Name _____ Student ID# _____

Midterm 1. Wednesday, 1-21-97 12:30-1:20. Math 101. Adam Coffman.

This is a closed book exam. You may not work together, or use notes or a calculator. Clearly indicate your answers on this test. You may use the back of each page for scratchwork. This test is worth 50 points out of a quarter total of about 450.

There are no questions on this page

Definition 1.1 $\mathbb{R}^2$ is the set $\{(x, y) : x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$, of "ordered pairs" of real numbers.

Definition 1.2 The x-axis is a subset of $\mathbb{R}^2$, defined by

$$\{(x, y) \in \mathbb{R}^2 : y = 0\}.$$

The y-axis is the set

$$\{(x, y) \in \mathbb{R}^2 : x = 0\}.$$

Definition 1.3 A subset $G \subseteq \mathbb{R}^2$ is "the graph of a function" if the following implication is true for any points (x_1, y_1) and (x_2, y_2) in the set G:

$$x_1 = x_2 \implies y_1 = y_2.$$

Definition 1.4 A "function" is the rule that assigns to the number x the number y such that (x, y) is on the graph.

There can't be two numbers y that get assigned to x by the definition of "graph of a function." However, there may be some $x \in \mathbb{R}$ that are not assigned a number y.

Definition 1.5 The "domain" of a function with graph G is the set $\{x \in \mathbb{R} : \text{there is a point } (x, y) \in G\}$.

(#1-8 1 pt each)

1. For the parabola $y = a(x-h)^2 + k$, $a \neq 0$, state the equation of the axis line, and the coordinates of the vertex.

2. For the parabola $y = ax^2 + bx + c$, state the equation of the axis line.

3. State the definition of "r is a root of the polynomial ax^3+bx^2+cx+d."

4. State the formula for the x-intercepts of the quadratic $y = ax^2+bx+c$.

5. Under what condition on the coefficients a, b, c can the polynomial $ax^2 + bx + c$ be factored as $a(x-h)^2$?

6. State the definition of "$R(x)$ is a rational function."

7. In terms of a focus $F = (q,p) \in \mathbb{R}^2$ and a directrix line $D = \{y = d\} \subseteq \mathbb{R}^2$, state the geometric definition of a parabola $P \subseteq \mathbb{R}^2$ by completing the sentence,
"A parabola is the set of points (x,y) such that ... "

8. There are several parabolas that go through the point $P = (a, b) \in \mathbb{R}^2$. Give an example of one of them.

9. Give examples of quadratic polynomials such that: $^{(1 each)}$

- _____ the polynomial has no real roots.
- _____ the polynomial has roots $\frac{1}{4}$ and 5.
- _____ the polynomial has the same roots as $x^2 - 5x + 6$, but it has different coefficients.

10. Here are seven functions, and on the next page are seven Maple printouts of their graphs, labeled A, B, ..., G. Match the function to the graph by writing the corresponding letter next to each function. (There are no repetitions, and don't leave any blank.) (1 pt each)

- _____ $y = (x-4)(x-5)$
- _____ $y = (x-1)^2(x-3)$
- _____ $y = (x-1)(x-3)^2$
- _____ $y = \frac{1}{x^2+10}$
- _____ $y = \frac{1}{x^2} + 10$
- _____ $y = (x-4)^2(x-5)^2$
- _____ $y = \frac{1}{10}(x-4)(x-5)(20(x-7)^2+2)$

11. (3 pts) On the same pair of axes, draw the three parabolas $y = x^2$, $y = -\frac{1}{2}x^2$, and $y = (x-4)^2$.

12. (3 pts) On the same pair of axes, draw the two hyperbolas $y = \frac{1}{x}$, $y = \frac{1}{x} + 1$. Calculate the x-intercept of the second hyperbola and indicate it on the graph.

13. (3 pts) Sketch a graph of the cubic $y = \frac{1}{2}(x+1)(x-1)(x-3)$, and calculate its y-intercept.

14. (2 pts) Factor $3x^2 - 42x + 120$ as $a(x - r_1)(x - r_2)$.

15. (2 pts) By completing the square, write $x^2 - 2x + 5$ in the $a(x - h)^2 + k$ form.

16. (3 pts) By completing the square, write $2x^2 - 20x + 53$ in the $a(x - h)^2 + k$ form.

17. (2 pts) State the domain of the function $y = \frac{2x}{x^2+8x-9}$.

18. (5 pts) Simplify the function $f(x) = \frac{x+1}{x^2-1} - \frac{2x}{x^2-2x+1} + 1$ by writing it as one fraction. Calculate its domain, its x-intercepts, and its y-intercept.

19. True or False. Write **T** or **F** next to each sentence. The numbers and lower-case variables refer to elements of $\mathbb{R}$ unless otherwise specified. P_1 and P_2 refer to parabolas

$$y = a_1x^2 + b_1x + c_1,\ a_1 \neq 0,$$

and

$$y = a_2x^2 + b_2x + c_2,\ a_2 \neq 0.$$

You need not explain your answer.$^{(1\ each)}$

_____ It is possible for P_1 and P_2 to intersect in exactly one point.

_____ If P_1 and P_2 both "open upward," then P_1 and P_2 intersect in at least one point.

_____ P_1 never intersects the parabola

$$y = a_1(x-1)^2 + b_1(x-1) + c_1.$$

_____ If $a_1 > 0$, then the y-coordinate of the vertex of P_1 is a minimum value.

_____ P_1 can intersect a cubic $\{y = ax^3 + bx^2 + cx + d,\ a \neq 0\}$ in exactly three points.

_____ P_1 can intersect a cubic $\{y = ax^3 + bx^2 + cx + d,\ a \neq 0\}$ in exactly four points.

_____ P_1 can intersect a circle $\{(x-a)^2 + (y-b)^2 = r^2\}$, in exactly four points.

_____ (Recall i is the complex number such that $i^2 = -1$)

$$(x-i)^3 = x^3 - 3ix^2 - 3x + i.$$

_____ If $f(x)$ is a polynomial with exactly one real root, and $g(x)$ is a polynomial with exactly one real root, then the product $f(x) \cdot g(x)$ is a polynomial with exactly one real root.