MWAA 2022
Purdue University Fort Wayne
October 7–9th

Friday		
15:30–16:00	**Coffee**	
16:00–17:00	**D. Hardin**	Linear programming bounds for packing, energy, and polarization problems: where geometry meets analysis
18:30–20:30	**Reception**	**Adam Coffman's Residence**
Saturday		
09:00–09:40	**Posters & Coffee**	
09:40–10:20	**Y. Zhang**	Unique continuation for $\bar{\partial}$ with square-integrable potentials
10:30–11:10	**M. Londhe**	A natural invariant measure for polynomial semigroups, and its properties
11:20–12:00	**M. Pilla**	One-parameter semigroups in the unit ball of $\mathbb{C}^2$
12:00–14:00	**Lunch**	
14:00–14:40	**Ch. Felder**	Limits of optimal polynomial approximants
14:50–15:30	**M. Stawiska-Friedland**	Gauss-Lucas theorem and its consequences in polynomial dynamics
15:30–16:00	**Coffee**	
16:00–16:40	**O. Vlasiuk**	Polarization and covering on sets of low smoothness
18:30–21:00	**Conference Dinner**	**Don Hall's Factory Restaurant**
Sunday		
09:00–09:40	**Posters & Coffee**	
09:40–10:20	**A. Barhoumi**	Asymptotics of rational solutions of Painlevé III
10:30–11:10	**A. Prokhorov**	Monodromy map under the confluence PIII$(D_6) \to$PIII(D_8)
11:20–12:00	**R. Orive**	Electrostatic models for orthogonal and multiple orthogonal polynomials

Plenary Talks

Ahmad Barhoumi
University of Michgan

Asymptotics of Rational Solutions of Painlevé III

Joint work with Oleg Lisovyy, Peter Miller, and Andrei Prokhorov

Painlevé equations are six nonlinear second order ODEs whose solutions are thought to be the special functions of the 21st century, and have already appeared in countless works in integrable systems, combinatorics, and random matrix theory amongst many others. In this talk, I will focus my attention on one equation, Painlevé III. While its generic solutions are transcendental, it is known to possess families of special-function solutions: solutions written in terms of elementary and/or classical special functions. I will discuss the rational solutions of Painlevé III and survey what is known of their large parameter behavior. In the last part of the talk, I will highlight the analysis of the rational solutions near the origin. Many of the previous results as well as our work rely on reformulating the rational solutions in terms of a 2×2 Riemann-Hilbert problem, which I will introduce and discuss.

Chris Felder
Indiana University

Limits of Optimal Polynomial Approximants

We will begin by discussing cyclic functions for the forward shift operator in a certain class of Hilbert function spaces. In the classical Hardy space on the unit disk, these functions are precisely those which are outer. Although the essence of cyclic functions in other spaces has been elusive, the study of optimal polynomial approximants has been devised as a possible inroad to establishing characterizations of cyclicity. These polynomials are solutions to the problem of minimizing the norm of $pf - 1$, where f is in a given Hilbert space and p is a polynomial of a fixed degree. The main objective of this talk is to present results on the limits of these polynomials, as their degrees tend to infinity. Time permitting, we will discuss some open questions.

Doug Hardin
Vanderbilt University

Linear Programming Bounds for Packing, Energy, and Polarization Problems: Where Geometry Meets Analysis

Joint work with P. Boyvalenkov, P. Dragnev, E. Saff, and M. Stoyanova

I will give an overview of linear programming bound methods for discrete point configurations in Euclidean space and the sphere. The talk will include an introduction to Maryna Viazovska's proof that the $E8$ lattice is the densest sphere packing in 8 dimensional Euclidean space and then move on to recent work with Boyvalenkov, Dragnev, Saff, and Stoyanova involving linear programming bounds for energy and polarization problems on the d dimensional sphere.

Mayuresh Londhe
Indiana University

A Natural Invariant Measure for Polynomial Semigroups, and its Properties

In this talk, we give a description of a natural invariant measure associated with a finitely generated polynomial semigroup (which we shall call the Dinh–Sibony measure) in terms of potential theory. The existence of this measure follows from a very general result of Dinh–Sibony applied to a holomorphic correspondence in $\mathbb{P}^1 \times \mathbb{P}^1$ that one can associate naturally with a semigroup of the above type. We obtain a complete description of this invariant measure. This requires the theory of logarithmic potentials in the presence of an external field, which, in our case, is explicitly determined by the choice of a set of generators. Our result generalizes the classical result of Brolin. Along the way, we establish the continuity of the logarithmic potential for the Dinh–Sibony measure, which might be of independent interest. If time permits, we shall also present some bounds on the capacity and diameter of the Julia sets of such semigroups, which uses the F-functional of Mhaskar and Saff.

Ramón Orive
Universidad de La Laguna

Electrostatic Models for Orthogonal and Multiple Orthogonal Polynomials

Joint work with A. Martínez Finkelshtein and J. Sánchez Lara

In this talk, the work of T. J. Stieltjes (1856–1894) on the electrostatic interpretation of zeros of classical orthogonal polynomials is revisited. Then we present an extension of this approach to the case of (type II) multiple (Hermite–Padé) orthogonal polynomials. We particularly focus on the well–known examples of Angelesco and Nikishin settings.

Michael Pilla
Ball State University

One-Parameter Semigroups in the Unit Ball of $\mathbb{C}^2$

For self-maps of the disk, it can be shown that under the right conditions one can embed a discrete iteration of the map into a continuous semigroup. One way of obtaining this result is to use a model theory of linear fractional maps. Under some restricted conditions, this can be extended to higher dimensions. In this talk, we began by discussing generalizations of linear fractional maps and their properties. We then discuss the restricted conditions under which our linear fractional models extend to the unit ball in two variables. Finally, we use this model to explore extensions of our results to higher dimensions.

Andrei Prokhorov
University of Michigan

Monodromy Map under the Confluence PIII(D_6) →PIII(D_8)

Joint work with Ahmad Barhoumi, Oleg Lisovyy, and Peter Miller

Painlevé equations are nonlinear ODEs which have many nice properties. There are six of them and they can be arranged in a confluence diagram, which tells how to obtain these equations from each other through a limiting procedure. Usually such procedure is described formally on the level of equations only. In this work we provide an example through which we can study the confluence on the level of solutions of Painlevé equations. In particular we describe the map between the monodromy data corresponding to the confluence PIII(D_6) →PIII(D_8).

More specifically consider the third Painlevé equation

$$u_0'' = \frac{(u_0')^2}{u_0} - \frac{u_0'}{x} + \frac{4\Theta_{0,0}u_0^2}{x} - \frac{4\Theta_{\infty,0}}{x} + 4u_0^3 - \frac{4}{u_0}.$$

Such an equation is usually called PIII(D_6). Assume that its solution $u_0(x)$ has generic two-parameter behavior at the origin $u_0(x) \simeq \alpha_0 x^{\beta_0}$. Parameters α_0 and β_0 can be expressed in terms of the so-called monodromy data. Introducing the Bäcklund transformation with

$$B : (a,b,u) \to \left(a+1, b-1, \frac{xu' + 2xu^2 + 2bu - u + 2x}{u\,(xu' + 2xu^2 + 2au + u + 2x)}\right),$$

denote $(\Theta_{0,n}, \Theta_{\infty,n}, u_n) = B^n(\Theta_{0,0}, \Theta_{\infty,0}, u_0)$. The function $u_n(x)$ satisfies the corresponding PIII(D_6) equation and has behavior $u_n(x) \simeq \alpha_n x^{\beta_n}$ at the origin. It turns out that solutions $u_n\,(x/n)$ model the confluence process PIII(D_6) →PIII(D_8). More specifically $u_{2n}\,(x/2n)$ converges to a solution $w(x)$ of the PIII(D_8) equation

$$w'' = \frac{(w')^2}{w} - \frac{w'}{x} + \frac{4w^2}{x} - \frac{4}{x}$$

with behavior $w(x) \simeq \alpha_\infty x^{\beta_\infty}$. We describe the monodromy map $(\alpha_0, \beta_0) \to (\alpha_\infty, \beta_\infty)$.

Margaret Stawiska-Friedland
Mathematical Reviews, AMS

Gauss-Lucas Theorem and its Consequences in Polynomial Dynamics

The classical Gauss-Lucas theorem says that for a (nonconstant) polynomial p with complex coefficients all zeros of the derivative p' lie in the convex hull of the zeros of p. We will start this talk by discussing two variants of this theorem, due respectively to L. Hörmander and W. P. Thurston. Using these variants, we will prove that for every complex polynomial p of degree $d \geq 2$ the convex hull H_p of the Julia set J_p of p satisfies $p^{-1}(H_p) \subset H_p$. This settles positively a recent conjecture by P. Alexandersson. We will also characterize polynomials for which the equality $p^{-1}(H_p) = H_p$ is achieved. We will further outline some other related problems.

Oleksandr Vlasiuk
Vanderbilt University

Polarization and Covering on Sets of Low Smoothness

We study the asymptotic properties of point configurations that achieve optimal covering of sets lacking smoothness. Our results include proofs of existence of asymptotics of best covering and maximal polarization on d-rectifiable sets and maximal polarization on self-similar fractals. This resolves a conjecture by Graf and Luschgy, and strengthens the classical results of Kolmogorov and Tikhomirov.

Yuan Zhang
Purdue University Fort Wayne

Unique Continuation for $\bar{\partial}$ with Square-Integrable Potentials

Joint work with Yifei Pan

In this talk, we will discuss a unique continuation property for the inequality $|\bar{\partial}u| \leq V|u|$, where u is a vector-valued function from a domain in $\mathbb{C}^n$ to $\mathbb{C}^N$, and the potential $V \in L^2$. We show that the strong unique continuation property holds when $n = 1$, and the weak unique continuation property holds when $n \geq 2$. In both cases, the L^2 integrability condition on the potential is optimal.