COMMENTS ON: REAL CONGRUENCE OF COMPLEX MATRIX PENCILS AND COMPLEX PROJECTIONS OF REAL VERONESE VARIETIES

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1. Errata

The following typos appear in the published paper, [C_3].

Line 3 of the Proof of Theorem 2.13: should read "As in the proof of" ....

The formula for τ in Example 8.5 at the top of page 82 should read:

$$\tau : (z_1, iz_1, z_3, z_4)^T \mapsto (-(z_1 + z_3 + iz_4), -i(z_1 + z_3 + iz_4), z_3, z_4)^T$$

2. Updates

In addition to reference [19] by Morley and Morley, there is another book which considers quadratically parametrized real curves in the complex plane, as in Section 4. [D] arrives at essentially the same classification as that in Section 4, and goes into some detail on the relationship between the complex coefficients and the geometry of the curve.

The Veronese surface and its projections, including Steiner surfaces (Examples 2.8, 7.4) are discussed in [B] and [SR]. These books also describe ([B] §§II, IV; [SR] §III.1) the connection between projective equivalence classes of rational parametric maps and linear systems of varieties, which is briefly mentioned in Example 7.6. A complete list of the 13 types of maps from Example 7.6 is given by Degtyarev (reference [13]), and also appears on my web site: [C_4].

Reference [5] will not appear under that title. However, due to a convenient editorial error, reference [5] is not cited in the paper anyway. The 4-manifold M in $\mathbb{C}^5$ from Example 8.5, and other varieties defined by similar quadratic expressions, are considered in [C_1], [C_2], [C_6], and [C_7].

References [6] and [8] have appeared as cited below: [C_5], [CF].

A second edition of reference [24] has appeared as [SK], with a new list of authors. It still describes the **vech** notation but omits the congruence classification of real symmetric matrices.

My contact information has changed, and the web address at the end of the article is now obsolete. My current home page can be found at:

`http://users.pfw.edu/CoffmanA/`

3. Citations

The article is cited in this paper: [W].

References

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[C_1] A. Coffman, *Enumeration and Normal Forms of Singularities in Cauchy-Riemann Structures*, dissertation, University of Chicago, 1997. MR 2716702.

[C_2] A. Coffman, *Complexification of the CR cross-cap*, unpublished lecture notes based on [C_1], 1997.
`http://users.pfw.edu/CoffmanA/pdf/complexif.pdf`

[C_3] A. Coffman, *Real congruence of complex matrix pencils and complex projections of real Veronese varieties*, Linear Algebra and its Applications, **370** (2003), 41–83. MR 1994320 (2004f:14026), Zbl 1049.14042.

[C_4] A. Coffman, *Linear systems of conics*, web page.
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[C_5] A. Coffman, *Analytic normal form for CR singular surfaces in* $\mathbb{C}^3$, Houston J. Math (4) **30** (2004), 969–996. MR 2110245 (2006d:32048), Zbl 1074.32013.

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[D] R. Deaux, *Introduction to the Geometry of Complex Numbers*, transl. from the revised French ed. by H. Eves, Ungar Pub. Co., New York, 1957. MR 0086745 (19,236f), Zbl 0077.14502.

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[W] W. Waterhouse, *Real congruences of complex subspaces of* 2×2 *symmetric complex matrices*, Linear Algebra and its Applications (2–3) **414** (2006), 502–505. MR 2214402 (2006j:15038), Zbl 1091.15033.

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