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Environmental considerations on the optimal product mix

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Abstract

Several types of regulations limit the amount of different emissions that a firm may create from its production processes. Depending on the emission, these regulations could include threshold values, penalties and taxes, and/or emission allowances that can be traded. However, many firms try to comply with these regulations without a systematic plan, often leading not only to emission violations and high penalties, but also to high costs. In this paper, we present two mathematical models that can be used by firms to determine their optimal product mix and production quantities in the presence of several different types of environmental constraints, in addition to typical production constraints. Both models are comprehensive and incorporate several diverse production and environmental issues. The first model, which assumes that each product has just one operating procedure, is a linear program while the second model, which assumes that the firm has the option of producing each product using more than one operating procedure, is a mixed integer linear program. The solutions of both models identify the products that the firm should produce along with their production quantities. These models can be used by firms to quickly analyze several "what if" scenarios such as the impact of changes in emission threshold values, emission taxes, trading allowances, and trading transaction costs.

Keywords: Environmental management; Production planning; Linear and mixed-integer programming

1. Introduction

The European Parliament adopted the world's first multi-national emissions trading scheme (ETS) covering greenhouse gases in the Union on 2 July. The Directive, taking effect in 2005, will cap emissions from 10,000 plants in the oil refining, smelting, steel, cement, ceramics, glass and paper sectors, and allow trading of their emissions

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allowances. The emissions trading market is estimated to grow to up to eight billion EUR per year. Commenting on the ETS, EU Environment Commissioner Margot Wallström said that "it means that the largest emissions trading scheme in the world to date will be a reality from 2005 and that the architecture foreseen under the Kyoto Protocol is coming to life. Companies across 25 countries must now start incorporating climate change into day-to-day commercial decisions, and begin assessing what innovative steps they can take to reduce emissions".

The above excerpt ("European Parliament Adopts Emissions Trading Scheme", EC Press Release, July 2003) clearly illustrates that environmental concerns are becoming increasingly relevant for firms as regulations on pollutants become more stringent and customer awareness of these issues increases. To reduce environmental liabilities and violations, some of which may lead to high penalties and even production shut downs, many firms are being forced to comply with several types of regulations that may severely limit the amount of different emissions that they may create. Depending on the emission, these regulations include threshold values that cannot be exceeded, penalties and taxes that are based on the output amount, and/or emission allowances that can be traded between firms. In addition, there's increasing evidence that environmental concerns may cause some customers to influence the demand for a firm's products, and stakeholders to insist that the firm contribute to efforts that help the environment (Hutchinson and Hutchinson, 1997).

Several recent developments in environmental awareness and control have contributed to this increased environmental concern, as detailed in the next few paragraphs. Firstly, it is now recognized that the greenhouse effect can be slowed down only if emissions of carbon dioxide and other harmful gases are reduced drastically (Houghton, 1996). A major step in this direction is the Kyoto Protocol (Bernstein et al., 1999; Matsuo, 1998), which designates a 7% drop in carbon dioxide emissions for the United States, and as much as a 21% drop for Germany compared to emissions in 1990. In addition, new regulations may soon obligate companies to take back and recycle their products, and to introduce closed loop cycles in their plants. For example, from the year 2007 all customers in the European Union can give back old cars to car manufacturers or their retailers without any disposal costs. As a main consequence of this European Directive, car manufacturers will have to recycle at least 85% of an end-of-life vehicle by 2006 and at least 95% by 2015.

The wastewater sector is another area in which further environmental control can be expected (Thomas and Ryan, 1999). Improvements in wastewater quantity and quality could be mandated by tightening current threshold values for emissions, or by penalizing hazardous substances that are discharged as sewage. For example, in Germany the wastewater charging act imposes charges per cubic meter of sewage depending on the amount of hazardous substances it contains (Berendes and Winters, 1995). As a result of this regulation, many firms have invested in their own purification plants and optimized their water usage.

Environmentally friendly products and processes also contribute to stakeholders' satisfaction since environmental attributes of products are becoming increasingly relevant for consumers' demand preferences. For instance, in the US more than 60% of customers avoid or consider avoiding a product for environmental reasons (The Economist, 2000). In this context, Nike is specifying that all parts of their shoes must be recyclable (Nike, 2000). This process, which is mainly customer-driven, may induce major changes in the purchasing and production policies of all suppliers of Nike. Such efforts may even transform apparent disadvantages into a competitive edge for environmentally progressive firms.

Many firms view these developments as potentially disadvantageous to their cost structures and competitiveness. As a result, several firms are becoming more "environmentally aware" and are starting to deal with these issues systematically. In the US, the strengthening environmental regulations have driven more than 3000 firms to introduce environmental management systems that meet the requirements of ISO-14001 (Peglau, 2003). Globally, nearly 55,000 organizations are certified according to ISO-14001, which has the overall goal of continually improving the environmental management of organizations (ISO 14001, 1996).

The environmental impact of a firm's products, processes, and resource usage are typically measured by the amount of emissions of wastewater and other industrial wastes and pollutants. Every unit of industrial waste, wastewater, and other pollutants that is created includes valuable raw materials, machine hours, and labor costs. Identifying product and process inefficiencies that result in high amounts of these emissions can be the key for improving productivity on the production floor. Therefore, a systematic environmental management system can also be seen as a driver for further innovation (Foster and Green, 1999). Note that this type of driver is consistent with the philosophies of modern manufacturing practices such as JIT and TQM, which also attempt to identify and eliminate any kind of waste through process and product redesign as well as continuous improvement.

It is logical that a firm's operational and strategic decisions would be affected if it considered their environmental impacts. However, many firms try to comply with environmental regulations without a systematic plan, in a more *reactive* manner rather than a *proactive* manner. This often leads not only to emission violations and high penalties, but also to high costs. One of the main reasons for this could be that simple planning methods that include emissions are still missing. Specifically, most production planning systems lack any capabilities that facilitate achieving legal compliance and meeting the requirements of stakeholders cost efficiently.

To address this issue, in this article we present two separate mathematical models that illustrate how different environmental concerns such as emission thresholds and tradable allowances can be integrated into production planning methods that currently consider only resource limitations. Both models are applicable for practical situations and can be solved rather easily since they are linear (in the first case) and integer (in the second case) models. The models identify the main environmental concerns for a firm, and determine which product mix should be produced in order to maximize the firm's profits, while ensuring environmental and legal compliance in addition to the usual resource limitations.

Although we have set up and solved both models on a PC, the primary focus of this paper is in the models themselves. Hence, while we illustrate the first model in this paper using a PC-based example and discuss the types of issues that can be studied using both models, we do not discuss the sensitivity of the solutions to the environmental and production parameters here. We are in the process of collecting comprehensive real-world data and, in a follow-up paper, hope to conduct detailed numerical experiments using this data. This would allow us to study the quality of the planning models, their performance under different operating scenarios, and their transferability to different industrial sectors.

This paper is organized as follows. In Section 2, we provide a literature review of prior work in the environmental management field, and describe our problem scenarios. In Section 3, we first define our notations, and then provide details of the mathematical formulations for both models. We then illustrate a numerical example of the first model in Section 4 and discuss the types of managerial insights that may be derived from solving both models. We conclude this paper in Section 5 with some suggestions for future research in this area.

2. Literature review

The importance and financial rewards for using environmental management planning systems (which would be facilitated by the models presented in this paper) has been well documented. For example, Klassen and McLaughlin (1996) show that public announcements of positive environmental events can lead to abnormal stock returns, while environmental crises can cause sizable negative returns. Gege (1997) studies about 1000 examples of cost reductions through systematic environmental management systems in 100 firms, and estimates that companies can achieve an average reduction of about 5% in their total cost with such systems. Reiborn et al. (1999) argue that the costs of implementing such systems are much lower than their short and long term benefits. Klassen and Whybark (1999) show that investments in environmental technologies may improve manufacturing performance.

Most published papers that address operations management issues in environmental planning deal with the design of production systems and the proper identification and solution of primary environmental problems. For example, Dreher et al. (1999) investigate the impact of intensified use and prolonged product lifetimes on manufacturing strategies and workforce. Bloemhof-Ruwaard et al. (1995) examine different links of supply chains and identify waste management, product recovery management, and source-directed product management as main problem areas. Daniel et al. (1997) emphasize special monitoring methods and impact assessment to identify and evaluate environmental aspects, and suggest using mathematical programming techniques to manage the use of natural resources.

Many authors such as Ulhoi (1995) and Nijkamp and van den Bergh (1997) emphasize reduction of the volume of material flows to improve environmental protection. Steven and Letmathe (1996) and Melnyk et al. (1999) suggest the integration of output related flows of material and energy such as waste, sewage, and pollutants into bills of materials. We note that the models proposed in this paper adopt this idea in the sense that outputs such as wastes, sewage, and pollutants are integrated in the coefficients that characterize different operating procedures.

Inman (1999) sees a lack of operations planning tools that cope with environmental problems in an effective manner. Such tools should include the main environmental constraints such as different types of threshold values, taxes on emissions, and the effect on demand of environmentally oriented customers. The models in this paper explicitly address these issues.

In recent years, several papers have proposed mathematical models to deal with different aspects of environmental management and control. For example, Wirl (1991) develops a non-linear dynamic model to illustrate that changing public environmental policies may lead to a more erratic production profile. Letmathe and Steven (1995) use an LP model to examine the impact of taxes on emissions, and of threshold values on the present value of investments, and present clear evidence that environmental constraints may have a significant impact on capital budgeting decisions. Kistner and Steven (1991) use a chance-constrained programming model to investigate the impact of environmental risks on the maximal product output. Penkuhn et al. (1997) use non-linear programming to study whether the profit-maximizing output of ammonia changes when the production planning considers emissions of substances such as nitrogen oxides and carbon dioxide.

Several authors have also developed models for special applications that may give deeper insights into the relevance of environmental constraints for production management. Remmers et al. (1990) use an LP model to analyze how to achieve optimal allocation of emission reduction measures in the energy production sector in Germany. In a recent paper, Curkovic (2003) discusses the growing importance of environmentally responsible manufacturing. He then uses an empirical approach to develop constructs and measures that are critical to the development and growth and research in this topic.

The above discussion clearly shows that systematic environmental planning is important, and that several authors have developed mathematical models to address specific issues in this regard. However, there have not been comprehensive models that simultaneously address multiple issues in production and environmental planning. The models presented in this paper attempt to bridge this significant gap.

3. Problem definition and formulation

The mathematical models presented in this paper show how important environmental constraints can be included in operations planning and scheduling on a regular and routine basis. The two separate models can be used by firms to determine the optimal product mix and production quantities in the presence of environmental as well as production constraints. Both models are comprehensive and incorporate several diverse issues such as:

- multiple products, each with its own resource usage and emission outputs per unit of the product, based on the production process used,
- product demands that could be (partially) influenced by the amount of emissions due to the product,
- finite resource availability,
- distinct resource costs,
- emission thresholds that cannot be exceeded on some emissions,
- taxes (penalties) based on the amount of output on some emissions,
- trading of output allowances for some emissions, with differences in transaction costs for purchasing and selling these allowances, and
- emission regulations that may be product based, process based, and/or resource based.

For each issue listed above, we present illustrative examples from real-world situations to prove their practical relevance and applicability. The objective in both mathematical models is to maximize the firm's profit.

The first model assumes that each product has its own *unique* operating procedure, which we define as the parameters of a production process that uses a known amount of each resource and results in a specific production yield of that product, and known emission amounts for each pollutant. Hence, if the firm decides to produce a certain product, the resource consumption (such as raw materials, machine times, etc.) are known from records such as the bill of materials, process specifications, etc. Likewise, since the operating procedure specifies the exact inputs used, we can assume that the resulting emission outputs may be identified and are known. The resulting mathematical model in this scenario is a linear program.

The second model assumes that the firm has the option of producing each product using more than one operating procedure, each of which may consume a different amount of resources, and result in different production yields and emission outputs. For example, usage of coal with 1% sulphur and coal with 3% sulphur could correspond to different operating procedures—each yielding different sulphur dioxide emissions. In the presence of multiple operating procedures, the firm can produce the required quantity of a product using a combination of procedures, so as to satisfy different environmental and production constraints. For instance, a high-yield process resulting in high emissions of some pollutants may be used in combination with a low-yield, low-emission process to satisfy production requirements as well as emission thresholds. The resulting model in this case is a mixed integer program.

The outputs of both models give the firm precise information regarding which operating procedures should be used (if relevant), which products should be produced, and in what quantities. Furthermore, both models show which environmental constraints limit the firm's profits so that the firm can carefully study these problem areas. For example, investments in new environmentally friendly technologies could help relax the impact of a critical environmental constraint. Since the models calculate the amount of different emissions in advance, these data can also be used for comparison between standard and actual amounts of these emissions. The models also reveal if allowances for traded emissions should be bought or sold. Since both models allow easy integration of environmental constraints into production planning problems, they have the potential to close an important gap in operations planning.

3.1. Notation

We first define the notations used in our models, and then use these notations to describe both model formulations. All emission allowances and thresholds are in terms of the unit for that specific emission (for example, wastewater allowance could be in liters, while sulphur dioxide allowance could be in cm^3). Likewise, all resource requirements and availabilities are measured in terms of the units for that specific resource (for example, machine time could be in hours, while raw material could be in kilograms).

Parameters (for which values are known)

I	number of products,
J	number of production resources,
M	total number of emissions,
N	number of emissions subject to emission taxes and/or threshold values ($N \leqslant M$),
S_i	unit selling price of product i ($i=1,\ldots,I$),
C_j	unit cost of resource j ($j=1,\ldots,J$),
Q_m	unit penalty for emission m ($m=1,\ldots,N$),
Q_m^+	unit purchase price for traded emission m ($m=N+1,\ldots,M$),
Q_m^-	unit selling price ($Q_m^- \leqslant Q_m^+$) for traded emission m ($m=N+1,\ldots,M$),
E_m^{T}	allowance units for traded emission m ($m=N+1,\ldots,M$),
R_j	maximum availability of resource j ($j=1,\ldots,J$),
E_m	maximum allowed of emission m (for relevant m, $m=1,\ldots,N$) (*overall emission limit*),
H_m	maximum average quantity allowed of emission m (for relevant m, $m=1,\ldots,N$), based on total production (*product based emission limit*),
G_{mj}	maximum average quantity allowed of emission m (for relevant m, $m=1,\ldots,N$), based on resource j (for relevant j, $j=1,\ldots,J$) (*resource based emission limit*),
A_{ji}	amount of resource j ($j=1,\ldots,J$) required per unit of product i ($i=1,\ldots,I$),
ε_{mi}	amount of emission m ($m=1,\ldots,M$) per unit of product i ($i=1,\ldots,I$).

Decision variables (for which values are obtained from the model solution)

x_i	production quantity of product i ($i=1,\ldots,I$),
r_j	total amount resource j that is required by the production plan ($j=1,\ldots,J$),
e_m	amount of emission m ($m=1,\ldots,M$),
e_m^+	allowance units purchased for emission m ($m=N+1,\ldots,M$),
e_m^-	allowance units sold for emission m ($m=N+1,\ldots,M$).

3.2. Model with single operating procedure for each product

As noted earlier, the first model assumes that each product has only a single operating procedure for its production. This implies that the exact quantity of resources required to manufacture each unit of each product is known (from BOM, etc.). Likewise, the exact amounts of different emissions that result from this unique operating procedure used are also known.

The objective is to maximize profit, which is calculated as the total revenue obtained by product sales and the sale of tradeable emission allowances, less the total cost of all resources used, emission penalties that are payable, and purchase cost of tradeable emission allowances. This may be expressed as follows:

$$\sum_i S_i x_i - \sum_j C_j r_j - \sum_{(m=1,\ldots,N)} Q_m e_m + \sum_{(m=N+1,\ldots,M)} Q_m^- e_m^- - \sum_{(m=N+1,\ldots,M)} Q_m^+ e_m^+. \tag{1}$$

It should be noted that we use the term "resource" here to signify not only raw materials but other production related issues such as machine capacity, labor availability, storage space, and process limitations. The unit cost of each resource j, expressed by C_j, could therefore also denote production related costs.

The first set of constraints identifies the total amount of each resource required to achieve the desired production levels, and also ensure that resource availability issues are satisfied. In view of our above definition of a "resource", note that R_j can also represent issues such as machine availability, capacity restrictions, etc.

$$r_j = \sum_{(i=1,\ldots,I)} A_{ji} x_i \leqslant R_j, \quad j = 1,\ldots,J. \tag{2}$$

Next, we identify the total amount of each emission resulting from the production of all products. Since ε_{mi} is a known parameter for each product, this is a straightforward computation. If emission reductions due to process integration are possible, it would be easy to incorporate a reduction constant (β_{mi}, where $\beta_{mi}<1$) in this expression. This of course assumes that β_{mi} is a known constant.

$$e_m = \sum_{(i=1,\ldots,I)} \varepsilon_{mi} x_i, \quad m = 1, \ldots, M. \tag{3}$$

Next, we define three different types of emission thresholds that may be relevant for different problem scenarios. We note that that all three types of emission thresholds may not be relevant for every problem scenario. Depending on the type of problem environment and the type of emission, only one of the three types of thresholds listed here may apply. However, the models discussed here are flexible enough to handle every type of threshold.

In its simplest form, the threshold may be an upper bound on the total quantity of an emission. It may often be defined as the average emission per time unit, e.g., per week, month, or year. For example, several thresholds imposed in Germany are documented in the *Technische Anleitung Luft* (Bundesministerium für Umwelt, Naturschutz, und Rektorsicherheit, 2002). The constraint may be expressed as follows:

$$e_m \leqslant E_m \quad \text{for all relevant } m. \tag{4}$$

Alternatively, the threshold may be a *product based* limit that imposes an upper bound on the *average* amount of an emission based on the total production of all products. This would be similar to the requirement that automobile firms face on the average miles per gallon required of their overall production. As another example, Switzerland and the European Union (EU) have different threshold values for insecticides in honey, with the EU having a tighter limit. For DDT and Lindan, the threshold values in Germany are 0.05 and 0.01 mg per kg, respectively (Bundesgesetzblatt, 1999), whereas such threshold values do not even exist in Switzerland (Eidgenössisches Department des Innern, 2002). Food producers, therefore, have the opportunity to mix honey purchased from different regions according to the threshold values of the different destinations where these products will be sold. This type of threshold constraints may be expressed as follows:

$$e_m \leqslant H_m \sum_{(i=1,\ldots,I)} x_i \quad \text{for relevant } m. \tag{5}$$

Finally, the threshold could also be a *resource based* limit that imposes an upper bound on the *average* amount of an emission per unit consumed of a specific resource. Several examples for this type of thresholds can be found in the *Technische Anleitung Luft* as well. This threshold may be written as:

$$e_m \leqslant G_{mj} r_j \quad \text{for relevant } m \text{ and } j. \tag{6}$$

As noted earlier, some of the emissions may have output allowances (usually allocated by the environmental regulatory authority of the country in which the firm operates) that can be traded with other firms. As mentioned in the introductory statement, members of the EU have committed to establish a scheme for greenhouse gas emission allowance trading within the community (Council Directive 96/61/EC). Emission trading in the EU will start in 2005 with the trading of carbon dioxide, and may be extended to other greenhouse gases three years later. Emission trading will be relevant for companies in the energy producing sector, companies in the steel and mineral industry, and other sectors like paper producers with a production of more than 20 tons per day.

It is still not clear how the market for emission trading will be organized in detail. For example, each country has to still define its national allocation plan. The European Commission is, however, convinced that efficient market structures will arise with an expected price per ton of carbon dioxide of between 14 and 30 Euro. Note that a price of 20 Euro would mean that the variable cost of electricity generation would increase by approximately 30% on average (Wietschel et al., 2002). Another system dealing with trading greenhouse gases will be introduced shortly at Chicago.

While estimations of transaction costs of emission trading do not yet exist, purchase costs will typically be higher than selling prices due to differences in transaction costs for selling and purchasing these allowances. We therefore use two variables, e_m^- and e_m^+, to represent the amount of emission allowances sold and purchased, respectively. The trading issue may sometimes pose a strategic challenge since, while it may be optimal for the firm to sell some of its allowances during the current year to realize higher profit, such an action may result in a smaller allowance from the regulatory authority the following year. If limits exist on either the allowance bought or the allowance sold (or both), a set of simple constraints can be added to the model.

$$e_m + e_m^- - e_m^+ = E_m^{\mathrm{T}}, \quad m = N+1, \ldots, M. \tag{7}$$

We now define the demand for each product. In some situations, product demand may be a *known value* based on forecasts. In such a case, the model can simply use this value for D_i in expression (8) below. Alternatively, demand could be an unknown quantity (e.g., it may be a function of other variables, or parameters such as the selling price and quality of the product). In such a case, its value would be automatically determined in the model. Additionally, if the firm's primary customers are environmentally minded, demand may *also* be influenced by the amount of different emissions due to the product. Examples of such scenarios worldwide are plenty. For instance, the market share of many products (such as indoor paints that contain no solvents) increased in Germany when they were awarded the "German Blue Angel" label (Müller, 2002). Moreover, some customers are willing to pay more for environmentally friendly products (Belz, 2001) as evidenced by the fact that growth rates for environmentally friendly products in the textile and food industries are significantly higher than average growth rates (Meyer, 2001; Ton et al., 1999; Bundesministerium für Verbraucherschutz, Ernährung, und Landwirtschaft, 2003).

As another example, in Germany, a current discussion deals with acrylamide, a substance found in french fries, cookies, potato chips, etc., and is estimated to cause thousands of cases of cancer each year in Europe and in the US. The amount of acrylamide included in different food products are regularly published in magazines and by public authorities, clearly impacting their demand from health oriented customers. Likewise, companies such as Miele household devices in Germany publish the amount of hazardous wastes and other environmental impacts caused by their products (Miele, 2002). The intent is to allow this information to be used by customers as one aspect in their comparison of products from different companies.

In our models, for computational purposes, we assume that the demand for each product is affected to some extent by the quantity of emissions created per unit of the product. That is, in addition to parameters such as selling price, product attributes, etc., the demand function also explicitly includes the emission quantities (with negative coefficients to represent the inverse relationship between demand and emissions). Although we model these demand functions as linear equations here, it is straightforward to model them as piecewise linear functions to account for any non-linear features in the relationship. In our tests, we assume that the base demand (based on all non-emission related issues) is a known constant BD_i. The actual demand is smaller than BD_i, depending on the emission quantities. Once the demand for each product has been expressed, the production of that product is limited by its demand.

$$x_i \leqslant D_i = \mathrm{BD}_i - f(\varepsilon_{1i}, \varepsilon_{2i}, \ldots, \varepsilon_{Mi}), \quad i = 1, \ldots, I. \tag{8}$$

Finally, we have the non-negativity constraints on production quantities, represented as:

$$x_i \geqslant 0, \quad i = 1, \ldots, I. \tag{9}$$

Since the above model is completely linear in nature, it can be easily solved using any standard LP software package.

3.3. Model with several operating procedures for each product

We now extend the above model to consider situations where each product can be manufactured using more than one of several different operating procedures (denoted by t). Each procedure could use a different amount of the resources per unit produced of the product, and yield different amounts of different emissions. For example, usage of coal with 1% sulphur and coal with 3% sulphur could correspond to different operating procedures, each yielding different sulphur dioxide emissions. To accommodate this feature in our formulation, we modify the definition of two of the known parameters in the previous model, and introduce new decision variables as shown below. Here again, all emission amounts and resource usages are expressed in terms of the units of the specific emission or resource.

Known parameters

T_i number of operating procedures for product i (could be different for different products),

A_{jit} amount of resource j ($j=1,\ldots,J$) required per unit of product i ($i=1,\ldots,I$) produced using operating procedure t ($t=1,\ldots,T_i$),

ε_{mit} amount of emission m ($m=1,\ldots,M$) per unit of product i ($i=1,\ldots,I$) produced using operating procedure t ($t=1,\ldots,T_i$).

Decision variables

x_{it} production quantity of product i ($i=1,\ldots,I$) using operating procedure t ($t=1,\ldots,T_i$),

z_{it} 1 if product i ($i=1,\ldots,I$) is produced using operating procedure t ($t=1,\ldots,T_i$), $=0$ otherwise.

Based on the resource usage and emission outputs of the operating procedures, note that it may be necessary for the firm to use more than one procedure to produce the total quantity required of a single product, so as to satisfy resource and emission constraints. That is, it is not necessary that each product be produced using just a single operating procedure. The total quantity produced of product i will then be the sum of all quantities produced using the different operating procedures.

As in the first model, the objective here too is to maximize the total profit. Likewise, as in the previous model, since the term "resource" signifies not only raw materials but other production related issues, the unit costs C_j also denote production related costs. The objective function may be expressed as:

$$\sum_i S_i \sum_{(t=1,\ldots,T_i)} x_{it} - \sum_j C_j r_j - \sum_{(m=1,\ldots,N)} Q_m e_m + \sum_{(m=N+1,\ldots,M)} Q_m^- e_m^- - \sum_{(m=N+1,\ldots,M)} Q_m^+ e_m^+. \tag{10}$$

The constraints defining the requirements and availability of all resources (including production related issues such as machine and labor capacities and storage space), total quantity that is output for each emission, trading of emission allowances, and three possible types of emission thresholds (simple, product based, and resource based) are similar to those described for the previous model, with minor changes to reflect the availability of multiple technologies. Here again, we note that all three types of emission thresholds may not be relevant for every problem scenario. These constraints are shown below:

$$r_j = \sum_{(i=1,\ldots,I)} \sum_{(t=1,\ldots,T_i)} A_{jit} x_{it} \leqslant R_j, \quad j = 1,\ldots,J, \tag{11}$$

$$e_m = \sum_{(i=1,\ldots,I)} \sum_{(t=1,\ldots,T_i)} \varepsilon_{\mathrm{mit}} x_{it}, \quad m = 1,\ldots,M, \tag{12}$$

Simple threshold:

$$e_m \leqslant E_m \quad \text{for relevant } m, \tag{13}$$

Product based:

$$e_m \leqslant H_m \sum_{(i=1,\ldots,I)} \sum_{(t=1,\ldots,T_i)} x_{it} \quad \text{for relevant } m, \tag{14}$$

Resource based:

$$e_m \leqslant G_{mj} r_j \quad \text{for relevant } m \text{ and } j, \tag{15}$$

$$e_m + e_m^- - e_m^+ = E_m^{\mathrm{T}}, \quad m = N+1,\ldots,M. \tag{16}$$

As in the first model, the demand for each product could be a known parameter based on forecasting models. Alternatively, demand could *partially* be a function of the quantity of each emission created per unit of the product. However, observe that a product could be produced using several different operating procedures each of which yields different emission outputs per unit. In this model, we assume that the demand for a product is affected by the *maximum* quantity of each emission due to that product. This is very realistic because in many cases mass media publications refer only to the maximum hazardous quantities in products (e.g., the acrylamide case mentioned above). That is, if several different operating procedures are used to produce a product and BD_i denotes its base demand (as in the first model), then:

$$D_i = \mathrm{BD}_i - f(\varepsilon^*_{1i}, \varepsilon^*_{2i}, \ldots, \varepsilon^*_{Mi}), \quad i = 1, \ldots, I, \tag{17}$$

$$\text{where} \quad \varepsilon^*_{mi} = \mathrm{Maximum}_{(\text{over all used } t)}(\varepsilon_{mit}), \quad m = 1, \ldots, M. \tag{18}$$

Observe that the ε^*_{mi} are also decision variables in this model. Using (17), the demand equations could be modified to incorporate the *maximum* emissions ε^*_{mi} per unit of a product as follows. The binary variables z_{it} are used in Eqs. (20) and (21) to link production quantities and emission amounts to whether an operating procedure is actually used.

$$\sum_{(t=1,\ldots,T_i)} x_{it} \leqslant D_i = \mathrm{BD}_i - f(\varepsilon^*_{1i}, \varepsilon^*_{2i}, \ldots, \varepsilon^*_{Mi}), \quad i = 1, \ldots, I, \tag{19}$$

$$\varepsilon^*_{mi} \geqslant \varepsilon_{mit} z_{it} \quad m = 1, \ldots, M, \ i = 1, \ldots, I, \ t = 1, \ldots, T_i. \tag{20}$$

Finally, we have the non-negativity constraints on the production quantities and the constraints that link the production quantities to usage of operating procedures.

$$0 \leqslant x_{it} \leqslant \infty z_{it}, \quad i = 1, \ldots, I, \ t = 1, \ldots, T_i. \tag{21}$$

The above model is a linear programming problem with some binary variables and can be easily solved for most situations. Observe that the number of binary variables equals the number of products times the number of operating procedures available for each product.

4. Numerical example

In this section, we discuss a numerical example to illustrate the first model. As mentioned earlier, the primary focus of this paper is the models themselves. Therefore, although we use this numerical example to describe "what-if" scenarios that can be studied using our models, we do not discuss detailed tests here to study the effect of various environmental and/or production parameters on the product mix and production quantities. The problem scenario is based on a real-world example but the numbers have been modified to illustrate our model in a more effective manner and to preserve confidentiality. As noted earlier, we are in the process of collecting more comprehensive real-world data and, in a follow-up paper we hope to conduct detailed numerical experiments using this data.

Both models are implemented and solved on a PC. Programs are written in C+ to accept the input parameters (listed under *Known Parameters* in the notations) for a given problem scenario, and automatically generate the mathematical formulation in a format that is directly acceptable to HYPERLINDO/PC. In our numerical example, the demand for each product is modeled as:

$$D_i = \mathrm{BD}_i - \sum_{(m=1,\ldots,M)} \alpha_{mi} \varepsilon_{mi}, \quad i = 1, \ldots, I. \tag{22}$$

Observe that by the judicious selection of values for the base demand BD_i, and the coefficients α_{mi} that represent the impact of each emission on demand, the above equation can be used to model and study the effect of different emissions on the demand and hence, the optimal product mix. It is also possible to include multiple versions of Eq. (22) for the same product in a model to represent different levels of interactions between various emissions.

Table 1 shows the input information that would be needed for formulating the first model, and Table 2 shows the resulting LP formulation.

When solved, this model yields the results shown in Table 3. For brevity, Table 3 shows only variables with non-zero values and constraints with non-zero dual (shadow) prices.

Several interesting observations may be derived from the results of this numerical example, as listed below:

- Environmental constraints clearly affect the composition of the optimal product mix. In our example, product 3 actually has a negative unit contribution to profit. However, its low output value for emission 2 makes it an attractive candidate to help the firm satisfy the

Table 1
Sample input data

Number of products, resources, total emissions, penalized emissions, tradable emissions = 12, 5, 5, 4, 1
Product selling prices (Products 1,...,12) = 800 700 500 1000 2200 2300 1600 2600 700 1000 1300 1800
Resource cost prices (Resources 1,...,5) = 50 100 3 5 2
Resource availability (Resources 1,...,5) = ∞ ∞ ∞ ∞ ∞
Emission penalties (Emissions 1,...,4) = 0 0 0 1
Tradable emission allowances, purchase prices, selling prices (Emission 5) = 50,000, 5, 4
Emission thresholds (Emission 1) = 60,000
Product based emission thresholds (Emission 3) = 6
Resource based emission thresholds (Emission 2, Resource 2) = 8

Unit emission outputs (Emissions 1–5)	
Products 1	4 10 6 4 2
Products 2	3 20 7 3 1
Products 3	2 1 1 1 1
Products 4	5 15 5 5 1
Products 5	3 50 4 16 4
Products 6	8 100 4 7 2
Products 7	10 50 5 7 3
Products 8	20 280 7 15 4
Products 9	4 40 5 1 2
Products 10	5 40 6 7 2
Products 11	12 50 6 2 2
Products 12	10 40 6 9 6
Resource usage (Resources 1–5)	
Products 1	3 1 20 30 50
Products 2	3 1 20 30 50
Products 3	3 1 10 30 50
Products 4	4 2 50 40 50
Products 5	10 6 80 100 50
Products 6	9 10 60 80 50
Products 7	8 4 60 70 50
Products 8	10 10 100 100 50
Products 9	3 1 30 30 50
Products 10	3 2 60 40 50
Products 11	5 3 60 50 50
Products 12	7 6 70 90 50
Demand functions (First number is D_{0i}; Remaining are D_{mi})	
Products 1	56,000 1000 0 8400 0 0
Products 2	74,000 1000 0 9800 0 0
Products 3	10,000 1000 0 500 0 0
Products 4	29,000 1000 0 4200 0 0
Products 5	12,000 2000 0 1000 0 0
Products 6	16,000 1000 0 1000 0 0
Products 7	15,000 500 0 1000 0 0
Products 8	12,000 100 0 500 0 0
Products 9	10,000 1000 0 1000 0 0
Products 10	24,000 1000 0 2800 0 0
Products 11	25,000 1000 0 2000 0 0
Products 12	47,000 2000 0 4200 0 0

resource based threshold value on this emission (row 26 in Table 2). That is, production of product 3 creates an opportunity for the firm to increase production of other products with higher contributions to profits. For this reason, product 3 is included in the optimal product mix. It also turns out that tightening this resource based threshold value leads to a further increase in the production of product 3. We note that this phenomenon of a firm selling products with negative profit contributions is not illogical and examples of such instances are available in the automotive and electronics industries (for example, see article titled "Flat-panel popularity could lead to LCD shortage", USA Today, 2 February 2002).

- As with any LP model, the reduced costs for decision variables and dual prices for constraints may be used (first model only) to conduct detailed sensitivity analysis. For instance, the dual price of row 24 in the model shows the impact on profit for each unit change in the threshold for emission 1. Clearly, loosening the threshold value of emission 1 will increase profits as long as this constraint is restrictive. Similar analysis can also be used to calculate the impact of investing in an emission reduction technology on profit.
- Tightening the threshold value for a specific emission may influence the level of other emissions. For instance, reducing the threshold for emission 1 from 60,000 to 55,000 in the sample problem causes the level of all other emissions to also decrease. In a different example (not shown here), however, decreasing one of the emission thresholds causes a different emission quantity to increase. That is, depending on the interaction between the levels of different emissions due to each product, tightening one threshold may cause the levels of other emissions to either increase or decrease.

The models described here can also be used by firms to quickly analyze several "what-if" scenarios such as the impact of changes in emission threshold values, emission penalties, trading allowances, and trading transaction costs. Although we have not discussed these detailed tests here,

Table 2
Sample HYPERLINDO formulation for Model #1

```
MAX 800X01 + 700X02 + 500X03 + 1000X04 + 2200X05 + 2300X06 + 1600X07 + 2600X08 + 700X09 +
1000X10 + 1300X11 + 1800X12 – 50R01 – 100R02 – 3R03 – 5R04 – 2R05 – 1E04 – 5EP05 + 4EM05
ST
(2) R01 – 3X01 – 3X02 – 3X03 – 4X04 – 10X05 – 9X06 – 8X07 – 10X08 – 3X09 – 3X10 – 5X11 – 7X12 = 0
(3) R02 – 1X01 – 1X02 – 1X03 – 2X04 – 6X05 – 10X06 – 4X07 – 10X08 – 1X09 – 2X10 – 3X11 – 6X12 = 0
(4) R03 – 20X01 – 20X02 – 10X03 – 50X04 – 80X05 – 60X06 – 60X07 – 100X08 – 30X09 – 60X10 – 60X11 – 70X12 = 0
(5) R04 – 30X01 – 30X02 – 30X03 – 40X04 – 100X05 – 80X06 – 70X07 – 100X08 – 30X09 – 40X10 – 50X11 – 90X12 = 0
(6) R05 – 50X01 – 50X02 – 50X03 – 50X04 – 50X05 – 50X06 – 50X07 – 50X08 – 50X09 – 50X10 – 50X11 – 50X12 = 0
(7) E01 – 4X01 – 3X02 – 2X03 – 5X04 – 3X05 – 8X06 – 10X07 – 20X08 – 4X09 – 5X10 – 12X11 – 10X12 = 0
(8) E02 – 10X01 – 20X02 – 1X03 – 15X04 – 50X05 – 100X06 – 50X07 – 280X08 – 40X09 – 40X10 – 50X11 – 40X12 = 0
(9) E03 – 6X01 – 7X02 – 1X03 – 5X04 – 4X05 – 4X06 – 5X07 – 7X08 – 5X09 – 6X10 – 6X11 – 6X12 = 0
(10) E04 – 4X01 – 3X02 – 1X03 – 5X04 – 16X05 – 7X06 – 7X07 – 15X08 – 1X09 – 7X10 – 2X11 – 9X12 = 0
(11) E05 – 2X01 – 1X02 – 1X03 – 1X04 – 4X05 – 2X06 – 3X07 – 4X08 – 2X09 – 2X10 – 2X11 – 6X12 = 0
(12) X01 <= 1600
(13) X02 <= 2400
(14) X03 <= 7500
(15) X04 <= 3000
(16) X05 <= 2000
(17) X06 <= 4000
(18) X07 <= 5000
(19) X08 <= 6500
(20) X09 <= 1000
(21) X10 <= 2200
(22) X11 <= 1000
(23) X12 <= 1800
(24) E01 <= 60,000
(25) E03 – 6X01 – 6X02 – 6X03 – 6X04 – 6X05 – 6X06 – 6X07 – 6X08 – 6X09 – 6X10 – 6X11 – 6X12 <= 0
(26) E02 – 8R02 <= 0
(27) E05 + EM05 – EP05 = 50,000
END
```

some of our observations from these tests are as follows:

- The influence of emission trading depends on the transaction costs of trading and the amount of basic allowances for the emission. If transaction costs are high (which implies that buying allowances is very expensive and selling allowances does not yield large revenue), the firm will not be very flexible in trading allowances. In the case of extremely high transaction costs, emission trading has the same effect as a threshold value. In contrast, if transaction costs are zero, traded emissions can be purchased or sold for the same price. That is, emission trading has the same effect as a taxed emission from the firm's perspective.
- If the costs of achieving legal compliance with environmental constraints are high (i.e., Q_m values are high), profit will be severely impacted.
- If the firm offers environmentally friendly products with low emission levels, the higher demand due to environmentally aware customers may improve profit.
- If there are several operating procedures available for a product, it is possible that the firm will use more than one procedure. In fact, the availability of several procedures can make it easier for the firm to comply with environmental constraints. For example, a procedure with a low output rate for a tightly constrained emission can be used in conjunction with a procedure that has a low resource usage per unit of the product to "balance" the two issues.

Table 3
Sample HYPERLINDO solution for Model #1

Variable	Value	Reduced cost
Objective function value		
(1) 1,781,188.00		
X01	1600.00	0.00
X02	2400.00	0.00
X03	3055.88	0.00
X04	3000.00	0.00
X05	2000.00	0.00
X11	107.35	0.00
X12	1800.00	0.00
R01	66,304.41	0.00
R02	36,177.94	0.00
R03	553,000.00	0.00
R04	699,044.10	0.00
R05	698,161.80	0.00
E01	60,000.00	0.00
E02	289,423.50	0.00
E03	63,900.00	0.00
E04	80,070.59	0.00
E05	30,670.59	0.00
EM05	19,329.41	0.00
Row	Slack	Dual prices
(2)	0.00	−50.00
(3)	0.00	−50.59
(4)	0.00	−3.00
(5)	0.00	−5.00
(6)	0.00	−2.00
(7)	0.00	−4.12
(8)	0.00	−6.18
(10)	0.00	−1.00
(11)	0.00	−4.00
(12)	0.00	199.18
(13)	0.00	46.53
(15)	0.00	126.59
(16)	0.00	203.29
(23)	0.00	65.23
(24)	0.00	4.12
(26)	0.00	6.18
(27)	0.00	4.00
No. iterations = 9		

5. Conclusions

In this paper, we present two mathematical models that can be used by firms to determine the optimal product mix and production quantities in the presence of several different types of environmental constraints, in addition to production constraints. The first model, which assumes that each product has just one operating procedure, is a linear program while the second model, with more than one operating procedure for a product, is a mixed integer program. The solutions of both models identify the products that the firm should produce, their production quantities, emissions amounts for each emission, allowances that should be traded (bought or sold) for emissions that permit trading, and resource levels required for all resources (raw materials, machine time, etc.). A small numerical example is used to offer insights into the impact of different environmental issues on profit for the first model.

This paper is an initial attempt to develop models that simultaneously addresses both production resource and several environment related issues in production planning. Using these models, firms can hopefully address environmental concerns and regulations in a proactive manner, rather than in a reactive manner.

The models presented here offer several avenues for further research. First, as noted earlier in this paper, both models need to be tested extensively using real-world data to study the quality of the planning models, their performance under different operating scenarios, and their transferability to different industrial sectors. Second, both models can be easily extended to multi-period scenarios to consider issues such as inventories and allowance transfers across periods. The second model can also be extended to analyze investments in environmentally friendly new technologies in future periods, or to study strategic implications of environmental decisions. For example, if a firm consistently sells a large portion of its allowance for a certain emission, it may find its allowance reduced in future years.

Other extensions to the research described here can analyze the implications of different product demand functions that consider environmental impacts. Information asymmetry between firms and their customers is another opportunity for further research.

References

Belz, F.-M., 2001. Integratives Öko-Marketing. Wiesbaden, Germany.

Berendes, K., Winters, K.P., 1995. Das Abwasserabgabengesetz. Beck, Munich, Germany.
Bernstein, P.M., Montgomery, W.D., Rutherford, T.F., 1999. Global impacts of the Kyoto agreement: Results from the MS-MRT model. Resource and Energy Economics 21, 375–413.
Bloemhof-Ruwaard, J.M., van Beek, P., Hordijk, L., van Wassenhove, L.N., 1995. Interactions between operational research and environmental management. European Journal of Operational Research 85, 229–243.
Bundesgesetzblatt, 1999. Rückstands—Höchstmengenverordnung, Teil 1, Köln, Germany.
Bundesministerium für Umwelt, Naturschutz, und Rektorsicherheit, 2002. Erste Allgemeine Verwaltungsvorschrift zum Bundes-Immissionsschutzgesetz (TA Luft), vom 24, July 2002, Gemeinsames Ministerialblatt, Heft 25–29, pp. 511–605.
Bundesministerium für Verbraucherschutz, Ernährung, und Landwirtschaft, 2003. Ernährungs- und agrarpolitischer Bericht der Bundesregierung 2003, Berlin, Germany.
Curkovic, S., 2003. Environmentally responsible manufacturing: The development and validation of a measurement model. European Journal of Operational Research 146, 130–155.
Daniel, S.E., Diakoulaki, D.C., Pappis, C.P., 1997. Operations research and environmental planning. European Journal of Operational Research 102, 229–243.
Dreher, C., Fleig, J., Arnold, F., 1999. Intensifying use and prolonging product lifetime in German companies—Impacts on manufacturing strategies and workforce. In: Greening the Industry Network Conference, Best Paper Proceedings, Chapel Hill, NC.
Eidgenössisches Department des Innern, 2002. Verordnung des EDI über Fremd und Inhaltsstoffe von Lebensmitteln, Bern, Switzerland.
European Council Press Release, 2003. European Parliament Adopts Emissions Trading Scheme.
Foster, C., Green, 1999. Greening the innovation process. In: Greening the Industry Network Conference, Best Paper Proceedings, Chapel Hill, NC.
Gege, M., 1997. Kosten senken durch Umweltmanagement. Vahlen, Munich, Germany.
Houghton, J.T., 1996. Climate Change 1995: The Science of Climate Change. Cambridge University Press, Cambridge, MA.
Hutchinson, A., Hutchinson, F., 1997. Environmental Business Management. McGraw-Hill, New York.
Inman, R.A., 1999. Environmental management New challenges for production and inventory managers. Production and Inventory Management Journal 40 (third quarter), 46–49.
ISO 14001, 1996. Environmental Management Systems. Specification with Guidance for Use.
Kistner, K.P., Steven, M., 1991. Management ökologischer Risiken in der Produktionsplanung. Zeitschrift für Betriebswirtschaft 61, 1307–1336.
Klassen, R.D., McLaughlin, C.P., 1996. The impact of environmental management on firm performance. Management Science 42, 1199–1214.
Klassen, R.D., Whybark, D.C., 1999. Environmental management in operations. Decision Sciences 30, 601–631.
Letmathe, P., Steven, M., 1995. Die Berücksichtigung von Maßnahmen der staatlichen Umweltpolitik bei betrieblichen Investitionsentscheidungen. Wirtschaftswissenschaftliches Studium 24, 120–123 and 167–172.
Matsuo, N., 1998. Key elements related to the emissions trading for the Kyoto Protocol. Energy Policy 26, 263–273.
Melnyk, S.A., Sroufe, R., Montabon, F., Calantone, R., Tummula, R.L., Hinds, T.J., 1999. Integrating environmental issues into material planning: Green MRP. Production and Inventory Management Journal 40 (third quarter), 36–45.
Meyer, A., 2001. What's in it for the customer? Successfully marketing green clothes. Business Strategy and the Environment 10, 317–330.
Miele, 2002. Nachhaltigkeitsbericht 2002, Gütersloh.
Müller, E., 2002. Environmental Labeling, Innovation, and the Toolbox of Environmental Policy—Lessons Learned from the German Blue Angel Program, Berlin.
Nijkamp, P., van den Bergh, J.C.J.M., 1997. New advances in economic modeling and evaluation of environmental issues. European Journal of Operational Research 99, 180–196.
Nike, 2000. Reuse-a-Shoe, Available from <www.nike.com/nikebiz/nikebiz.jhtml?page=27&cat=reuseashoe>.
Peglau, R., 2003. EMAS-registration and ISO 14001-certification (July 2003), Federal Environmental Agency, Berlin, Germany.
Penkuhn, T., Spengler, T., Püchert, H., Rentz, O., 1997. Environmental integrated production planning for ammonia synthesis. European Journal of Operational Research 97, 327–336.
Reiborn, C.A., Joyner, B.E., Logan, J.W., 1999. ISO 1400 and the bottom line. Quality Progress (November), 89–93.
Remmers, J., Morgenstern, T., Schons, G., Haasis, H.D., Rentz, O., 1990. Integration of air pollution control technologies in linear energy-environmental models. European Journal of Operational Research 47, 306–316.
Steven, M., Letmathe, P., 1996. Umweltstücklisten als Datengrundlage für umweltorientierte PPS-systeme. In: Albach, Dyckhoff (Eds.), Betriebliches Umweltmanagement (ZfB-Ergänzungsheft 2/96), Gabler, Wiesbaden, pp. 165–183.
The Economist, 2000. "How green is your market? 76.
Thomas, S., Ryan, M., 1999. Burning rivers. In: Yandle (Ed.), Economic Analysis of Environmental Policy. Rowman & Littlefield, New York.
Ton, P., Boone, N., Imhoff, D., Sweeney, K., 1999. The market for organic cotton. In: Myers, Stolton (Eds.), Organic Cotton—From Field to Final Product. Intermediate Technology Publications, London, pp. 101–119.
Ulhoi, J.P., 1995. Corporate environmental and resource management: In search of a new managerial paradigm. European Journal of Operational Research 80, 2–5.

USA Today, 2002. Flat-panel popularity could lead to LCD shortage, Available from <www.usatoday.com/tech/zd/zd5.htm>, 6 February.

Wietschel, M., Fichtner, W., Enzensberger, N., Rentz, O., 2002. Zur Relevanz eines europaweiten CO_2-Zertifikatehandels für strategische Unternehmensentscheidungen. In: Fichtner, Geldermann (Eds.), Einsatz von OR-Verfahren zur Techno-Ökonomischen Analyse von Produktionssystemen. Peter Lang, Frankfurt, Germany.

Wirl, F., 1991. Evaluation of management strategies under environmental constraints. European Journal of Operational Research 55, 191–200.