© 1995 American Accounting Association
Accounting Horizons
Vol. 9 No. 4
December 1995
pp. 48–61

Integrating Activity-Based Costing With the Theory of Constraints To Enhance Production-Related Decision-Making

Robert Kee

Robert Kee is Associate Professor at the University of Alabama.

SYNOPSIS: Activity-based costing (ABC) and the theory of constraints (TOC) represent alternative paradigms for modeling a firm's production structure. This paper discusses how aspects of the TOC may be integrated with ABC. The resulting model captures the interaction between the cost, physical resources, and capacity of production activities. The model enables an optimal production mix to be determined from simultaneous evaluation of ABC data and physical attributes of the production process. Equally important, it facilitates identifying a bottleneck activity that constrains the firm's production opportunities and may lead to excess resources in the firm's other production activities. Sensitivity analysis may be used to estimate the benefits that may accrue from relieving a constraint and identifying the subsequent set of activities that will become a bottleneck as prior constraints are relieved. Integrating ABC with principles from the TOC provides an expanded framework for understanding the economic consequences of production-related decisions.

Activity-based costing (ABC) and the theory of constraints (TOC) represent alternative paradigms for modeling a firm's production structure. Both paradigms are designed to aid managers in understanding the firm's production processes and to provide information for resource allocation decisions. While their objectives are similar, the means used to achieve these objectives differ significantly. ABC represents an extension of traditional cost systems and provides more accurate product cost information. Firms adopting ABC report an improved understanding of the profitability of their product lines and customer base (Cooper et al. 1992, 55). In addition, many firms report that ABC has stimulated improvements in their production processes (Cooper et al. 1992, 57). Conversely, the TOC represents a theory for optimizing production that provides insights into the manufacturing process that traditional cost systems ignore. Firms adopting the TOC indicate that it has aided in reducing lead time, cycle time, and inventory while improving productivity and quality (Goldratt and Fox 1987, 22).

One of the questions confronting many managers today is that of deciding which paradigm to select for production-related decisions. As noted earlier, ABC and the TOC represent radically different approaches to modeling the production process. However, as noted by Goldratt (1990, 110) in comparing the TOC with just-in-time and total-quality-management:

> Most of these new approaches usually provide a significant contribution. They may emerge from different angles, they may be based on different facets of the established base of knowledge.... It is no wonder that after some time the consolidation process must begin. People start to explore ways in which to mold the new ideas, which have passed the test of reality, into a new and uniform body of knowledge.

The author gratefully acknowledges the helpful comments provided by Robin Cooper, Thomas Albright, the associate editor, and anonymous reviewers. Financial support for this paper was provided by the Culverhouse School of Accountancy, University of Alabama.

Submitted February 1995
Accepted June 1995

Similarly, ABC and the TOC may represent different aspects of the same underlying phenomenon.

The purpose of this paper is to discuss how the principles of ABC and the TOC may be used in conjunction with one another. The paper demonstrates that ABC and the TOC reflect different aspects of the production process and that concepts of both models may be integrated to provide deeper insights into a firm's underlying production process. The remainder of the paper is organized as follows: the next two sections evaluate many of the conceptual aspects and limitations of ABC and the TOC, respectively. This is followed by a discussion of how the principles of ABC and the TOC may be integrated within a common framework for modeling a firm's production structure. A methodology is then proposed for implementing the model. The next section presents a numerical example to illustrate the application of the model and the methodology used for its implementation. The summary and conclusions of the paper are presented in the final section.

ACTIVITY-BASED COSTING

ABC was developed independently by General Electric and other firms to improve the usefulness of accounting information (Johnson 1992, 27). Cooper, Kaplan, and others studied organizations that developed innovative accounting systems and introduced ABC to the academic literature. ABC differs from traditional cost systems in two important respects. First, it traces indirect costs to cost objects such as products and customers on the basis of factors (cost drivers) that cause or correlate highly with indirect costs. The use of multiple-cost drivers results in product costs that more accurately reflect the quantity and diversity of resources used by products in the manufacturing and support processes used in their production. Second, ABC traces indirect costs on the basis of the structural or hierarchical level at which costs are incurred in the production process. For example, many indirect costs are incurred at the batch, product, and facility levels (Cooper 1990). The use of multiple-cost drivers and tracing cost at the hierarchical level at which it is incurred enable ABC to more accurately model the relationship between resources used in production activities and the products they are used to produce. ABC, thereby, provides a better estimate of product cost as well as the cost of the individual activities used in its production.

ABC has been criticized for its failure to identify and remove constraints that lead to delay, excess, and variation in the production process (Johnson 1992, 32). Johnson notes that removing constraints is one of the primary avenues firms should exploit to reduce costs and become more globally competitive. Production constraints also play a significant role in many of the decisions in which ABC is used. For example, a bottleneck activity in the firm's production structure plays a critical role in decisions such as the optimal-production mix, pricing, make-buy, and special order. Similarly, a production constraint plays a key role in determining the opportunity cost of the firm's resources and in determining where process improvement would be the most beneficial to the firm.

Cooper and Kaplan (1992, 12) note that ABC measures the usage of resources with respect to the demand placed on a production activity. If demand for an activity is below the level of services supplied, ABC assigns the cost of these excess resources to "unused capacity" (Cooper and Kaplan 1992a, 1). Under ABC, the cost of unused capacity is used for resource allocation decisions to better match the supply and demand for an activity's resources. The excess capacity of production activities is determined, in part, by a constraint or bottleneck activity in the firm's production structure. A constraint or bottleneck restricts production, thereby limiting resource usage by non-constrained activities and leading to excess or unused capacity. A constraint, therefore, plays a pivotal role in understanding why unused capacity exists in the firm's production structure and in decisions involving its disposition. For example, a constraint represents an activity where resources may be added to expand production and use the excess capacity of non-constrained activities. Conversely, for a resource reduction decision, a bottleneck may be used to determine the level of unused capacity that may be eliminated without adversely impacting production. Consequently, identifying a constraint and understanding its impact on the firm's production opportunities is crucial for resource allocation decisions using ABC to maximize the firm's production and profitability.

THEORY OF CONSTRAINTS

The theory of constraints was developed by Goldratt as a process of ongoing improvement (Sheridan 1991, 48). The objective of the TOC is to maximize the goal of an organization which is limited by a constraint (Goldratt 1990, 4). The system is managed with respect to the constraint or bottleneck, while resources are expended to relieve this limitation on the system (Goldratt and Cox 1992, 301). When the constraint is removed and the firm moves to a higher level of goal attainment, a new bottleneck will appear, and the cycle of managing the system with respect to the new constraint is repeated.

The simplicity and compact nature of Goldratt's theory conceal many of the underlying assumptions that make it so powerful. First, the TOC recognizes both the interdependent nature of production activities and how the most limited of these activities controls the performance of the larger system. Equally important, the TOC focuses attention on constraints from an organizational perspective, i.e., how does removing the constraint impact the goal of the firm. Consequently, the assumptions underlying a constraint and its core problems rather than symptoms may be addressed. Finally, the TOC's process of continuously removing bottlenecks will impact successively larger subsystems of the firm and promote change in the firm's management culture. For example, the TOC encourages communication and problem-solving across functional areas based on an organizational rather than a local perspective.

The TOC is implemented through three measurements: throughput, the rate at which the system generates money through sales; inventory, all money the system invests in purchasing items the system intends to sell; and operating expenses, all the money the system spends in turning inventory into throughput (Goldratt and Fox 1986, 29). Under the TOC, direct material is treated as a variable cost, while direct labor and all other costs are treated as fixed. The objective of the TOC is to maximize throughput subject to the capacity of the individual production activities of the firm.

Kaplan has criticized the application of the TOC to production decisions as an extreme form of direct costing or the contribution margin approach to decision-making (Robinson 1990, 3). Contribution margin, in its traditional and the TOC forms, has been criticized for its short-run focus. Shank suggests that the contribution margin approach to decision-making will lead a firm to never drop a product, always make instead of buy, charge inadequate prices, and support the short-run status quo (Robinson 1990, 19). Supporters of the contribution margin approach suggest that these problems are the result of inappropriate applications of the method rather than the method itself. However, Shank notes that use of the contribution approach is pernicious and that it's "a snare, a trap, and a delusion" (Robinson 1990, 19). Finally, the use of contribution margin in either its traditional or TOC forms may lead to a series of short-term maximization decisions that fail to maximize the long-term profitability of the firm. For example, Shank suggests that the contribution margin approach led the airlines and trucking industries to the verge of bankruptcy after deregulation (Robinson 1990, 17).

Another limitation of the TOC concerns its use of global operational measures to model the relationship between resources used in the production process and output. As noted earlier, the TOC assumes a short-term decision horizon. However, even in the short run, many decisions involve trade-offs between increases in throughput, inventory, and operating expenses. Therefore, using throughput maximization as a decision criterion may lead to suboptimal decisions in some circumstances. For intermediate and longer-run decisions in which management has discretionary power over direct labor or overhead cost items, the global operational measures of the TOC ignore factors relevant to the decision process.

COMPETING AND COMPLEMENTARY ASPECTS OF ABC AND TOC

ABC and the TOC model different aspects of a firm's production structure. ABC models the economic aspects of how resources at the unit-, batch-, and product-level activities are transformed into the firm's products. ABC, thus, represents a long-term perspective of how costs vary with production. Conversely, the principles of the TOC reflect how the physical resources consumed by production activities and their production capacity play a critical role in the production process. The global operational measures used to implement these principles reflect a direct-cost approach to decision-making. Unlike ABC, they represent a short-term perspective of the relationship between a change in cost and production.

As noted earlier, one of the limitations of ABC is its failure to explicitly reflect the physical usage of resources by production activities and the capacity of these activities.[1] The inability to incorporate physical measures of resource usage and the production capacity of activities fosters an inability to identify constraints and predict their effect on the firm as noted by Johnson (1992, 32). Conversely, the TOC is based, in large part, on managing production constraints. One of the limitations of the TOC is its use of global operational measures to guide production decisions. As noted earlier, the global operational measures of the TOC will lead to optimal decisions under very restrictive circumstances. Alternatively, ABC provides a comprehensive framework for modeling the economic attributes of the production process. It thereby provides managers with a means of predicting the economic consequences of alternative resource allocation decisions. The strengths of ABC and TOC are complementary in nature. The strengths of each model overcome a major limitation of the other.

Many of the conceptual aspects of ABC and the TOC may be integrated to form a larger model that simultaneously links the cost and physical attributes of a firm's production structure.[2] This objective may be accomplished by expanding the framework provided by ABC to incorporate the resource usage of individual products and the capacities of the processes used in their production. However, the cost and physical usage of resources by production activities must be modeled at the level at which an activity contributes to the production process. Resources used by unit-level activities vary proportionally with production and may be represented with a linear relationship. Batch- and product-level activities, however, use resources in large discrete quantities at periodic intervals as production is increased. Consequently, they must be modeled with a step function to reflect the non-

[1] ABC incorporates the physical resources and capacity of an activity indirectly into the analysis of a product's cost. An activity's costs are divided by a measure of its practical capacity to compute a cost driver rate for tracing costs to products using an activity's resources. Over a short to intermediate time horizon, an activity's production capacity may be fixed. That is, the firm may have trouble adding or deleting capacity. Consequently, an activity's production capacity represents a potential constraint on the firm's production opportunities. However, the cost driver rates used by ABC fail to reflect this limitation. In the long run, the capacity of production activities is not restricted. The firm can add or delete capacity as needed. However, even then, cost driver rates are predicated on specific levels of production capacity. Integrating ABC with the capacity levels used to compute its cost driver rates may be useful for understanding the production opportunities inherent in cost driver rates and evaluating whether these capacity levels are optimal for the firm.

[2] Spoede et al. (1994, 43) have suggested that ABC may be used "to generate the data necessary to support the Theory of Constraints." ABC cost data, in effect, would be used as an input into the TOC for making product-mix decisions. The Spoede et al. (1994) suggestion for using ABC with TOC contrasts with the proposal presented in this paper that expands the ABC model to explicitly recognize the physical usage of resources and the capacity of production activities. It is demonstrated that the expanded ABC model overcomes limitations of the traditional ABC model and that it may be used in conjunction with principles from the TOC to make product-mix and other resource allocation decisions. However, unlike Spoede et al. (1994), product mix and other resource allocation decisions are made within the framework of the ABC model.

linear manner in which their resources are used in the production process.

A METHODOLOGY FOR INTEGRATING RESOURCE CONSTRAINTS INTO ABC

Mixed-integer programming

To integrate ABC with the physical usage and capacity of production activities, a mixed-integer programming model can be used. A brief review of mixed-integer programming is provided in Appendix A. A mixed-integer programming model may be used to represent unit-level cost and resources as continuous variables, while batch- and product-level activities are represented as discrete variables.[3] The resulting model captures the interaction among the cost, physical resources, and capacity of production activities. It thereby enables many of the principles of the ABC and the TOC to be implemented within a common framework.

The solution to the mixed-integer programming model gives the optimal-production mix subject to the capacity of the individual activities comprising the firm's production structure. The solution identifies non-constrained activities and their excess resources. These are the resources that Cooper and Kaplan (1992) note may result in excess spending. However, identification of these activities prior to actual production may assist management in reallocating these resources to other uses. The solution to the mixed-integer programming model also may be used to identify the constraint that limits production and leads to excess spending. Constraint identification provides a starting point for the application of Goldratt's principles (1990) for managing production bottlenecks. Sensitivity analysis of the mixed-integer model may be used to estimate the expected increase in production and profit from relieving the bottleneck activity. Sensitivity analysis also may be used to identify the activity that will become a constraint when the current bottleneck is relieved as well as the economic benefits of its removal.

A Numerical Example

To illustrate the integration of ABC with the TOC, consider the example provided in table 1. XYZ Inc. is a medium-size firm with two production departments, assembly and finishing, and three support departments, set-up, purchasing, and engineering. XYZ is currently considering adding products X_1, X_2, X_3, or X_4 or some combination to expand its current product mix. ABC cost, price, and expected annual demand for these products are given in panel I.[4] Direct material and labor are traced directly to individual products. Unit-level overhead was estimated using direct labor cost as the cost driver. Set-up and purchasing costs are incurred at the batch level, while engineering is incurred at the product level. These costs, their cost driver, and batch- and product-level rates are given in panels II and III of table 1, respectively. Based on how products use batch- and product-level resources, their costs are converted to an equivalent unit cost.[5]

As noted earlier, a mixed-integer programming model may be used to integrate ABC data with the physical usage of resources and their productive capacities. The mixed-integer equations for modeling the production structure in table 1 are listed in table 2. The first equation or objective function reflects the goal of maximizing the profit (Z) that may be achieved from the different production mixes that XYZ may produce. The contribution of each product was computed by subtracting

---

[3] Unit-level activities use resources in discrete quantities. However, the requirement of using integer values to represent these activities may be relaxed if the range of the variable is sufficiently large so that rounding the solution to nearby integer values will lead to approximately the same objective function value (Harvey 1979, 245).

[4] XYZ does not have sufficient capacity in its support activities to produce each of the four new products. Consequently, if customers demand a full product line, then XYZ is confronted with outsourcing the new products it cannot produce or adding capacity to expand production.

[5] For example, set-up (see panel II of table 1) is expected to provide 500 hours of set-up time while incurring a cost of $200,000. Set-up costs are, therefore, $400 per set-up hour. A batch of product X_1, 1,000 units, requires two hours of set-up time. Therefore, the set-up cost for a unit of X_1 is approximately $.80. Purchasing and engineering costs were converted from batch- and product-levels to a unit basis in a similar manner.

TABLE 1
XYZ Inc.
Revenue, Cost and Operating Structure

Panel I: Activity-Based Cost, Price, and Demand

	Product				
	X_1	X_2	X_3	X_4	
Assembly Labor Hours	1.00	1.00	2.00	5.00	200,000
Finishing Labor Hours	.50	.50	2.00	4.00	180,000
Direct Material Cost	$ 4.00	$ 7.00	$ 15.00	$ 28.00	
Direct Labor Cost	12.00	12.00	32.00	72.00	
Overhead Cost*	36.00	36.00	96.00	216.00	
Unit-Level Cost	$52.00	$55.00	$143.00	$316.00	
Batch-Level Cost**					
Set-up	.80	.80	3.20	10.00	
Purchasing	.25	.40	2.40	6.00	
Product-Level Cost**					
Engineering	1.00	1.00	10.00	25.00	
Total Activity-Based Cost	$54.05	$57.20	$158.60	$357.00	
Price	70.00	80.00	223.00	516.00	
Profit	$15.95	$22.80	$ 64.40	$159.00	
Maximum Annual Expected Demand	100,000	100,000	30,000	20,000	

* 300% of direct labor cost
** Converted to an equivalent unit level cost

Panel II: Batch-Level Activities

	Product				
	X_1	X_2	X_3	X_4	
Set-Up Department					
Batch Size (units)	1,000	1,000	500	200	
Hours/batch	2	2	4	5	
Expected Cost					$200,000
Expected Capacity (hours)					500
Cost Per Set-Up Hour					$400
Purchasing Department					
Batch Size (units)	4,000	4,000	1,000	500	
Orders/batch	5	8	12	15	
Expected Cost					$160,000
Expected Capacity (orders)					800
Cost Per Purchase Order					$200

Panel III: Product-Level Activities

	Product				
	X_1	X_2	X_3	X_4	
Engineering Department					
Drawings/product	100	100	300	500	
Expected Cost					$1,000,000
Expected Capacity (drawings)					1,000
Cost Per Engineering Drawing					$1,000

direct material, labor, and unit-level costs from its price. Set-up and purchasing costs are deducted at the batch level, while engineering costs are deducted at the product level. The constraints of XYZ's production processes are listed below the objective function. Constraints 1 and 2 reflect the usage of resources in assembly and finishing measured in direct labor hours. Constraints 3 through 7 reflect the batch level at which set-up activities are used in production. For example, constraint 3 computes the number of set-ups (S_1) for product X_1, where the number of set-ups is restricted to an integer value. The hours of set-up time used for a batch of each product, subject to available set-up time, are modeled in constraint 7. Constraints 8 through 12 are formulated to reflect the batch-level behavior of the purchasing department in a similar manner. Constraints 13 through 17 reflect the product-level behavior of engineering services. For example, constraint 13 is used to compute the amount of engineering (E_1) required to produce product X_1, where E_1 is restricted to a value of zero or one.[6] Constraint 17 measures the number of engineering drawings required to support each product and the capacity of the department. The last four constraints, 18 through 21, reflect the maximum expected annual demand for each of XYZ's products.

The solution to the mixed-integer problem in table 2 is given in table 3.[7] The first panel of table 3, labeled "Solution Variables," gives the production mix that maximizes the objective function in table 2. As indicated, the optimal-production strategy consists of producing 30,000 units of X_1, 100,000 units of X_2, and 30,000 units of X_3. Variables S_1, S_2, S_3, P_1, P_2, and P_3 measure the number of set-up and purchase batch-level activities required to support this production strategy. Similarly, variables E_1, E_2, and E_3 measure the product-level support required from the engineering department. Finally, the profit (Z) projected for the optimal-production mix is $4,620,000.

The second panel of table 3, labeled "Slack Variables," measures the slack or excess capacity of the non-constrained activities in XYZ's production structure. As indicated, assembly and finishing are projected to have 10,000 (constraint 1) and 55,000 (constraint 2) excess labor hours of capacity, respectively. Similarly, constraints 12 and 17 suggest that purchasing and engineering will have excess capacity of 200 purchasing orders and 500 engineering drawings. The slack variables for constraints 18 and 21 indicate that the firm will be unable to supply 70,000 units of demand for product X_1 and 20,000 units of demand for product X_4.[8]

To provide a benchmark for assessing how well the expanded ABC model performs, the data in table 1 were analyzed with the ABC and TOC models. The product mix, excess resources, and profit from these models are provided in table 4. The product mix for the ABC model was computed by ranking each product in terms of its profitability and producing

---

[6] If product X_1 is produced, 100 engineering drawings are required before the first unit of X_1 is manufactured. However, once developed, these drawings can be used to produce one to 100,000 units of X_1 without the use of additional engineering resources. Conversely, if product X_1 is not produced, no engineering drawings are required. Consequently, the use of engineering is a product-level cost dependent solely upon whether or not X_1 is produced. Therefore, the use of engineering resources is modeled in constraint 13 with the binary variable E_1 that assumes a value of one if X_1 is produced or zero if it is not produced. The manner in which E_1 is computed can be seen by moving the second term in constraint 13 (-100,000E_1) to the right hand side of the equation. The revised equation for constraint 13 states that the units of X_1 produced is less than or equal to the maximum number of X_1 that may be produced times E_1. Consequently, if X_1 is not produced, constraint 13 in conjunction with the objective function will assign a value of zero to E_1 indicating that no engineering resources will be used for product X_1. On the other hand, if X_1 is produced, E_1 assumes a value of one indicating that resources for 100 engineering drawings will be required. Constraints 14 through 16 compute the engineering effort required for manufacturing products X_2, X_3, and X_4 in a similar manner, respectively.

[7] The mixed-integer equations in table 2 were solved using STORM, a software product of Storm Software, Inc. The STORM package may be used to solve linear, integer, and mixed-integer programs as well as other statistical and quantitative models.

[8] The slack variable for constraint 8 measures the number of additional units that could be produced with the last batch of purchase orders for product X_1. Similarly, the slack variable for constraint 13 represents the additional units that could be produced from the engineering drawings for product X_1.

TABLE 2
XYZ Inc.
Mixed-Integer Programming Equations

Objective Function: Maximize Z =		$18X_1$	$+25X_2$	$+80X_3$	$+200X_4$	$-800S_1$ $-1{,}000P_1$ $-100{,}000E_1$	$-800S_2$ $-1{,}600P2$ $-100{,}000E_2$	$-1600S_3$ $-2{,}400P_3$ $-300{,}000E_3$	$-2{,}000S_4$ $-3{,}000P_4$ $-500{,}000E_4$	
Subject to:										
Assembly Dept.	1	$1X_1$	$+1X_2$	$+2X_3$	$+5X_4$					$\leq 200{,}000$
Finishing Dept.	2	$.5X_1$	$+.5X_2$	$+2X_3$	$+4X_4$					$\leq 180{,}000$
Set-up Dept.	3	X_1				$-1000S_1$				≤ 0
	4		X_2				$-1000S_2$			≤ 0
	5			X_3				$-500S_3$		≤ 0
	6				X_4				$-200S_4$	≤ 0
	7					$2S_1$	$+2S_2$	$+4S_3$	$+5S_4$	≤ 500
Purchasing Dept.	8	X_1				$-4{,}000P_1$				≤ 0
	9		X_2				$-4000P_2$			≤ 0
	10			X_3				$-1000P_3$		≤ 0
	11				X_4				$-500P_4$	≤ 0
	12					$5P_1$	$+8P_2$	$+12P_3$	$+15P_4$	≤ 800
Engineering	13	X_1				$-100{,}000E_1$				≤ 0
	14		X_2				$-100{,}000E_2$			≤ 0
	15			X_3				$-30{,}000E_3$		≤ 0
	16				X_4				$-20{,}000E_4$	≤ 0
	17					$100E_1$	$+100E_2$	$+300E_3$	$+500E_4$	$\leq 1{,}000$
Product Demand	18	X_1								$\leq 100{,}000$
	19		X_2							$\leq 100{,}000$
	20			X_3						$\leq 30{,}000$
	21				X_4					$\leq 20{,}000$

Where: Z = net profit
X_i = units of product i produced, $X \geq 0$, $i = 1, \ldots, 4$
S_i = number of set-ups for product i, $S_i = 0, 1, 2, \ldots$, $i = 1, \ldots, 4$
P_i = number of batches for which purchases for product i are required, $P_i = 0, 1, 2, \ldots$, $i = 1, \ldots, 4$
E_i = engineering for product i, $E_i = 0$ or 1, $i = 1, \ldots, 4$

TABLE 3
XYZ Inc.
Mixed Integer Programming Solution

Panel I: Solution Variables

Variable	Optimal Value
X_1	30,000
X_2	100,000
X_3	30,000
S_1	30
S_2	100
S_3	60
P_1	8
P_2	25
P_3	30
E_1	1
E_2	1
E_3	1
Z	4,620,000

Panel II: Slack Variables

Constraint	Slack	Explanation
1	10,000	Excess capacity in the assembly department
2	55,000	Excess capacity in the finishing department
8	2,000	Excess capacity of last batch of purchasing orders for product X_1
12	200	Excess purchase order capacity
13	70,000	Excess capacity of engineering for product X_1
17	500	Excess capacity in engineering
18	70,000	Excess demand for product X_1
21	20,000	Excess demand for product X_4

the products with the highest profitability subject to available resources. As indicated in table 4, this procedure would lead to producing 20,000 units of product X_4. Producing 20,000 units of product X_4 uses the resources of the set-up department thereby preventing the production of other products.

The product mix for the TOC in table 4 was computed using procedures proposed by Plenert (1993, 126).[9] As indicated, the TOC would suggest producing 50,000 units of product X_1, 100,000 units of X_2, and 25,000 units of X_3. The product mix for the expanded ABC model was taken from table 3. The excess resources resulting from the production mix under the ABC, TOC, and expanded ABC models are given in the second section of table 4. The ABC model resulted in more excess resources than the TOC and expanded ABC models. This result was to be expected since the TOC and expanded ABC models explicitly incorporate resource constraints into the determination of an optimal-production mix. The income of each model is given in the last section of table 4. The projected income for the ABC, TOC, and expanded ABC models are \$3,180,000; \$2,280,000; and \$4,620,000, re-

[9] A product mix is selected under the TOC, based on throughput maximization. Operationally throughput is defined as a product's price less its direct material cost. Direct labor and overhead are treated as a fixed operating expense and therefore irrelevant to the product-mix decision. Fox (1987) outlines a set of procedures for selecting a product mix that maximizes throughput. However, Plenert (1993, 126) suggests these procedures may be inefficient when the firm's production structure contains multiple constrained resources. Plenert (1993, 126) demonstrates that linear-integer programming overcomes the limitations of Fox's procedures. With unit-, batch-, and product-level activities mixed-integer programming would be appropriate for selecting a product mix rather than the linear-integer model. A mixed-integer model was used to select the product mix for the TOC in table 4. The objective function, $Z = 66X_1 + 73X_2 + 208\,X_3 + 488X_4 - 13{,}520{,}000$, was maximized subject to the constraints listed in table 2.

TABLE 4
Comparative Analysis
ABC, TOC, and Expanded ABC Models

	ABC Model	TOC Model	Expanded ABC Model
Product Mix:			
X_1	–0	50,000	30,000
X_2	–0	100,000	100,000
X_3	–0	25,000	30,000
X_4	20,000	–0	–0
Excess Resources:			
Assembly (labor hours)	100,000	–0	10,000
Finishing (labor hours)	100,000	55,000	55,000
Set-up (hours)	–0	–0	–0
Purchasing (orders)	200	235	200
Engineering (drawings)	500	500	500
Profit:			
Projected Income	$3,180,000	$2,280,000	$4,620,000
Cost Saving Available From Excess Resources*	–0	2,307,000	–0
Available Income	$3,180,000	$4,587,000	$4,620,000

* The cost savings from excess resources was computed by multiplying the excess capacity of each activity by its respective cost driver rate. The cost of excess resources are excluded from income under ABC and the expanded ABC models.

spectively. The incomes for the three models are not comparable since the ABC and expanded ABC models exclude the cost of excess resources while the TOC includes these costs. Adding back the potential cost saving from the excess resources with the TOC model, its available income is $4,587,000.[10]

The performance of the three models listed in table 4 is a result of the opportunity cost of the resources used to determine an optimal-product mix. ABC assumes that production capacity is unconstrained. Therefore, there are no opportunity costs associated with using the capacity of production activities. A product's price less its ABC costs is used for evaluating production-related decisions. Conversely, the TOC and expanded ABC models assume that production capacity is constrained and that producing one set of products precludes the production of other products. The TOC and

[10] The ABC, TOC, and expanded ABC models discussed throughout the paper are being proposed as planning models for making product-mix and other resource allocation decisions. Identification of the optimal-product mix, maximum income potentially available to the firm, and excess resources are crucial for understanding the opportunities and problems facing the firm in developing its product-mix strategy. Consequently, the income of the ABC, TOC, and expanded ABC models in table 4 exclude the cost of excess resources. Identifying and planning how these resources may be used in more productive activities is an essential aspect of developing an optimal-product-mix strategy. In cases where the excess resources of the optimal-product mix cannot be reallocated or the firm's management chooses not to reallocate these resources, the coefficients of the mixed-integer set of equations used to model the firm's revenue, cost, and production structure may be adjusted to assess the production opportunities available to the firm given the committed nature of its resources. For financial reporting purposes, the cost of excess resources or unused capacity is expensed in the period incurred. See Cooper and Kaplan (1992) for an extended discussion of this and related issues.

expanded ABC models select products with the highest contribution margin per unit and highest profit per unit for a bottleneck activity, respectively. The opportunity cost from using the resources of the bottleneck activity is reflected in the relative profitability computed for each product.

The difference in the income of the TOC and expanded ABC model is the result of the costs that are assumed to be relevant for the product-mix decision. The TOC assumes that all resources other than direct material are committed and therefore irrelevant for decision making. Conversely, ABC assumes that all costs are relevant for understanding the costs of resources used to perform the activities used to produce the firm's output (Cooper and Kaplan 1992, 12). Based on these assumptions, the TOC selects products to the point that the marginal revenue from the last unit produced is equal to the cost of the direct material used, while the expanded ABC model selects products to the point that the marginal revenue from the last unit produced is equal to the marginal cost of the resources used in its manufacturing.

The relative superiority of the ABC, TOC, and expanded ABC models for product-mix decisions is dependent upon the economic circumstances of the firm. When demand for the firm's products exceeds the capacity of at least one production activity, the expanded ABC model will lead to a more or equally profitable product-mix decision relative to ABC.[11] The expanded ABC model incorporates the opportunity cost of a production constraint, thereby identifying those products with the highest profit potential given the firm's limited production opportunities. Similarly, when demand for the firm's products exceeds the capacity of at least one of its production activities and the firm has some discretionary power over the excess resources in its production structure, the expanded ABC model will lead to a more or equally profitable product-mix decision relative to the TOC. As noted earlier, the expanded ABC model selects products to the point where the marginal revenue from the last unit produced is equal to the marginal cost of all the resources used in its production. Conversely, the TOC will continue to produce beyond the point where the marginal revenue is equal to the marginal cost of the resources used in production, thereby consuming additional resources, and reducing available income. The expanded ABC model selects a product mix based on the resources used rather than the resources supplied to the production process. The expanded ABC model, thereby, measures the maximum potential income available to the firm and identifies resources that are unable to generate a return sufficient to justify their use.[12]

Further Analysis and Interpretation

The expanded ABC model provides a framework for implementing many of the principles of the TOC. As noted earlier, under the TOC a system is managed with respect to a constraint while resources are expended to remove a bottleneck activity. An examination of the slack variables in panel II of table 3 indicates that the set-up department is the only production or support activity without excess resources. Therefore, it represents the constraint in XYZ's production structure limiting further production and profitability. The optimal-product mix for XYZ Inc. listed in table 3 was based, in part, upon the limited resources in the set-up department. This may be seen by computing the profit per hour of set-up time for each product. The profit per hour of set-up time is \$7,975 for X_1, \$11,400 for X_2, \$8,050 for X_3, and \$6,360 for X_4.[13] The

[11] When a ranking of each product's profitability is the same under the ABC and expanded ABC models, both models will lead to identical product-mix decisions when there is a single constrained resource.

[12] In cases where the firm has no discretionary power over production resources other than direct material, the TOC will provide a product mix with a higher or equal income relative to the expanded ABC model. The TOC includes only direct material cost in selecting an optimal-product mix, while the expanded ABC model includes all production costs in developing a product mix. Consequently, the expanded ABC model may result in a lower level of production, higher level of excess resources, and lower income relative to the TOC.

[13] Each product's profit per hour of set-up time was computed from its profit per unit in table 1 multiplied by the units and divided by the hours of set-up time per batch.

relative rates of profit per hour of constrained resources are reflected in the optimal-product mix, i.e., demand for products X_2 and X_3 and part of the demand for X_1 are met in the optimal solution.

The slack or excess resources identified in the second section of table 3 are the direct result of the limited capacity of the set-up department to support further production. The impact of expanding the resources in the set-up department may be evaluated by increasing the amount of set-up time in the original mixed-integer programming model, solving the revised model, and comparing the original and revised solutions. Adding two additional hours to set-up would increase production of X_1 by one batch, profit by $17,200, and would decrease excess resources in assembly and finishing by 1,000 and 500 hours, respectively. The TOC would suggest this has several implications for the management of XYZ Inc. First, an hour of resources lost in the set-up department is much more critical than an equivalent amount of capacity lost in another department. Every pair of hours lost in the set-up department results in one less batch of product X_1, reduced profit of $17,200, and increased excess resources in the assembly and finishing departments of 1,000 and 500 hours, respectively. A comparable inefficiency in assembly, finishing, purchasing, or engineering would have no impact on the firm's production and profitability. Consequently, monitoring the efficiency of the set-up department is critical for maximizing production and profitability and minimizing XYZ's excess production capacity.

A second implication of the TOC is that adding resources to the set-up department will dramatically impact the production and profitability of the firm. However, when one constraint is relieved, another activity will become a bottleneck. To evaluate how much capacity can be added to the set-up department, the resource usage from expanding production and the excess capacity available in other departments must be evaluated. Every additional batch of X_1 produced uses 1,000 and 500 hours of capacity in assembly and finishing, respectively, and takes five additional purchase orders for every 4,000 additional units of production. Since there are 10,000 hours in assembly, 55,000 hours in finishing, and 200 orders in purchasing of excess capacity, assembly will become constrained after 20 additional hours of resources are added to the set-up function. Adding these resources will increase profit by $170,000, while using 10,000 hours in assembly, 5,000 hours in finishing, and 10 purchase orders that are currently unused.[14] At this point, assembly will become a constraint. Therefore, to expand production and profit further, additional resources would need to be added to the assembly and set-up departments. The subsequent set of activities in XYZ's production structure that will become a constraint and the economic impact of their removal can be determined in a similar manner.

Identification of a constraint plays a key role in a program of continuous improvement. The TOC indicates that process improvement should be targeted at a constraint (Sheridan 1991, 46). Improving a constrained activity relieves a bottleneck, thereby increasing throughput. Conversely, improving non-constrained activities simply increases their excess capacity. Applying these principles to the data in table 3, XYZ Inc. should target the set-up department for improvement. Prior sensitivity analysis indicates that this strategy will increase production, profitability, and use excess resources in other departments. However, once set-up time has been increased more than 20 hours, process improvement must be directed at the assembly department to expand production and profitability further. The expanded ABC model, thus, provides a system for prioritizing process improvement activities and estimating their potential economic impact.

SUMMARY AND CONCLUSIONS

ABC and the TOC incorporate facets of the production process ignored by traditional cost models. Accordingly, both paradigms provide

[14] The increase in profit may be computed by executing the mixed-integer model in table 2 with 520 set-up hours or manually by multiplying the additional units by their unit-level contribution less batch- and product-level costs.

insights into the production process that enhance production decisions. An examination of their cost measures and time frames for making production-related decisions suggests that they are diametrically competing paradigms. However, an examination of their respective strengths and limitations suggest that they are more complementary than they may first appear. The product costs of ABC may be combined with the physical resources used by production activities and their capacity to form a more comprehensive model of a firm's production structure. The resulting model provides an expanded framework for understanding the economic consequences of production-related decisions and implementing the principles of the TOC.

The expanded ABC model may be operationalized using mixed-integer programming to integrate a firm's ABC data with physical resource used by production activities and their production capacity. The mixed-integer programming model provides information that may be used for making a variety of marketing and production-related decisions. The solution to the expanded ABC model identifies the firm's optimal-production mix from simultaneous evaluation of the cost, physical resources, and marketing opportunities available to the firm. Perhaps equally important, it aids in identifying a production bottleneck and assessing its economic impact upon the firm. The solution to the expanded ABC model also facilitates identifying excess resources that can be reallocated to more productive uses. Finally, the expanded ABC model may be used to identify activities where a program of continuous improvement has the highest potential for enhancing organizational productivity and profitability.

The expanded ABC model may have several limitations when used in practice. The expanded ABC model is formulated in terms of quantitative measures of the firm's revenue, cost, physical resources, and production capacity. However, product-mix and other resource allocation decisions frequently involve numerous non-quantitative factors. Consequently, the expanded ABC model provides only a subset of the information needed for many marketing and production decisions. The expanded ABC model requires numerous estimates and assumptions of the firm's business opportunities and resources over an extended time period. The usefulness of much of the information developed from the expanded ABC model is dependent, in part, upon the accuracy of these assumptions and estimates. However, assumptions and estimates are required for other approaches to making product-mix and resource allocation decisions and impact the usefulness of their information as well. Finally, the cost of using the expanded ABC model may be relatively high compared to other approaches because of the substantial time required to model the firm's revenue, cost, and production structure and interpret and analyze the resulting mixed-integer programming solution. The additional costs of applying the expanded ABC model may be relatively insignificant, however, compared to the potential benefit of the information that it may provide for making more informed marketing and production decisions.

APPENDIX A
Mixed-Integer Programming

Mixed-integer programming is a subset of mathematical programming designed to optimize an objective function subject to constraints with continuous and integer variables. A business problem is formulated as a mixed-integer model identical to that of the more familiar linear-programming technique. An objective function is used to represent an organizational goal such as profit maximization. A second set of equations is used to model constraints that limit the attainment of this objective. Once formulated, a mixed-integer programming algorithm is used to solve the resulting set of equations.

The algorithm will provide the variables that maximize the objective function, the slack variables that measure the excess resources of non-constrained activities, and the value of the objective function. Unlike linear programming, the mixed-integer programming model will not generate information about the sensitivity of the optimal solution to changes in a constraint or to a change in the value of an objective function coefficient. However, sensitivity analysis may be performed by changing the value of one or more variables in the original mixed-integer model, solving the revised model, and observing the change between the original and revised models.

A variety of computerized algorithms are available for solving integer and mixed-integer programming problems. However, an integer or mixed-integer problem requires more computational time than a comparable linear-programming problem. For example, the branch-and-bound algorithm, which is widely used for solving mixed-integer problems, takes a prohibitive amount of time to solve if the problem has over one hundred integer variables and the continuous solution is distant from the optimal integer solution (Budnick et al. 1988, 413). However, even under these conditions, the manner in which a mixed-integer problem is formulated or structured significantly affects its computational time. For a more in-depth discussion of mixed-integer programming and factors affecting its computational time see Budnick et al. (1988) and Eppen et al. (1993).

REFERENCES

Budnick, F., D. McLeavey, and R. Mojena. 1988. *Principles of Operations Research for Management*, 2nd edition. Homewood, IL: Irwin.

Cooper, R. 1990. Cost classification in unit-based and activity-based manufacturing cost systems. *Journal of Cost Management* 4 (Fall): 4–14.

———, and R. Kaplan. 1992. Activity-based systems: Measuring the costs of resource usage. *Accounting Horizons* 6 (September): 1–13.

———, and ———, L. Maisel, E. Morrissey, and R. Oehm. 1992. From ABC to ABM does activity-based management automatically follow from an activity-based costing project? *Management Accounting* 74 (November): 54–57.

Eppen, G., F. Gould, and C. Schmidt. 1993. *Introductory Management Science*, 4th edition. Englewood Cliffs, NJ: Prentice Hall.

Fox, R. 1987. Theory of constraints. *NAA Conference Proceedings* (September): 41–52.

Goldratt, E., and R. Fox. 1986. *The Race*. Croton-on-Hudson, NY: North River Press.

———, and ———. 1987. Goldratt & Fox: Revolutionizing the factory floor. *Management Accounting* 68 (May): 18–22.

———. 1990. *What is This Thing Called Theory of Constraints and How Should it be Implemented*. Croton-on-Hudson, NY: North River Press.

———, and J. Cox. 1992. *The Goal A Process of Ongoing Improvement*, 2nd edition. Croton-on-Hudson, NY: North River Press.

Harvey, C. 1979. *Operations Research An Introduction to Linear Optimization and Decision Analysis*. New York, NY: North Holland.

Johnson, H. 1992. It's time to stop overselling activity-based concepts. *Management Accounting* 74 (September): 26–35.

Plenert, G. 1993. Optimizing theory of constraints when multiple constrained resources exist. *European Journal of Operational Research* 70 (October): 126–133.

Robinson, M. ed. 1990. Contribution margin analysis: No longer relevant/strategic cost management: The new paradigm. *Journal of Management Accounting Research* 2 (Fall): 1–32.

Sheridan, J. 1991. Throughput with a capital T. *Industry Week* 240 (March 4): 44–49.

Spoede, C., E. Henke, and M. Umble. 1994. Using activity analysis to locate profitability drivers ABC can support a theory of constraints management process. *Management Accounting* 75 (May): 43–48.