Name ________________________ Student ID# ____________

Midterm 1. Wednesday, 4-23-97 12:30-1:20. Math 102. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

There are no questions on this page

θ radians	θ degrees	$\sin(\theta)$
0	0	0
$\pi/6$	30	$1/2$
$\pi/4$	45	$\sqrt{2}/2$
$\pi/3$	60	$\sqrt{3}/2$
$\pi/2$	90	1
$7\pi/12$	105	$\frac{1}{2}\sqrt{2+\sqrt{3}}$
π	180	0

Definition 1.1 $\sin^2(\theta)$ is an abbreviation for $(\sin(\theta))^2$, and there are similar abbreviations for the other trigonometric functions: cos, tan, cot, sec, csc.

Theorem 1.2 $\sin^2(\theta) + \cos^2(\theta) = 1$.

Theorem 1.3 $\cos(\theta) = \sin(90^\circ - \theta)$. ($\theta$ degrees)

Theorem 1.4 $\sin(\theta) = \cos(90^\circ - \theta)$. ($\theta$ degrees)

Theorem 1.5 $\sin(-\theta) = -\sin(\theta)$.

Theorem 1.6 $\cos(-\theta) = \cos(\theta)$.

Theorem 1.7 $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

Theorem 1.8 $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.

Questions 1, 2, 3 are derivations of trigonometric identities. Show all work, using the page 1 theorems *one at a time*. (3 pts each) Here's an example:

Example 1.9 Show $\cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta)$.

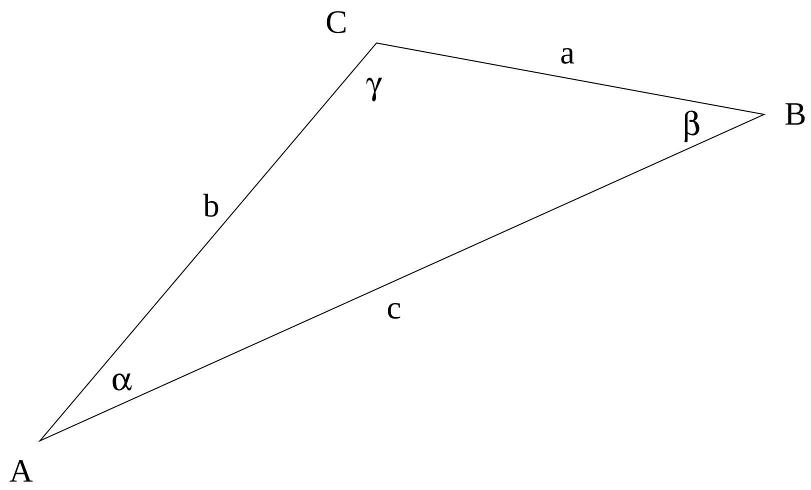
$$\begin{aligned} \cos^2(\theta) - \sin^2(\theta) &= (1 - \sin^2(\theta)) - \sin^2(\theta) \quad (Thm.\ 1.2) \\ &= 1 - 2\sin^2(\theta) \end{aligned}$$

1. Use Theorem 1.8 to derive the "double-angle" formula for $\sin(2\theta)$.

2. Derive the "cosine of a difference" formula for $\cos(\alpha - \beta)$.

3. Show $\sin(180° - \theta) = \sin(\theta)$ for θ degrees. Hint: start with

$$\sin(180° - \theta°) = \sin(90° - (\theta - 90°))$$



4. (3 pts) In this triangle, γ is an obtuse angle. Use the definition of the sine function to show that

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}.$$

5. (1 pt each) Suppose, in the above triangle, $\beta = 45°$ and $\gamma = 105°$ and $a = 10$. Calculate $\angle\alpha$ and side lengths b and c exactly (no decimals, and not necessarily simplifying fractions):

6. Calculate the following numbers exactly, with angles in radians (unmarked θ) or degrees ($\theta°$). You do not have to simplify fractions or radicals in your answer. (1 pt each, best 8 of 10)

$$\begin{aligned}
\sin(120°) &= \underline{\hspace{3cm}} \\
\sin(\frac{3\pi}{4}) &= \underline{\hspace{3cm}} \\
\sin^{-1}(\frac{1}{2}\sqrt{2+\sqrt{3}}) &= \underline{\hspace{3cm}}\ \textit{radians} \\
\sin(210°) &= \underline{\hspace{3cm}} \\
\cos(210°) &= \underline{\hspace{3cm}} \\
\cos(15°) &= \underline{\hspace{3cm}} \\
\sin(52.5°) &= \underline{\hspace{3cm}} \\
\sin(41°)\sin(4°) - \cos(41°)\cos(4°) &= \underline{\hspace{3cm}} \\
\sin(\pi/3) + \sin(\pi/6) &= \underline{\hspace{3cm}} \\
2\sin(\pi/12)\cos(\pi/12) &= \underline{\hspace{3cm}}
\end{aligned}$$

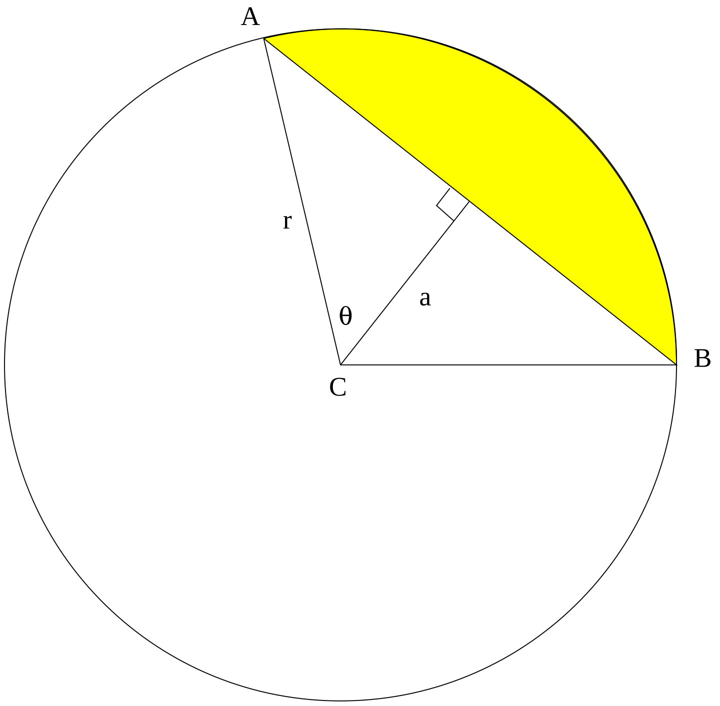
7. (1 pt) State the definition of $\sin^{-1}(x) = y$.

8. (2 pts) Sketch a graph of $y = \sin^{-1}(x)$, indicating its domain and range.

9. (1 pt) State the definition of $\tan^{-1}(x) = y$.

10. (2 pts) Sketch a graph of $y = \tan^{-1}(x)$, indicating its domain and range.

11. (2 pts each) In the following circle with center C and radius $r > 0$, assume θ is an acute angle.



a. Using the fact that the area of the sector ACB (the triangle plus the shaded region) is $\theta \cdot r^2$, calculate the area of the shaded region in terms of a, r, and θ.

b. Using inverse trig functions, express the area of the shaded region in terms of only a and r.

c. If D is the midpoint of the (shorter) arc AB, calculate the areas of triangles $\triangle ADB$ and $\triangle ADC$.

12. True or False. Write **T** or **F** next to each sentence. The variables α, β, and θ are always angles in radians. You need not explain your answer.$^{(1\ each)}$

__________ $\cos^2(\alpha/2) = (1 + \cos(\alpha))/2$.

__________ $\cos(\alpha/2) = \left|\sqrt{(1 + \cos(\alpha))/2}\right|$.

__________ $|\cos(\alpha/2)| = \sqrt{(1 + \cos(\alpha))/2}$.

__________ $\tan(-\theta) = \tan(\theta)$ for $\theta \in$ domain of tan.

__________ $\sin(\alpha)\sin(\beta) = \sin(\frac{\alpha+\beta}{2})$.

__________ $\sin^4(\theta) - 2\sin^2(\theta)\cos^2(\theta) + \cos^4(\theta) = \cos^2(2\theta)$.

__________ $\sin^4(\theta) + 2\sin^2(\theta)\cos^2(\theta) + \cos^4(\theta) = \cos^2(2\theta)$.

__________ A triangle with side lengths 3, 4, 5 must be a right triangle.

__________ A triangle with one side twice as long as another must be a $30°$, $60°$, $90°$ triangle.

In an equilateral triangle $\triangle ABC$, side AB is divided into three equal segments AD, DE, EB. True or false, continued:

__________ Area $\triangle ACD =$ Area $\triangle DCE$.

__________ Perimeter $\triangle ACD =$ Perimeter $\triangle DCE$.

__________ $\angle CDB = \angle CEA$.

__________ $\tan\angle CDB = 3\sqrt{3}$.

13. By drawing and labeling a right triangle, show that $\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$ for $x \geq 0$. (2 pts)