Name ______________________________ Student ID# _______________

Midterm 2. Friday, 11-8-96 12:30-1:20. Math 100. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

Definition 2.1 The logical symbols NOT, AND, OR, and $\Rightarrow$ are defined by the following rules for combining the labels **T** and **F** for sentences A and B.

A	B	NOT A	A AND B	A OR B	$A \Rightarrow B$
T	**T**	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**
F	**F**	**T**	**F**	**F**	**T**

Definition 2.2 A number system $\mathbb{F}$ is called *ordered* if the set $\mathbb{F}$ contains a subset $P \subseteq \mathbb{F}$ with the following properties: (Recall $x \in P$ is the same as "x is positive" or "$x > 0$.")

- Additive Order: $(x > 0$ AND $y > 0) \Rightarrow (x + y) > 0$.
- Multiplicative Order: $(x > 0$ AND $y > 0) \Rightarrow (x \cdot y) > 0$.
- Trichotomy: Every $x \in \mathbb{F}$ has *exactly* one of the following properties:
  - ⋄ $x > 0$,
  - ⋄ $-x > 0$,
  - ⋄ $x = 0$.

Note that $\mathbb{Q}$ and $\mathbb{R}$ are ordered, and so are the integers $\mathbb{Z}$, but $\mathbb{C}$ is not ordered.

Questions #1–7 one point each.

1. Recall that, for $a < b$, the "open interval" of real numbers between a and b is denoted (a, b), and defined by the set $\{x \in \mathbb{R} : a < x < b\}$. State the similar set notation for all the intervals:

- $[a, b] =$
- $[a, b) =$
- $[a, \infty) =$
- $(-\infty, b) =$

2. State the Theorem on Transitivity of Inequality.

3. State the definition of $|x|$.

4. State the Triangle Inequality.

5. Give an example of a pair of numbers x, y where the Triangle Inequality happens to be an equality. (show your example is correct) Then demonstrate another pair where the Triangle Inequality is a strict inequality. (show the two numbers you get are different)

6. For two points x, $y \in \mathbb{R}$, state the definition of the "distance" between x and y.

7. State the formula for the average of two real numbers a and b.

8. Fill in this table with **T** or **F**. $^{(4\ pts)}$

A	B	NOT(A OR B)	(NOT A) OR (NOT B)	$A \iff B$	(NOT B) $\Rightarrow$ (NOT A)
T	**T**				
T	**F**				
F	**T**				
F	**F**				

9. Solve the following inequalities, that is, find **all** numbers $x \in \mathbb{R}$ that satisfy the given statement. You don't have to explain your reasoning or unnecessarily simplify your answer, but you must phrase your answer in **interval notation**, e.g., $[4, 5)$. The solutions x might involve both numbers and real constants such as m, b, c, δ. The solutions $\emptyset$ and $\mathbb{R}$ might also occur, and the solutions might be disconnected unions of intervals, e.g., $[4, 5) \cup (10, \infty)$. Check your answers! (Give an answer for each question; I will count your twelve best answers out of fifteen for one point each.)

- ______________ $2x - \frac{4}{3} > 0$.
- ______________ $2x - \frac{4}{3} \neq 0$.
- ______________ $mx + b > 0,\ m > 0$.
- ______________ $mx + b > 0,\ m < 0$.
- ______________ $-11 \leq 2x - 3 < 7$.
- ______________ $\frac{4x}{3} - 2 > \frac{x}{4}$.
- ______________ $\sqrt{x - c} \in \mathbb{R}$.
- ______________ $|x - 3| \leq 5$.
- ______________ $|2x + 1| \geq 1$.
- ______________ $x \in (0, 3)$ AND $x \in (2, 6]$.
- ______________ $x \in (-1, 3)$ OR $x \in (2, 6]$.
- ______________ $x \in (-1, 3)$ OR ($x \in (2, 4]$ AND $x \neq 2.5$).
- ______________ ($x \in (-1, 3)$ OR $x \in (2, 4]$) AND $x \neq 2.5$.
- ______________ $|x - c| < \delta$ AND $x \neq c$.
- ______________ $|\frac{1}{2}x - \frac{3}{2}| < \frac{3}{4}$ AND $x \notin \mathbb{Z}$.

10. Solve the following equalities. Same instructions as #9, but if there are finitely many solutions, just list them.

- ______________ $|x - 2| = 0.1$.
- ______________ $|x^2 - 5x| = 6$.
- ______________ $|x| + x = 0$.
- ______________ $|2x| = x^2$.

11. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. The capital letters A, B, C refer to sets. You need not explain your answer.$^{(1\ each)}$

____________ $0 < a < b \Rightarrow a^2 < ab$.

____________ $(c > 0$ AND $|x - c| < \frac{c}{2}) \Rightarrow x > 0$.

____________ $(4, 5] \cap [4\frac{1}{2}, 7)$ is a closed interval.

____________ $(4, 5] \cup [4\frac{1}{2}, 7)$ is a closed interval.

____________ $0 < a \Rightarrow \sqrt{a} < a^2$.

____________ $(A \subseteq B$ and $B \subseteq C) \Rightarrow A \subseteq C$.

____________ $(A \subseteq B$ and $C \subseteq B) \Rightarrow A \subseteq C$.

____________ $a^2 + b^2 = 0 \implies a = 0$.

____________ $\{a, b\} \subseteq (a, b)$.

____________ $(A \subseteq B$ AND $B \cap C = \emptyset) \Rightarrow A \cap C = \emptyset$.

____________ $(A \subseteq B$ AND $A \cap C = \emptyset) \Rightarrow B \cap C = \emptyset$.

____________ $(A \subseteq B$ AND $A \cup C = \emptyset) \Rightarrow B \cup C = \emptyset$.

____________ $\frac{a}{b} \leq \frac{c}{d}$ AND $b \neq 0$ AND $d \neq 0 \implies abd^2 \leq b^2cd$.

____________ $a^2 < b^2 \implies a < b$.

12. Here is a list of statements which could be used to justify steps in a proof. Some of them might get used twice in the proof below, or not at all. Before each line, write the reason why that line follows from the previous line. I have already written one of the answers.(3 pts total)

Theorem 2.3 $|a| - |b| \leq |a - b|$.

A. Commutative Law for +.

B. Additive identity: $x + 0 = x$.

C. Triangle Inequality.

D. Theorem: $x \leq y \Rightarrow x + z \leq y + z$.

E. Additive inverse: $x + (-x) = 0$.

F. Definition of subtraction: $x - y = x + (-y)$.

G. Distributive Law.

H. Associative Law for +.

I. Definition of Absolute Value.

PROOF:

______________ $|a| = |a + 0|$

______________ $|a| = |a + (b + (-b))|$.

______________ $|a| = |a + ((-b) + b)|$.

______________ $|a| = |(a + (-b)) + b|$.

______________ $|a| = |(a - b) + b|$.

______________ $|a| \leq |a - b| + |b|$.

______________ $|a| + (-|b|) \leq (|a - b| + |b|) + (-|b|)$.

______________ $|a| + (-|b|) \leq |a - b| + (|b| + (-|b|))$.

______________ $|a| + (-|b|) \leq |a - b| + 0$.

______________ $|a| + (-|b|) \leq |a - b|$.

______________ $|a| - |b| \leq |a - b|$. ∎

13. (2 pts) Adam buys groceries and has a coupon for "5% off". The cashier rang up the 5% discount first, and then computed the tax $\$.18 + .28 + .53$ on the discounted subtotal. Without calculating anything, explain why (one or two sentences in English) the order of these operations didn't matter. (Why would the total be the same if the tax were computed first?)

(4 pts) Suppose instead the coupon is for \$.99 off, and that the tax is still a fixed percentage of the total. Would Adam pay less:

- if the coupon is subtracted, and then the subtotal taxed, or
- if the subtotal is taxed, and then the \$.99 coupon is subtracted, or
- does the total come out the same?

Pick one of the choices, and again explain, in English or by setting up a formula comparing the two methods, which is correct, but you don't need to calculate anything.