(200) =$

12. (3 pts) This is a graph of the function $y = Ae^{kx} + C$, together with its horizontal asymptote line. Using the coordinates of the two marked points, solve for the constants:

- $C =$
- $A =$
- $k =$

13. Manipulate the following expressions, assuming x and y are positive: (1 pt each)

- simplify: $e^{(5\ln(x)-3\ln(y))} =$
- expand as a sum or difference: $\ln\left(\sqrt{\frac{4}{4+x}}\right) =$
- simplify: $e^{(\frac{1}{2}\ln(x^2-2x+1))} =$
- expand: $(e^x - e^{-x})^2 =$
- Solve for x: $e^{5\ln(x)} - 32 = 0$:
- Find the two solutions for x: $e^{2x} - 5e^x + 6 = 0$:

14. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. Assume a, c, M, and N are all positive, and b is an integer greater than 1. ($\frac{1}{2}$ pt each)

______________ $(b^x)^y = b^{(x^y)}$.

______________ $(b^x)^y = b^{(x \cdot y)}$.

______________ $(a + c)^b = a^b + c^b$.

______________ $(a + c)^b = a^b + bac + c^b$.

______________ $(ac)^b = a(c^b)$.

______________ $a^{(b+1)} = a^b + a$.

______________ $b^{\log_b c} = c$.

______________ $\log_b(b^y) = y$

______________ $\log_b(M \cdot N) = \log_b M + \log_b N$.

______________ $\log_b(M \cdot N) = (\log_b M) \cdot (\log_b N)$.

______________ $\log_b(M \cdot N) = (\log_b M)^{(\log_b N)}$.

______________ $\log_b(M + N) = \log_b M + \log_b N$.

______________ $\log_b(M + N) = (\log_b M) \cdot (\log_b N)$.

______________ $\log_b(M + N) = (\log_b M)^{(\log_b N)}$.

______________ $\log_b(1/c) = 1 - \log_b c$.

______________ $\log_b(1/c) = (\log_b 1) - (\log_b c)$.

______________ $\log_b(1/c) = -\log_b c$.

True or False, continued:

____________ $y = \ln(x)$ is an increasing function.

____________ If $k \neq 0$ then $y = e^{kx}$ is an increasing function of x.

____________ $\ln(e^c) = e^{\ln(c)}$.

____________ $y = \ln|x|$ has domain $\mathbb{R}$.

____________ $c^{-\frac{1}{2}} = \sqrt{\frac{1}{c}}$.

____________ $c^{-\frac{1}{2}} = \sqrt{-c}$.

____________ $c^{-\frac{1}{2}} = \frac{1}{\sqrt{c}}$.

____________ $c^{-\frac{1}{2}} = \frac{1}{c^2}$.

____________ $0 < w < 1,\ x > 1 \implies \log_w(x) < 0$.

____________ $2^{35} = 34359738369$.

____________ $10^{-4} = .0004$

____________ For a given interest rate, continuous compounding earns a greater return than annual compounding.

____________ If $1 < k < K$, then $e^{kx} < e^{Kx}$.

____________ If $1 < k < K$, and $x > 0$, then $e^{kx} < e^{Kx}$.

____________ If $1 < k < K$, and $x > 0$, and $A > 0$, then $Ae^{kx} < Ae^{Kx}$.